PHYS 225 – Spring 2011 **Marie Lopez del Puerto**

Exam 3 Review

<u>Information you may need:</u>

$h = 6.626 \times 10^{-34}$ J s	$hc = 1240$ eV nm	$k_B = 8.62 \times 10^{-5}$ eV / K
$k = 8.99 \times 10^{9}$ N m^2 / C^2	1 eV $= 1.602 \times 10^{-19}$ J	$k_B = 1.38 \times 10^{-23}$ J / K
$e = 1.602 \times 10^{-19}$ C	$m_e = 9.11 \times 10^{-31}$ kg	

$$-\frac{\hbar^2}{2m}\frac{\partial^2\Psi(x,t)}{\partial x^2} + V(x,t)\Psi(x,t) = i\hbar\frac{\partial\Psi(x,t)}{\partial t} \qquad -\frac{\hbar^2}{2m}\frac{d^2\varphi(x)}{dx^2} + V(x)\varphi(x) = E\varphi(x)$$

$$\Psi(x,t) = (Ae^{ikx} + Be^{-ikx})e^{-iEt/\hbar}$$

$$\Psi(x,t) = (Ae^{\alpha x} + Be^{-\alpha x})e^{-iEt/\hbar}$$

Tunneling: $$\alpha^2 = \frac{2m(V_0 - E)}{\hbar^2}$$

Infinite potential well: $$E_n = \frac{\pi^2\hbar^2 n^2}{2mL^2}$$

Hydrogenic atom: $$E_n = -\frac{me^4Z^2}{8\varepsilon_0^2 h^2 n^2} = -\frac{Z^2(13.6eV)}{n^2}$$

$$f(E) = \frac{1}{1 + e^{(E-E_F)/kT}}$$

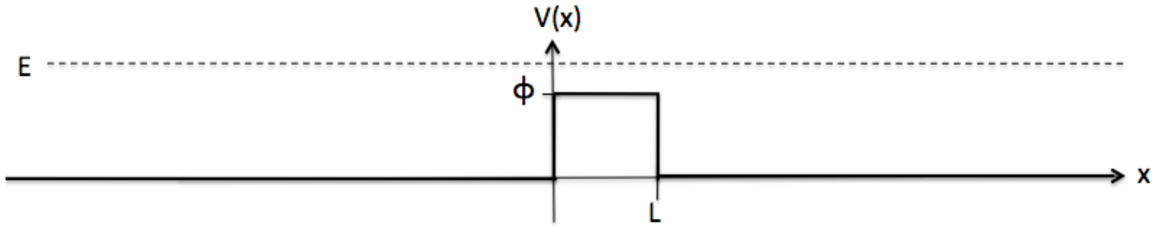
For a particle in an infinite potential box: $g(E) = 8\pi 2^{1/2}(m_e/h^2)^{3/2}E^{1/2}$

$E_{F0} = (h^2 / 8m_e)(3n/\pi)^{2/3}$ $E_F(T) = E_{F0}\,[1 - (\pi^2/12)(kT/E_{F0})^2]$

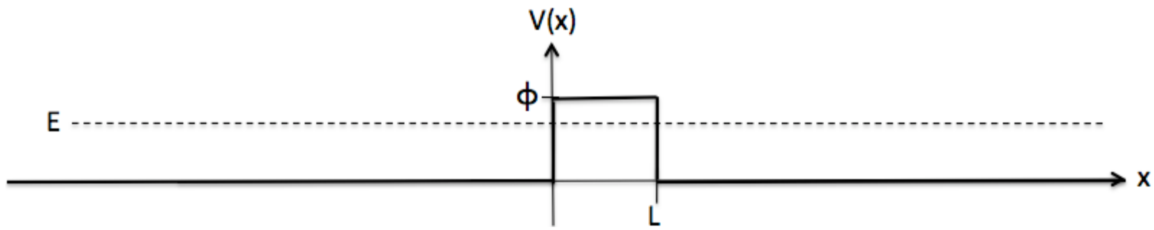
$E_{av}(0) = (3/5)\,E_{F0}$ $E_{av}(T) = (3/5)E_{F0}\,[1 - (5\pi^2/12)(kT/E_{F0})^2]$

