Laboratory Exercises in Astronomy — The Orbit of Mars

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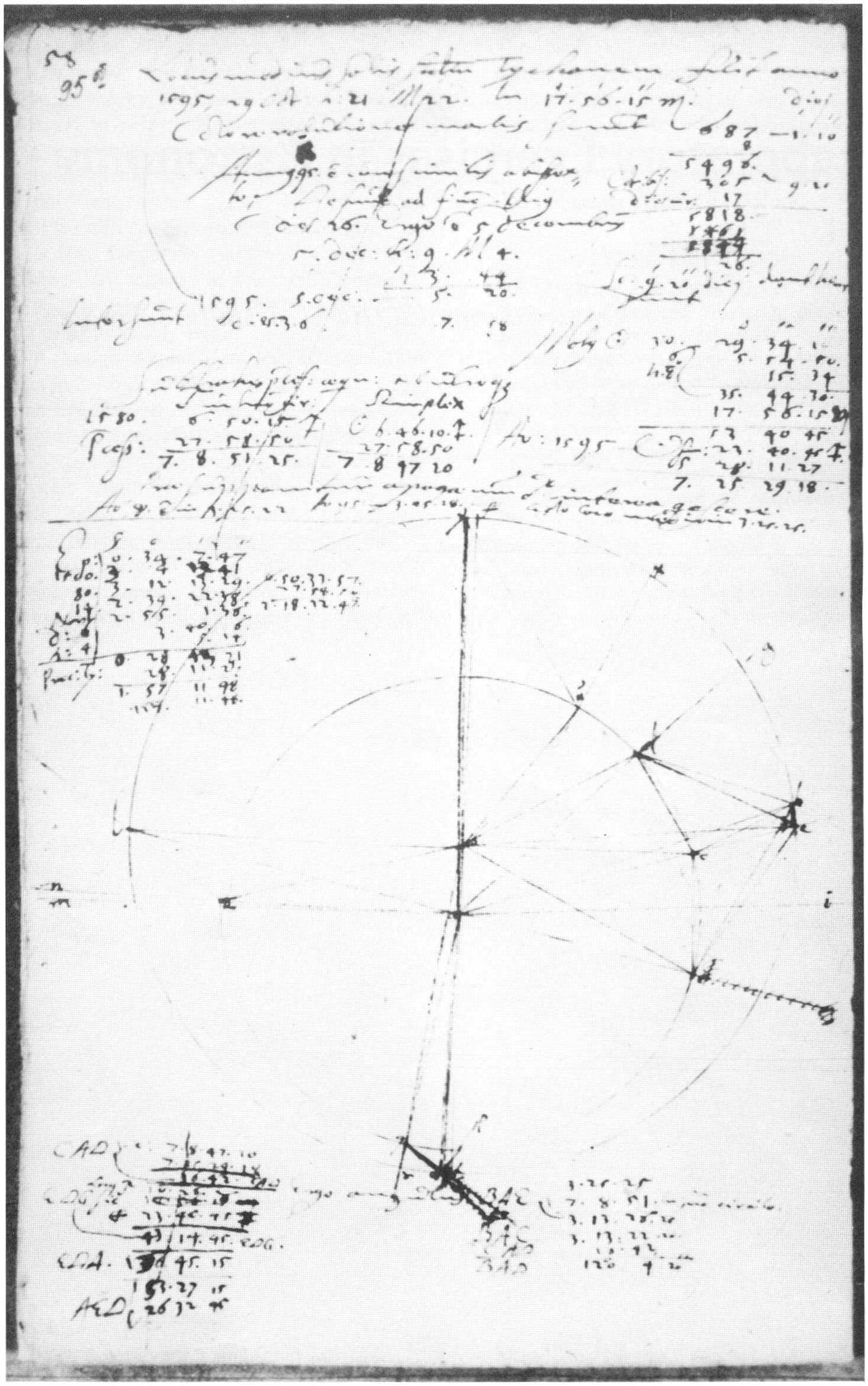
From our earthbound viewpoint, the eccentricity of Mars' orbit provides the most interesting aspect of the planet's motion. This relatively large eccentricity makes the elliptical shape of Mars' orbit comparatively easy to study. As Johannes Kepler said, to discover the secret of the cosmos we must use the motion of Mars; "otherwise, it would remain eternally hidden."

