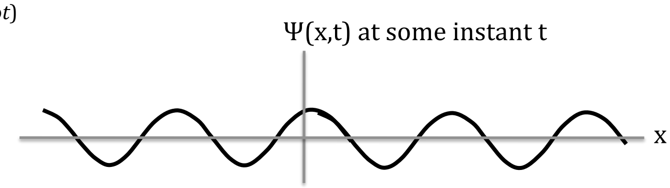
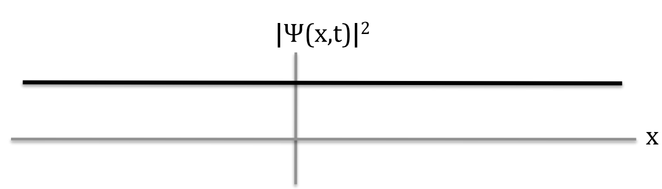
3.3 Infinite potential well and 3.4 Heisenberg uncertainty principle

The free electron: $V(x) = 0$

$\Psi(x,t) = A\, e^{i(kx - \omega t)}$

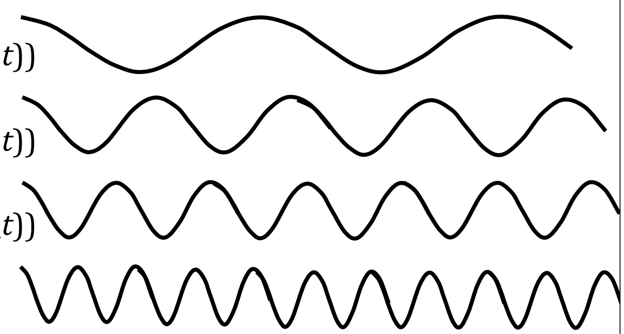


The probability density: $|\Psi(x,t)|^2 = A^2$



Plane Waves

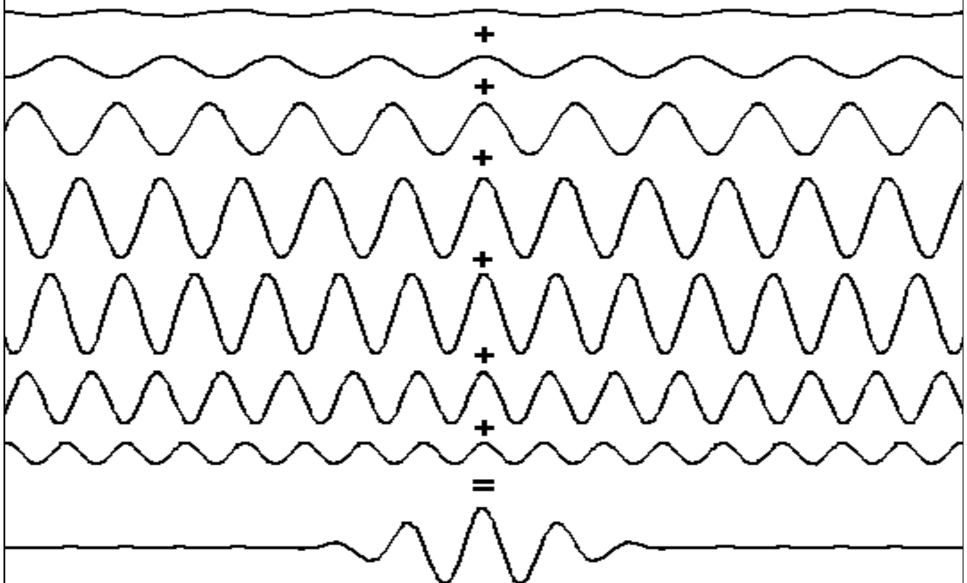
- Ψ_1(x,t) = exp($i(k_1x\text{-}\omega_1t)$)
- Ψ_2(x,t) = exp($i(k_2x\text{-}\omega_2t)$)
- Ψ_3(x,t) = exp($i(k_3x\text{-}\omega_3t)$)
- Ψ_4(x,t) = exp($i(k_4x\text{-}\omega_4t)$)
- etc…



Superposition principle

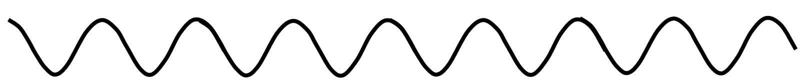
- If Ψ_1(x,t) and Ψ_2(x,t) are both solutions to wave equation, so is Ψ_1(x,t) + Ψ_2 (x,t).

Superposition



Plane Waves vs. Wave Packets

Plane Wave: $\Psi\,(x,t) = A\exp(i(kx\text{-}\omega t))$



Wave Packet: $\Psi_n(x,t) = \Sigma_n A_n \exp(i(k_n x\text{-}\omega_n t))$

Which one looks more like a particle?

