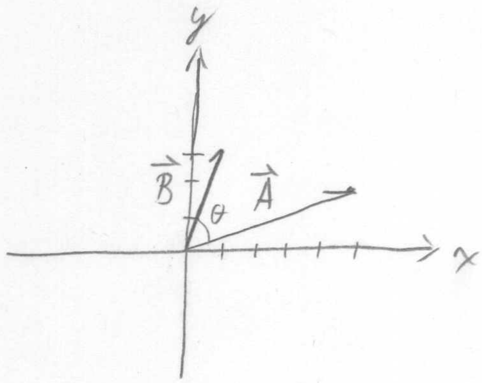
Work-Energy

1. For each vector pair below, sketch the pair and calculate $\vec{A}\cdot\vec{B}$.

a. $\vec{A}=5\hat{\imath}+2\hat{\jmath}$	$\vec{B}=1\hat{\imath}+3\hat{\jmath}$
b. $\lvert\vec{A}\rvert=2\cdot\sqrt{3},\theta=45^o$	$\vec{B}=-1\hat{\imath}+4\hat{\jmath}$
c. $\vec{A}=-6\hat{\imath}+6\hat{\jmath}$	$\vec{B}=11\hat{\imath}+8\hat{\jmath}$
d. $\vec{A}=-2\hat{\imath}+6\hat{\jmath}$	$\vec{B}=-5\hat{\imath}+2\hat{\jmath}$
e. $\lvert\vec{A}\rvert=2\cdot\sqrt{10},\theta=-71.6^o$	$\vec{B}=-3\hat{\imath}+1\hat{\jmath}$
f. $\vec{A}=-5\hat{\imath}+2\hat{\jmath}$	$\vec{B}=-3\hat{\imath}+1\hat{\jmath}$

a)



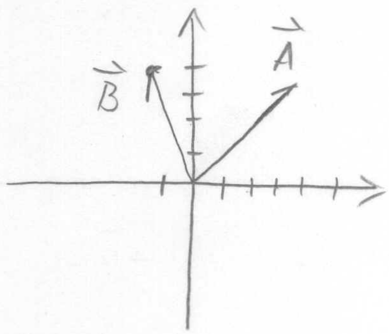
$\vec{A}\cdot\vec{B}=(5)(1)+(2)(3)=11$

$|\vec{A}|=\sqrt{29}$

$|\vec{B}|=\sqrt{10}$

$\theta=\cos^{-1}\left(\frac{11}{\sqrt{10}\sqrt{29}}\right)=50^\circ$

b)



$\vec{A}=2\sqrt{3}\cos(45)\hat{\imath}+2\sqrt{3}\sin(45)\hat{\jmath}$

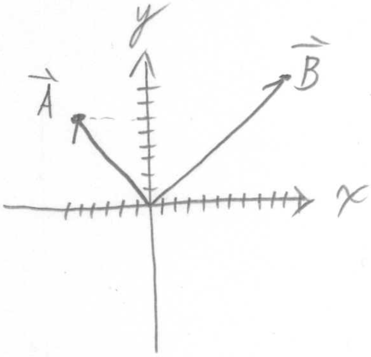
$=2.45\hat{\imath}+2.45\hat{\jmath}$

$\vec{A}\cdot\vec{B}=(2.45)(-1)+(2.45)(4)$

$=\underline{7.35}$

$\theta=\cos^{-1}\left(\frac{7.35}{2\cdot\sqrt{3}\cdot\sqrt{17}}\right)=\underline{59^\circ}$

c)

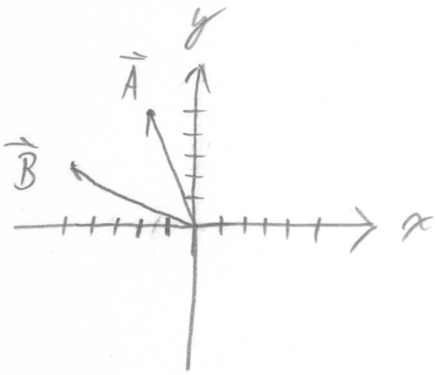


$$\vec{A}\cdot\vec{B} = (-6)(11) + (6)(8) \qquad |\vec{A}| = 6\sqrt{2}$$

$$= -18 \qquad |\vec{B}| = \sqrt{185}$$

$$\theta = \cos^{-1}\left(\frac{-18}{6\sqrt{2}\cdot\sqrt{185}}\right) = \underline{98^\circ}$$

d)