$\sigma = (1/3)\,e^2 v_F^2\,\tau\,g(E_F)$ $\sigma = e^2 n\tau / m_e$

1. Sketch the wave function for an electron with energy E traveling from left to right in the potential shown below:



2. Sketch the wave function for an electron with energy E traveling from left to right in the potential shown below:



3. Explain what we mean when we say a particle can “tunnel” through a potential barrier.

4. For questions 1 and 2, what are the kinetic, potential and total energy of the electron (in terms of E) to the left of the barrier, inside the barrier, and to the right of the barrier?

5. Explain what happens (in terms of the wave functions and energy levels) as two hydrogen atoms approach each other to form a hydrogen molecule.

6. How many terms would the potential energy in the Schrödinger equation have for an electron in a hydrogen molecule? What are they?

7. Draw an energy diagram that shows the electron energy in an isolated hydrogen atom and the energies of the bonding and anti-bonding orbitals.

8. Explain what happens (in terms of the wave functions and energy levels) as two helium atoms approach each other. Why don't the helium atoms form a molecule?

9. When three (or more) hydrogen atoms are brought together, the resulting molecular orbitals must be either symmetric or anti-symmetric with respect to the center atom. Why?

10. Sketch the 1s wave function (as a function of r) for an isolated hydrogen atom, and the possible linear combinations for three hydrogen atoms that are brought together. Rank the linear combinations from lowest energy to highest energy.

11. N atoms with N valence electrons (1 electron per atom) are brought together. How many orbitals are there? How many distinct quantum states?

12. Explain what an energy band is and how it forms.

13. Explain why, when N lithium atoms are brought together, the 2s band is only half full.

14. In terms of energy bands, what is the difference between a metal, an insulator and a semiconductor?

15. Explain the concept of Fermi energy at zero temperature.

16. Explain how conduction occurs in a metal.

17. Draw an energy band diagram for a metal at zero temperature that shows the valence band, the conduction band, the Fermi energy and the work function.

18. Plot energy vs. momentum for the electrons in the valence band. Which states are occupied/empty? How does the diagram you just drew change when a battery is connected across the metal? Where does the electric current come from?

19. Explain how silicon bonds and how this leads to it being a semiconductor.

20. List the three ways in which an electron can be moved from the valence band to the conduction band in a semiconductor.

21. What is meant by “effective mass”?

22. Why is the density of states highest near the center of a band?

23. Starting from the energy levels for a particle in a box, calculate the density of states in one, two and three dimensions.

24. We obtained Fermi-Dirac statistics by looking at the probability that two particles would interact. Initially one particle is in level E_1 and one in level E_2, and in the end one particle is in level E_3 and one in level E_4. We wrote the probability as: $f(E_1)\, f(E_2)[1- f(E_3)]\, [1-f(E_4)]$ Explain how this expression is obtained.

25. Why can’t we use Boltzmann statistics to describe the conduction electrons in a metal?

26. Sketch f(E) vs. E at zero temperature. Mark the Fermi energy in your sketch.

27. Sketch f(E) vs. E at T>0. Explain the differences between this sketch and the previous one (question 25).

28. The number of electrons per unit volume, n, can be calculated from the density of states per unit volume, g(E), and the probability that a state with energy E is occupied, f(E).
    i. Write down the integral that would give you n.
    ii. Why is it reasonable to approximate the integral by changing the top limit to infinity?

29. Calculate the average energy of an electron at T = 0K in terms of the Fermi energy E_{F0}.

30. The average energy of an electron at T = 0K is $3E_{F0}/5$. By considering the density of states in a band and the Pauli exclusion principle, explain why this result makes sense.

31. The effective speed of electrons in a metal is higher that it would be if the electrons behaved like a classical ideal gas. Explain why this is.

32. Why does the conductivity depend on the density of states?