a) What is oscillating as light propagates?

i) the air molecules

ii) the electric field

iii) the magnetic field

iv) nothing

b) If two waves are out of phase by a __________ number of wavelengths, they interfere destructively.

i) integer

ii) half-integer

Show that $E_y(x,t) = E_0 \sin(kx - wt)$ is a solution to the wave equation

$$\frac{\partial^2 E}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2}$$

$$\frac{\partial E}{\partial x} = kE_0 \cos(kx - \omega t)$$

$$\frac{\partial^2 E}{\partial x^2} = -k^2 E_0 \sin(kx - \omega t)$$

$$\frac{\partial E}{\partial t} = -\omega E_0 \cos(kx - \omega t)$$

$$\frac{\partial^2 E}{\partial t^2} = -\omega^2 E_0 \sin(kx - \omega t)$$

$$-k^2 \cancel{E_0} \sin(\cancel{kx - \omega t}) = -\frac{\omega^2}{c^2} \cancel{E_0} \sin(\cancel{kx - \omega t})$$

$$k^2 = \frac{\omega^2}{c^2}$$

for waves, $v = \lambda f = \frac{\omega}{k}$, so for light $c = \frac{\omega}{k}$, then

$$k^2 = \frac{\omega^2}{(\omega/k)^2}$$

$$k^2 = \frac{\cancel{\omega^2} k^2}{\cancel{\omega^2}}$$

$$k^2 = k^2$$

$$1 = 1$$

i.e. the left hand side and right-hand sides are equal, so $E = E_0 \sin(kx - \omega t)$ is a solution.

Show that the right-hand-side of the equation for the intensity of light, $I = \frac{1}{2} c \varepsilon_0 E_0^2$, has units of energy per unit area per second.

$[c] = m/s$

$[\varepsilon_0] = As/Vm$

$[E_0] = V/m$

$[I] = [c \varepsilon_0 E_0^2] = \frac{m}{s} \cdot \frac{As}{Vm} \cdot \frac{V^2}{m^2} = \frac{AV}{m^2}$

current is charge over time, so $A = C/s$

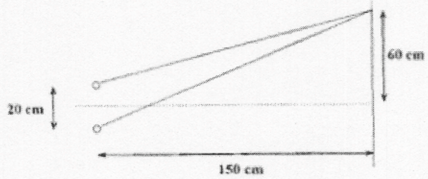
and potential is energy over charge $(\Delta V = \frac{\Delta U}{q})$, so $V = \frac{J}{C}$

then

$[I] = \frac{AV}{m^2} = \frac{C}{s} \cdot \frac{J}{C} \cdot \frac{1}{m^2} = \frac{J}{m^2 \cdot s} = \frac{\text{energy}}{\text{area} \cdot \text{second}}$

1. Two coherent, in-phase microwave sources separated by d = 20.0 cm create an interference pattern. At a center-line distance of D = 150 cm from the sources, the separation between the central axis and fifth maximum (m = 5) equals 60.0 cm. What is the wavelength of the microwave source?

Note: Find the path length difference directly from the geometry since the approximation d << D is of questionable validity here.



$\Delta L = L_2 - L_1 = m\lambda$ for constructive interference

$$\lambda = \frac{\Delta L}{m} = \frac{(150^2 + 70^2)^{1/2} - (150^2 + 50^2)^{1/2}}{5} = 1.48\,\text{cm}$$