As Mars and the Earth move in their orbits, they come close to each other about every two years. But, because Mars' path around the Sun is much more off-center than ours, some approaches are much nearer than others. The very closest ones occur approximately 15 years apart, at so-called perihelic oppositions. Thus it was at a particularly close opposition in 1877 that Asaph Hall discovered the satellites of Mars and that Giovanni Schiaparelli first observed the *canali* on its surface. At one of the next favorable close approaches, in 1894, Percival Lowell made a big splash with his theory of intelligent life on Mars, and again in 1907 the "Mars furor" resumed briefly.

Although the Martian orbit has an appreciable eccentricity, it is a mistake to think of its shape as an obvious ellipse; rather, it looks very much like a circle that is off-centered from the Sun. In this exercise we shall investigate precisely this feature of Mars' orbit, employing some of the same data from Tycho Brahe that Kepler used in the early 1600's, and we shall follow the method invented by Kepler.

As a boy, Tycho had been greatly impressed by the astronomers' ability to foretell an eclipse. However, at age 16 he was somewhat disillusioned to learn that they could do no better than predict within a few days the great conjunction between Jupiter and Saturn. Thereafter, Tycho decided to devote his life to systematic observations of the stars and planets. Using "more than a ton" of King Frederick's gold, he built a splendid observatory on the island of Hven and equipped it with elaborate measuring instruments. These were, of course, all pretelescopic, so the best accuracy he could achieve was around one arc minute.

Like Tycho, Kepler also remembered several skywatching experiences from youth; when he was six years old, for example, his mother showed him the Great Comet of 1577. But Kepler really discovered astronomy when he was studying for his master's degree at Tübingen in 1590. Later, in his great book on Mars, the *Astronomia nova,* he gave an autobio-



When Kepler came to work with Tycho in 1600, he kept a special notebook concerning his investigations of the orbit of Mars. On page 58 there appears for the first time an attempt to triangulate from the Earth's orbit (the smaller circle) to the orbit of Mars (the larger circle). The exercise presented on the following pages uses a similar procedure and similar data. Note on this reproduction of page 58 the Julian date 1595 29 Oct. on the second line and the geometrical position of the Sun, 17°56′ 15″ of Scorpius. (For this exercise the Gregorian calendar is used.) Today this manuscript can be found in the Leningrad Archives of the Academy of Sciences of the U.S.S.R., Kepler Vol. XIV, folio 95v. This photograph was supplied by the author.

graphical account of the events that ensued:

When I was old enough to taste sweetness of philosophy, I embraced it all with an intense passion, without taking a particular interest in astronomy. I have for it, certainly, a sufficient intelligence, and I understood without difficulty the geometry and astronomy imposed by the series of courses, which depend on figures, numbers, and proportions. But these were the prescribed studies, and nothing indicated to me a particular inclination for astronomy. Now I was supported by a scholarship from the Duke of Württemberg, and when I saw that my fellow students would excuse themselves when the Prince was soliciting for foreign countries, although in fact they simply refused for love of their native land, I fully decided, being of a tougher nature, to go immediately where I might be sent.

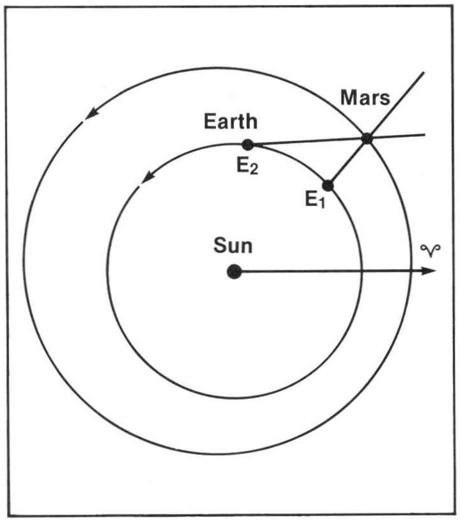
The first place offered to me was an astronomical position into which, frankly, I was pushed only because of the authority of my teachers, not that I was frightened by the distance of the place — a fear I had condemned in the others (as I have said) — but because of the unexpected character and lowness of the position as well as the weakness of my education in this part of philosophy. I accepted, therefore, being richer in ingenuity than in knowledge, and protesting highly that I would by no means abandon my right to another kind of life [an ecclesiastical position] that appeared to me much better.

