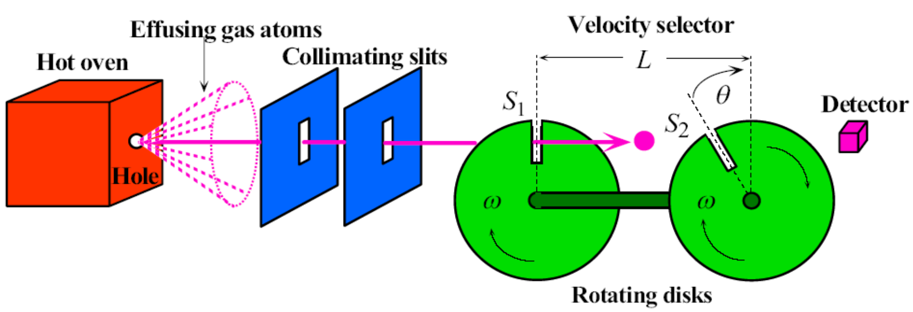
1.5 Molecular velocity and energy distribution



Schematic diagram of a stern type experiment for determining the distribution of molecular velocities

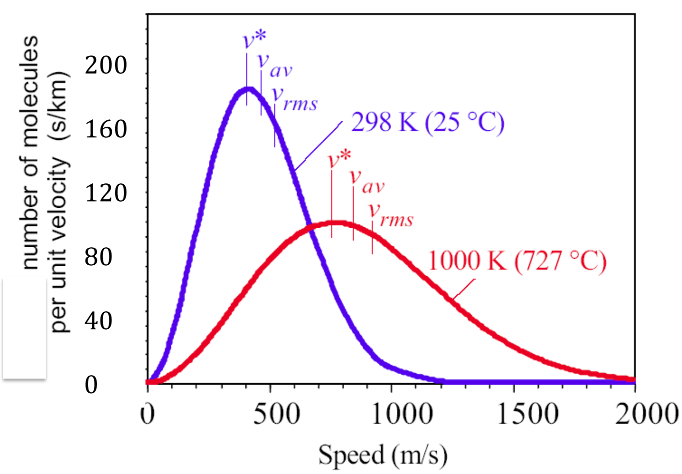
*** Show gas demo

Maxwell-Boltzmann Distribution for Molecular Speeds

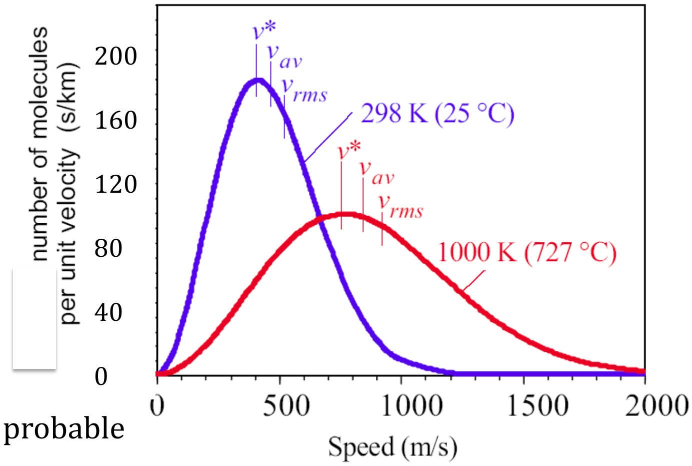
$$n_v = 4\pi N\left(\frac{m}{2\pi kT}\right)^{3/2} v^2 e^{-\frac{mv^2}{2kT}}$$

n_v = velocity density function,
N = total number of molecules,
m = molecular mass,
k = Boltzmann constant,
T = temperature,
v = velocity

$$\frac{dN}{dv} = n_v$$



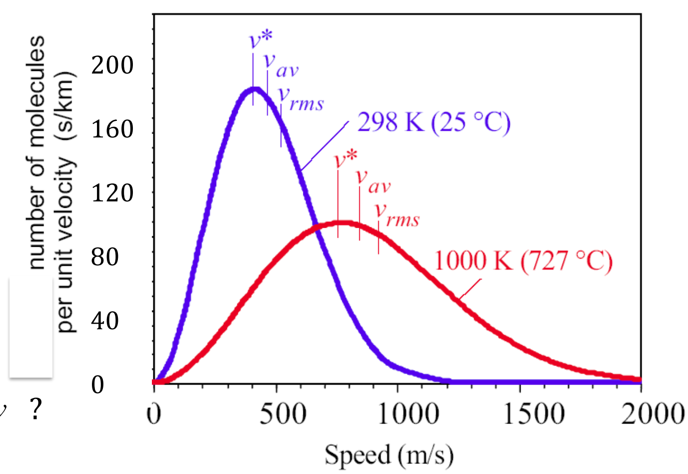
$$\frac{dN}{dv} = n_v$$



What is the most probable velocity?