Wave packets

A wave packet is:

A. a bunch of electrons of different energies all traveling together

B. a short snippet of a wave produced from a wave with a single wavelength

C. a wave with a localized region of non-zero probability produced by the superposition of many waves of differing wavelengths

D. all of the above

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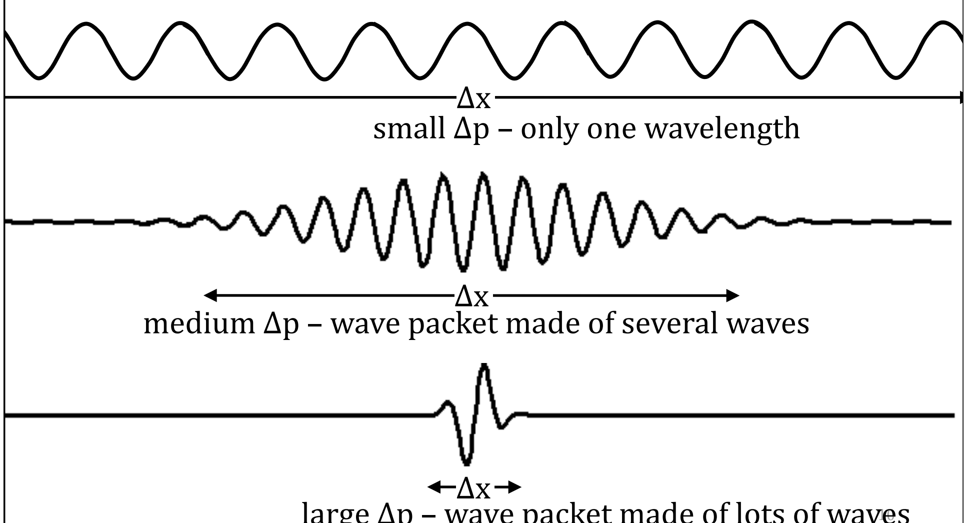
A wave packet is NOT a bunch of electrons! It is a single electron whose wave function can be expressed as a sum of plane waves.

Heisenberg Uncertainty Principle

- $\Delta x \Delta p_x \geq \hbar/2$
- $\Delta y \Delta p_y \geq \hbar/2$
- $\Delta z \Delta p_z \geq \hbar/2$

- $\Delta E \Delta t \geq \hbar/2$

Heisenberg Uncertainty Principle



For copper, work function = 4.7 eV. What is the probability that the electron will be outside of the well? (if copper is at room temperature)

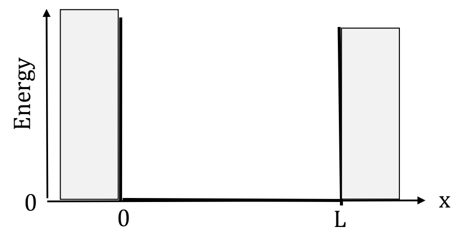
A. zero chance
B. very small chance
C. small chance
D. likely

ans c.

b. 1/40 eV<< 4.7eV. So very small chance ($e^{-4.7/.02}$) an electron could have enough energy to get out.

What does that say about boundary conditions on $\psi(x)$?

a. $\psi(x)$ must be same $x<0$, $0<x<L$, $x>L$,
b. $\psi(x<0)\sim 0$, $\psi(x>0)\neq 0$
c. $\psi(x)\sim 0$ except for $0<x<L$



V(x) ~∞, x<0
V(x) =0, 0<x<L
V(x) ~ ∞, x>L

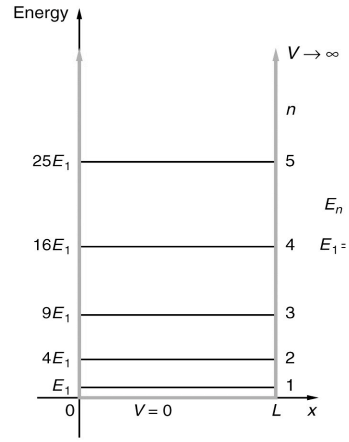
Solving the Schrodinger equation for electron wave in 1 D

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2} + V(x)\psi(x) = E\psi(x)$$

1. Figure out what V(x) is for situation given.
2. Guess or look up functional form of solution.
3. Check solution.
4. Figure out what boundary conditions must be to make sense physically.
5. Figure out values of constants to meet boundary conditions and normalization $\int_{-\infty}^{\infty} |\psi(x)|^2 dx = 1$
6. Multiply by time dependence $\Phi(t) = \exp(-iEt/\hbar)$ to have full solution.

E quantized by boundary conditions:

$$E = \frac{p^2}{2m} = \frac{n^2\pi^2\hbar^2}{2mL^2}$$



Does this L dependence make sense?