Kepler then goes on to describe briefly how, while teaching high school in southern Austria, he had written his first book, the *Mysterium cosmographicum,* with encouragement from his former teacher, Michael Maestlin. He continues:

In 1597 I wrote to Tycho Brahe [in Denmark] asking him to tell me what he thought of my little work, and when in his answer he mentioned, among other things, his observations, he fired me with an enormous desire to see them. Moreover, Tycho Brahe, himself an important part in my destiny, did not cease from then on to urge me to visit him. But since the distance of the two places would have deterred me, I ascribe it to divine Providence that he came to Bohemia. I thus arrived there just before the beginning of the year 1600, with the hope of obtaining the correct eccentricities of the planetary orbits. . . . Now at that time his personal aide, Christian Severinus [Longomontanus], had taken up the theory of Mars. Had Christian been occupied with another planet, I would have started on the same one.

This is why I consider it again an act of divine Providence that I arrived at the time when he was studying Mars; because for us to arrive at the secret knowledge of astronomy, it is absolutely necessary to use the motion of Mars; otherwise it would remain eternally hidden.

Kepler had been in Prague only a few weeks when he began to develop a particularly ingenious geometrical method to establish the position of Mars' path. He knew that it took 687 days for Mars to complete one heliocentric circuit. On the other hand, the Earth would return to the same place in its own orbit every year: 365 days for one circuit, 730 days for two. Hence, if Mars is observed at a particular



As described in the text, a position of Mars in its orbit can be determined by triangulation from sightings made at two different positions along the Earth's orbit, here labeled E_1 and E_2.

place, he knew that it would be back at the same spot in 687 days, whereas the Earth would not quite have completed its second revolution; therefore Mars would be sighted from a different vantage point. This clever triangulation technique is seen in the facing photograph of Kepler's earliest diagram showing his method.

In order to use this scheme, Kepler had to have a series of Martian observations 687 days apart, and here Tycho's observing books provided a veritable gold mine. We can follow this procedure using some of the same Tychonic observations that Kepler used; these are found in the table.

PROCEDURE

For this exercise we will first need a protractor, a centimeter ruler, a compass, and a regular notebook-size piece of graph paper. If graph paper is unavailable, a piece of lined paper will do, because the lines will be handy for making sure that the base of the protractor is always parallel when angles are measured from various positions.

Later it will be desirable to have a pair of tacks or pins and a string for drawing a fairly large circle and an ellipse.

Find the center of your graph paper (the Sun) and draw a straight line from the center to the right-hand edge, parallel to the grid lines. This represents the basic 0° direction in space, the direction of the Sun as seen from the Earth at the time of the vernal equinox. (This position is technically known as "the first point of Aries" and is designated with the Ram's horns ♈.)

Draw a circle of five centimeters radius to represent the Earth's orbit. (Although the orbits of Mars and the Earth are actually ellipses, we will see that circles are pretty good approximations.) Using the protractor centered on the *Sun* and with the vernal equinox as the starting line, lay

Tycho's Observations of the Planet Mars

The following pairs of Martian positions are found in Kepler's *Astronomia nova* (Chapters 26-28 and 52). They are based on Tycho's observations, but sometimes Kepler had to interpolate to get the dates he needed.

Note: If a limited amount of time is available for doing this exercise, be sure to include at least the first two pairs of observations.

Date	*Heliocentric long. of Earth*	*Geocentric long. of Mars*
1585 Feb. 17	159° 23′	135° 12′
1587 Jan. 5	115° 21′	182° 08′
1591 Sep. 19	5° 47′	284° 18′
1583 Aug. 6	323° 26′	346° 56′
1593 Dec. 7	85° 53′	3° 04′
1595 Oct. 25	41° 42′	49° 42′
1587 Mar. 28	196° 50′	168° 12′
1589 Feb. 12	153° 42′	218° 48′
1585 Mar.10	179° 41′	131° 48′
1587 Jan. 26	136° 06′	184° 42′

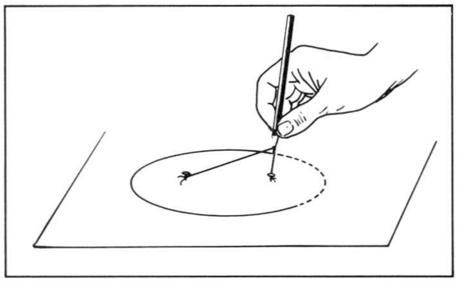
off counterclockwise the *heliocentric longitude* of the Earth for each date in the table, and label the date.