A) v_{av}
B) v_{rms}
C) v*
D) ???

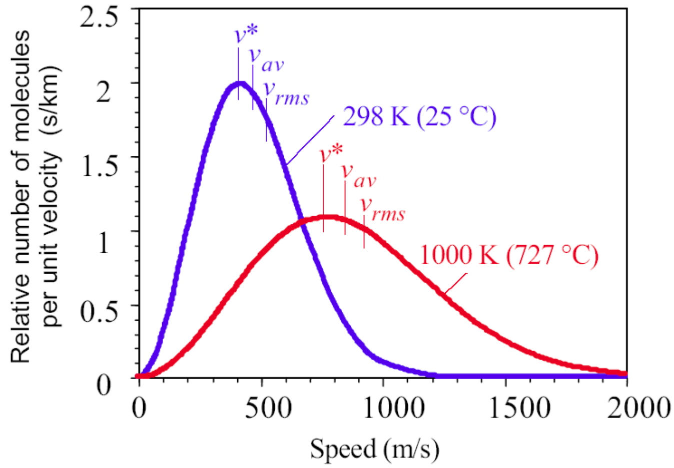
$$\frac{dN}{dv} = n_v$$



What is $\int n_v dv$?

A) v_{av}
B) v_{rms}
C) N
D) T

$$\frac{1}{N}\frac{dN}{dv} = \frac{1}{N} n_v$$

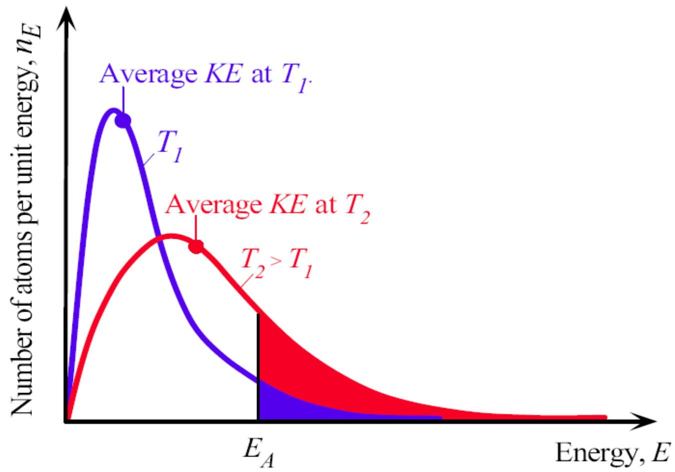


Maxwell-Boltzmann Distribution for Translational Kinetic Energies

$$n_E = \frac{2}{\sqrt{\pi}} N \left(\frac{1}{kT}\right)^{3/2} E^{1/2} e^{-\frac{E}{kT}}$$

Boltzmann factor

n_E = number of atoms per unit volume per unit energy at an energy E,
N = total number of molecules per unit volume,
k = Boltzmann constant,
T = temperature.



What is $\int_0^\infty \frac{n_E}{N} dE$?

A) N

B) 1

C) ½

D) ???

What is the probability than an atom will have energy greater than E_A?

A) 1

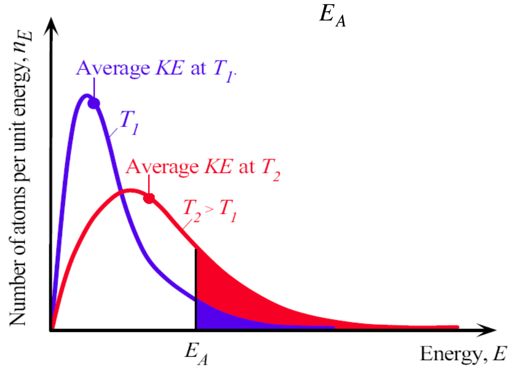
B) ½

C) $\int_0^{\infty} \frac{n_E}{N} dE$

D) $\int_{E_A}^{\infty} \frac{n_E}{N} dE$

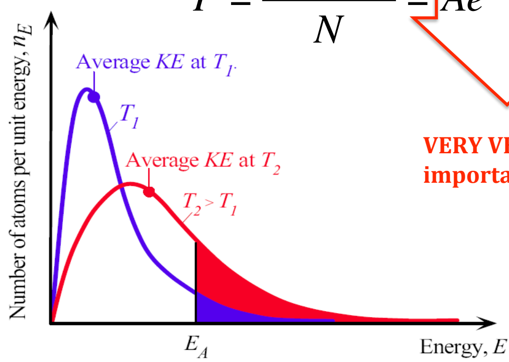
- Number of atoms with E > E_A:

