Derivation for Exam 4:

Number of $e^-$ per unit volume in the conduction band:

$$n = \int_{E_c}^{E_c + X} g_{cb}(E)\, f(E)\, dE$$

since $n <<$ number of available states, $f(E) \simeq e^{-(E-E_F)/kT}$

and approximating $g_{cb}$ by the 3D infinite-potential-well density of states, $g_{cb} \simeq 8\pi\sqrt{2}\left(\frac{m_e^*}{h^2}\right)^{3/2}(E-E_c)^{1/2}$

then

$$n = 8\pi\sqrt{2}\left(\frac{m_e^*}{h^2}\right)^{3/2}\int_{E_c}^{\overset{\text{since } f(E)\to 0 \text{ as } E\to\infty}{\infty}} (E-E_c)^{1/2}\, e^{-(E-E_F)/kT}\, dE =$$

$$= 8\pi\sqrt{2}\left(\frac{m_e^*}{h^2}\right)^{3/2}\frac{\pi^{1/2}}{2}(kT)^{3/2}\, e^{-(E_c-E_F)/kT} = \underbrace{2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2}}_{N_c:\ \text{effective density of states at the conduction band edge}} e^{-(E_c-E_F)/kT}$$

for the holes:

$$p = \int_0^{E_v} g_{vb}(E)\left[1-f(E)\right] dE = \int_0^{E_v} 8\pi\sqrt{2}\left(\frac{m_e^*}{h^2}\right)^{3/2} E^{1/2}\left[1-e^{-(E_F-E)/kT}\right] dE =$$

$$= 8\pi\sqrt{2}\left(\frac{m_e^*}{h^2}\right)^{3/2}\frac{\pi^{1/2}}{2}(kT)^{3/2}\, e^{-(E_F-E_v)/kT} = 2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/2} e^{-(E_F-E_v)/kT}$$

for an intrinsic semiconductor $n=p$. then

$$np = n_i^2 = N_c N_v\, e^{-(E_c-E_F)/kT}\, e^{-(E_F-E_v)/kT} =$$

$$= N_c N_v\, e^{-(E_c-\cancel{E_F}+\cancel{E_F}-E_v)/kT} = N_c N_v\, e^{-E_{gap}/kT}$$

finally,

$$\boxed{n_i = \sqrt{N_c N_v}\, e^{-E_{gap}/2kT}}$$