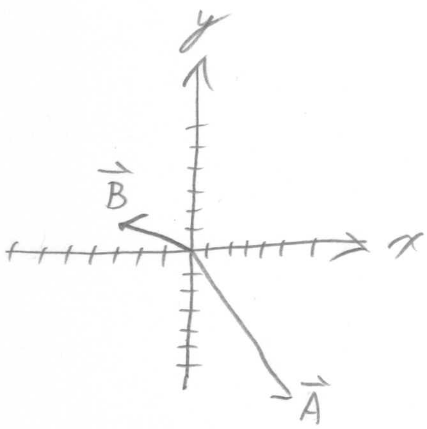


$$\vec{A}\cdot\vec{B} = (-2)(-5) + (6)(2) \qquad |\vec{A}| = 2\cdot\sqrt{10}$$

$$= 22 \qquad |\vec{B}| = \sqrt{29}$$

$$\theta = \cos^{-1}\left(\frac{22}{2\cdot\sqrt{10}\cdot\sqrt{29}}\right) = 50^\circ$$

e)



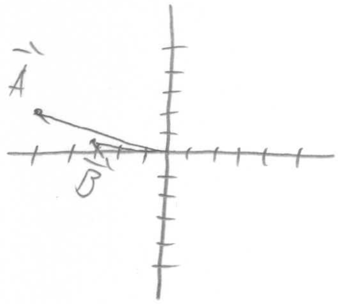
$$\vec{A} = 2\cdot\sqrt{10}\cos(-71.6)\hat{i} + 2\sqrt{10}\sin(-71.6)\hat{j}$$

$$= 2.0\hat{i} + {}^{-}6.0\hat{j}$$

$$\vec{A}\cdot\vec{B} = (2)(-3) - 6 = \underline{-12}$$

$$\theta = \cos^{-1}\left(\frac{-12}{2\cdot\sqrt{10}\cdot\sqrt{10}}\right) = 126^\circ$$

F)



$\vec{A}\cdot\vec{B} = (-5)(-3) + (2)(1)$

$= \underline{17}$

$|\vec{A}| = \sqrt{29}$

$|\vec{B}| = \sqrt{10}$

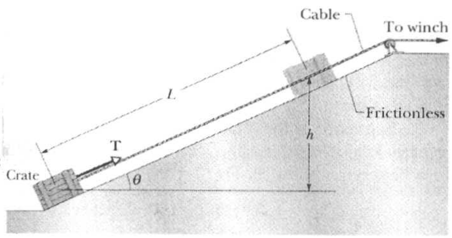
$\theta = \cos^{-1}\left(\frac{17}{\sqrt{29}\sqrt{10}}\right) = \underline{3.4°}$

None are orthogonal

$\vec{A}\cdot\vec{B} \neq 0$

Work-Energy

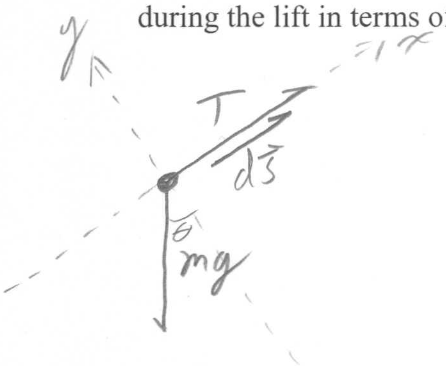
An initially stationary crate of mass m is pulled a distance L up a frictionless ramp to a height h where it stops.



a) Find an expression for the work W_g done on the crate by gravity during the lift in terms of m, h, and g.

b) Find an expression for the work W_T done on the crate by the tension T in the cable during the lift in terms of m, h and g.

