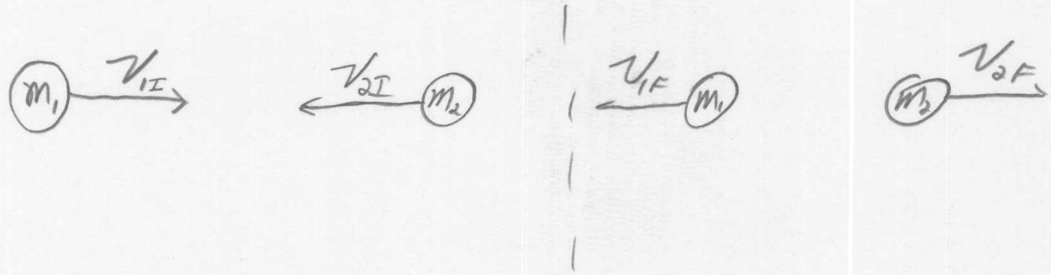
1D Elastic Collision Derivation

Two masses collide



Conserve momentum

$$\text{①} \quad m_1 v_{1I} + m_2 v_{2I} = m_1 v_{1F} + m_2 v_{2F}$$

Conserve Energy

$$\text{②} \quad \cancel{\tfrac{1}{2}} m_1 v_{1I}^2 + \cancel{\tfrac{1}{2}} m_2 v_{2I}^2 = \cancel{\tfrac{1}{2}} m_1 v_{1F}^2 + \cancel{\tfrac{1}{2}} m_2 v_{2F}^2$$

Rearrange both ① and ② so that m_1 is on the left and m_2 is on the right. Then, divide ② by ① to get:

$$\text{③} \quad \frac{\cancel{m_1}(v_{1I}^2 - v_{1F}^2)}{\cancel{m_1}(v_{1I} - v_{1F})} = \frac{\cancel{m_2}(v_{2F}^2 - v_{1I}^2)}{\cancel{m_2}(v_{2F} - v_{1F})}$$

Now, note that: $(a+b)(a-b) = a^2 + \cancel{ab} - \cancel{ab} - b^2$
$= a^2 - b^2$

apply to both numerators

$$\frac{(v_{1I} + v_{1F})\cancel{(v_{1I} - v_{1F})}}{\cancel{(v_{1I} - v_{1F})}} = \frac{(v_{2F} + v_{2I})\cancel{(v_{2F} - v_{2I})}}{\cancel{(v_{2F} - v_{2I})}}$$

contin. ↓

And we have:

$$④\ V_{1I} + V_{1F} = V_{2I} + V_{2F}$$

Now, solve ④ for V_{2F} and plug back into ①

$$V_{2F} = V_{1I} + V_{1F} - V_{2I}$$

$$m_1 V_{1I} + m_2 V_{2I} = m_1 V_{1F} + m_2 (V_{1I} + V_{1F} - V_{2I})$$

$$m_1 V_{1I} + m_2 V_{2I} = m_1 V_{1F} + m_2 V_{1I} + m_2 V_{1F} - m_2 V_{2I}$$

Gather all Initial V_s on right and all Final V_s on left.

$$V_{1F}(m_1 + m_2) = (m_1 - m_2) V_{1I} + 2m_2 V_{2I}$$

$$* \boxed{V_{1F} = \frac{(m_1 - m_2)}{(m_1 + m_2)} V_{1I} + \frac{2m_2}{(m_1 + m_2)} V_{2I}}$$

To get the other one, change all subscripts

$$* \boxed{V_{2F} = \frac{(m_2 - m_1)}{m_2 + m_1} V_{2I} + \frac{2m_1}{m_2 + m_1} V_{1I}}$$