Next, with the *Earth* as the center and with 0° in the direction of, and along a parallel to, the vernal equinox, mark off counterclockwise the directions for the *geocentric longitude* of Mars. For each pair of observations, the position of Mars in its orbit lies at the intersection of lines drawn from both Earth positions through the corresponding geocentric longitudes of Mars.

Kepler himself chose the orbital positions of Mars as given by the first two sets of observations to indicate the planet's distance from the Sun at perihelion and aphelion (the smallest and greatest distances, respectively). Find the midpoint of these two positions and draw a circle centered on the midpoint to represent the orbit of Mars. The *major axis* of Mars' orbit goes through the Sun and the midpoint, and a straight line through these points should come out close to the intersections of both triangulations from the first two sets of observations. Measure this line and by means of ratios calculate its length in astronomical units. (The astronomical unit is the distance from the Earth to the Sun, here represented as five centimeters.)

The eccentricity e of the orbit is the ratio of the distance between the Sun and the midpoint to the *semimajor axis* of the orbit. Determine this value, and write it in the lower left corner of the diagram, together with the value of the semimajor axis for Mars' orbit.

As you examine the orbits, can you see



How to draw an ellipse using two pins and a piece of string.

why the closest approaches of Mars always take place in August? When do the least favorable oppositions occur?

We now wish to compare an elliptical orbit with our circular approximation. The Sun provides one focus for the ellipse, and the so-called "empty focus" lies along the major axis in the direction of the midpoint but twice as far away from the Sun. Fix pins or tacks at each focus. Now place a loop of string around the pins, with the loop adjusted so that a pencil point just reaches the perihelion or aphelion points when the loop is fully extended. Be sure to use string or thread that does not stretch easily, and maintain the same tension as much as possible when drawing the ellipse. The circle should prove to be a rather good approximation to the ellipse. Are you surprised?

SOME FURTHER THOUGHTS

The first two pairs of observations help establish the aphelion and perihelion of Mars, and the remaining three pairs fall elsewhere around the orbit. From these would you be able to decide between the circular and elliptical paths?

Of course, the Earth's orbit is only approximated by the circle. By working his method in reverse Kepler established that the eccentricity of the Earth's orbit was only about half the value of 0.036 given by Tycho (the actual value is just under 0.0168) and that its perihelion fell near heliocentric longitude 100°. (His value was correct to within a few arc minutes.) Would a better orbit for the Earth have given appreciably better results?

Although Kepler used this triangulation method to demonstrate that Mars' orbit was not circular, he carried out his calculations numerically, not graphically. Nevertheless, he was unable to distinguish the precise shape of the Martian orbit, even with many more data points, because of their observational errors. In the end it was necessary to guess what shape might most accurately represent the observations — a method of "votes and ballots" as Kepler once picturesquely described it.

Even so, he was unwilling to guess its shape without some kind of theoretical model to justify his choice. He wrote that at first "I could not find why the planet would rather go on an elliptical orbit. Oh ridiculous me! . . . With reasoning derived from physical principles agreeing with experience there is no figure left for the orbit except a perfect ellipse." Eventually Kepler rationalized that a magnetic emanation driven by a rotating Sun could indeed yield an orbit with an elliptical shape, and with that he had concluded that Tycho's observations were compatible with an ellipse.

FURTHER READING

Owen Gingerich, "Johannes Kepler and the New Astronomy," *Quarterly Journal* of the Royal Astronomical Society, Vol. 13, page 346, 1972; Curtis Wilson, "How Did Kepler Discover His First Two Laws?" *Scientific American,* Vol. 226, page 92, March, 1972; Owen Gingerich, "Ptolemy, Copernicus, and Kepler," in *Great Ideas Today 1983* (Chicago, 1983), page 137.

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