a)



$$W_g = \int_0^L m\vec{g}\cdot d\vec{s} = \int_0^L mg\cos(90+\theta)\,ds$$

$$= -\int_0^L mg\sin\theta\,ds$$

$$= -mgL\sin\theta$$

$$\boxed{W_g = -mgh}$$

b) $W_{net} = W_g + W_T = \Delta K$

Since $V = 0$ at the top and the bottom, $\Delta K = 0$

$$W_g + W_T = 0 \Rightarrow W_T = -W_g$$

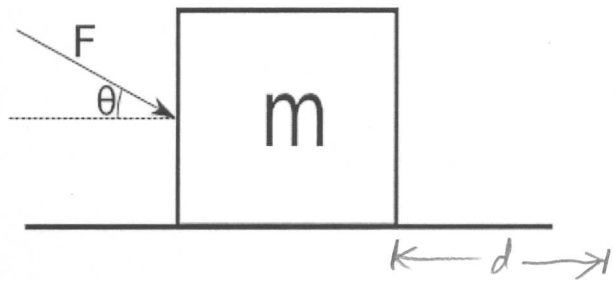
$$\boxed{\therefore W_T = mgh}$$

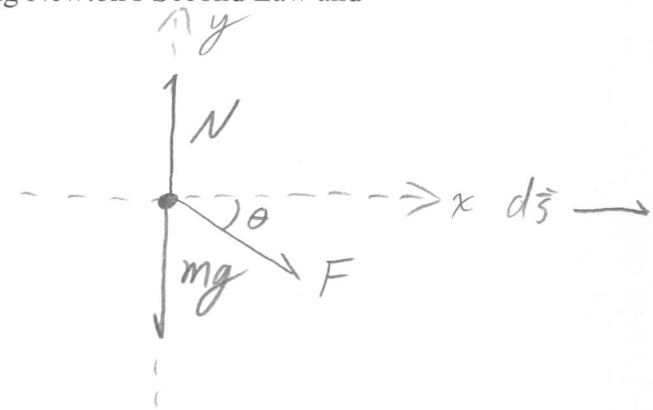
Work-Energy

A box with mass m, initially at rest, is pushed a distance d along a surface with a force F making and angle θ with the horizontal. The coefficient of friction between the box and the surface is μ_k.

a) Find an expression for the final velocity of the box, V_f, using Work-Energy techniques.

b) Find an expression for final velocity of the box using Newton's Second Law and kinematics and show that the answer is the same.





a) $W_{net} = W_N + W_g + W_F = \Delta K$

$$= \int_0^d \cancel{\vec{N}\cdot d\vec{s}}^{\,0} + \int_0^d \cancel{m\vec{g}\cdot d\vec{s}}^{\,0} + \int_0^d \vec{F}\cdot d\vec{s}$$

$\vec{N} \perp d\vec{s}$ and $\vec{g} \perp d\vec{s}$

$$W_{net} = \int_0^d F\cos\theta \, dx = Fd\cos\theta = \tfrac{1}{2} m V_f^2$$

$$\boxed{V_f^2 = \frac{2Fd\cos\theta}{m}}$$

b) $x: F\cos\theta = ma_x \Rightarrow \boxed{a = \dfrac{F\cos\theta}{m}}$

$y: N - mg = 0$

$v_f = \cancel{v_0}^{0} + at$

$v_f = \dfrac{F\cos\theta}{m} t$

$\Rightarrow t = \dfrac{v_f m}{F\cos\theta}$

$x_f = \cancel{x_0}^{0} + \cancel{v_0 t}^{0} + \frac{1}{2}at^2$

$d = \frac{1}{2}\dfrac{F\cos\theta}{m} t^2$

$$d = \frac{1}{2}\frac{\cancel{F\cos\theta}}{\cancel{m}}\frac{v_f^2 m^{\cancel{2}}}{F^{\cancel{2}}\cos^{\cancel{2}}\theta}$$

$$\boxed{v_f^2 = \frac{2Fd\cos\theta}{m}}$$