$$\int_{E_A}^{\infty} n_E dE$$



- Probability that one atom has E > E_A:

$$P = \frac{\int_{E_A}^{\infty} n_E \, dE}{N} = Ae^{-E_A/kT}$$

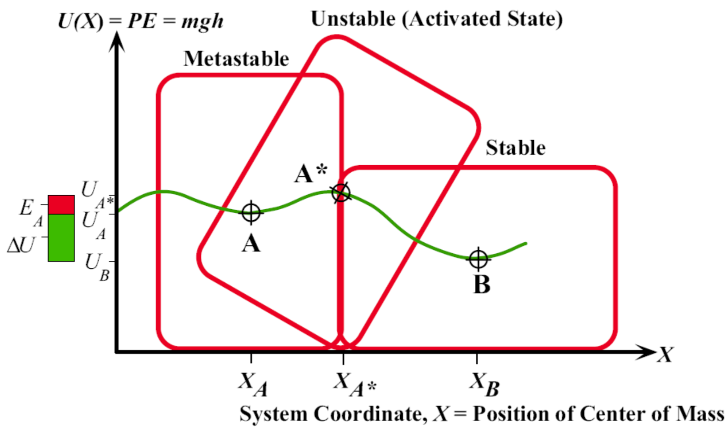


VERY VERY VERY important!

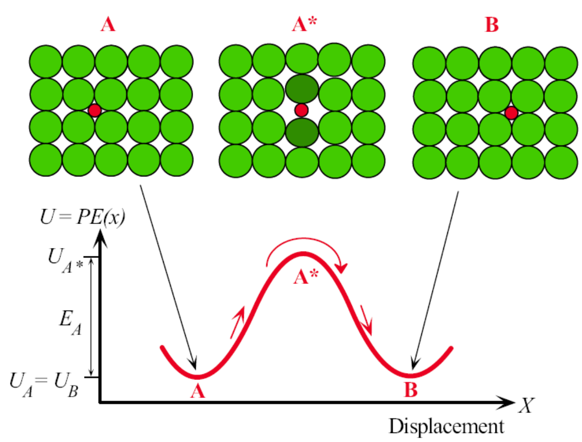
Boltzmann Energy Distribution

$$\frac{n_E}{N} = Ce^{-\frac{E}{kT}}$$

n_E = number of atoms per unit volume per unit energy at an energy E,
N = total number of atoms per unit volume in the system,
C = a constant that depends on the specific system (weak energy dependence),
k = Boltzmann constant,
T = temperature



Tilting a filing cabinet from state *A* to its edge in state *A** requires an energy E_A. After reaching *A**, the cabinet spontaneously drops to the stable position *B*. *PE* of state *B* is lower than *A* and therefore state *B* is more stable than *A*.



Diffusion of an interstitial impurity atom in a crystal from one void to a neighboring void. The impurity atom at position A must posses an energy E_A to push the host atoms away and move into the neighboring void at B.

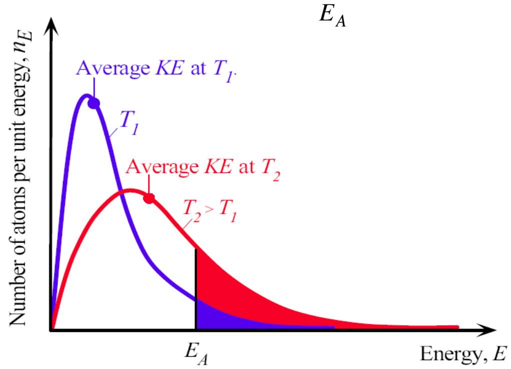
- Suppose there are N impurity atoms, with energies distributed according to Boltzmann's distribution:

$$\frac{n_E}{N} = Ce^{-\frac{E}{kT}}$$

- How many impurity atoms have E > E_A?

- Number of impurity atoms with E > E_A:

$$\int_{E_A}^{\infty} n_E \, dE$$



- Probability that one impurity atom has E > E_A:

$$P = \frac{\int_{E_A}^{\infty} n_E \, dE}{N} = Ae^{-E_A/kT}$$

VERY VERY VERY important!