Work-Energy

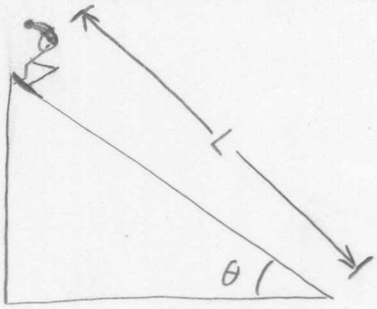
A skier of mass m skis a distance L down a frictionless hill that has a constant angle of inclination θ. The top of the hill is a vertical distance h above the bottom of the hill.

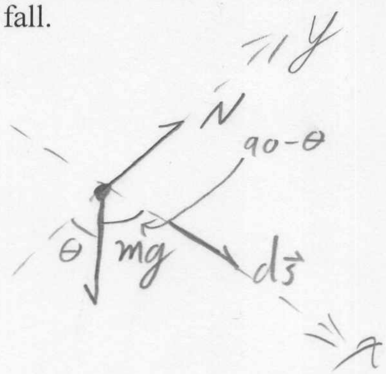
a. Use the integral form of the definition of work to find an expression for the work done on the skier by each of the forces involved.

b. Find an expression for the **total** work, W_{net}, done on the skier. Your expression should be in terms of m, g, and h only.

c. Use the **W**ork **E**nergy **T**heorem to find the skier's speed, V_f, at the bottom of the hill.

d. Use any method you like to find an expression for the final speed of the skier if she were to simply drop from a height h in free fall.





$$W_N = \int \vec{N}\cdot d\vec{s} = \int N\,ds\cos(90) = \boxed{0}$$

$$W_g = -\int_0^L m\vec{g}\cdot d\vec{s} = +\int_0^L mg\cos(90-\theta)\,dx$$

$$= +\int_0^L mg\sin\theta\,dx$$

$$= +mgL\sin\theta$$

$$\boxed{W_g = +mgh}$$

b) $W_{net} = W_N + W_g = +mgh$

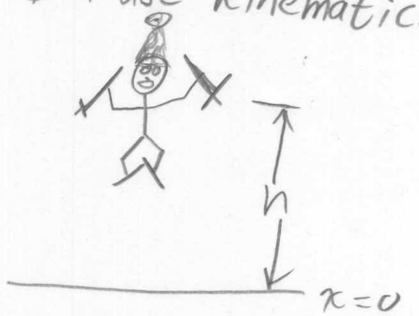
c) $W_{net} = \Delta K$

$+\cancel{m}gh = \frac{1}{2}\cancel{m}v_F^2 - \cancel{\frac{1}{2}mv_I^2}^{\,0}$

start from rest

$$\boxed{v_F^2 = 2gh}$$

d) I'll use kinematics



$\cancel{x_F}^{\,0} = x_0 + \cancel{v_0 t}^{\,0} + \frac{1}{2}at^2$ $\qquad v_F = \cancel{v_0}^{\,0} + at$

$0 = h - \frac{1}{2}gt^2$ $\qquad v_F = -gt$

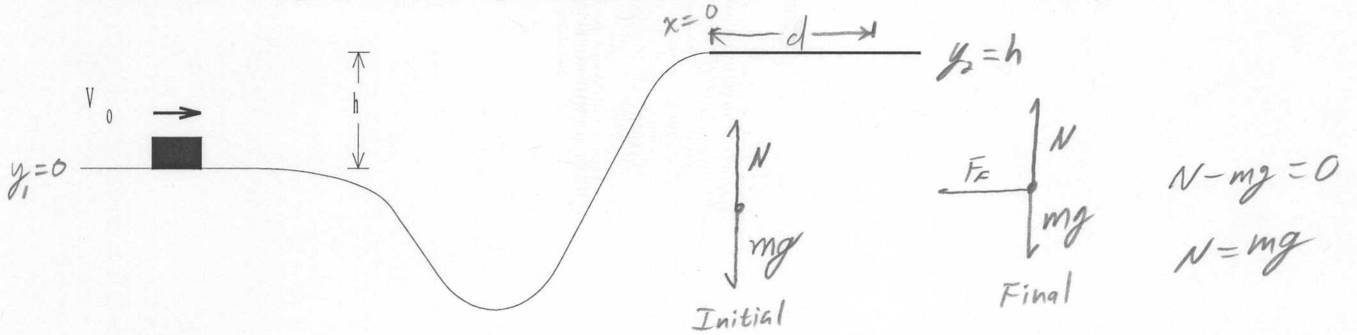
$t = -\frac{v_F}{g}$

$h = \frac{1}{2\cancel{g}}\frac{v_F^2}{\cancel{g^2}}$

$$\boxed{v_F^2 = 2gh}$$

Work-Energy

A block slides along the track shown below. The track is frictionless until the block reaches the level portion at the top of the hill, where the coefficient of friction is μ_k.



a. Write expressions for U_I, K_I, U_F, K_F, and W_f.

$U_I = mgy_1 = 0$ $\qquad$ $U_F = mgy_2 = mgh$

$K_I = \frac{1}{2}mV_0^2$ $\qquad$ $K_F = \frac{1}{2}mV_F^2 = 0$

$$W_F = \int_0^d \vec{F}_F \cdot d\vec{s} = -\int_0^d \mu_k N\,dx = -\mu_k mgd$$

b. Use the Conservation of Energy and your expressions from part a to find an expression for the distance, d, that the block slides across the level surface at the top of the hill.

$$U_I + K_I + W_F = U_F + K_F$$

$$\cancel{mgy_1}^{0} + \frac{1}{2}\cancel{m}V_0^2 - \mu_k\cancel{m}gd = \cancel{m}gh$$

$$\boxed{d = \frac{\frac{1}{2}V_0^2 - gh}{g}}$$

c. If the $V_0 = 6.0$ m/s, $h = 1.1$ m, and $\mu_k = 0.60$, what is d?

$$d = \frac{\frac{1}{2}(6)^2 - (9.8)(1.1)}{9.8} = \boxed{0.7\text{ m}}$$

Work-Energy

The force provided by a spring is given by Hooke's Law:

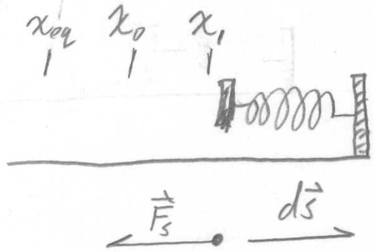
$$F = -k\Delta x$$

where

$$\Delta x = x - x_{eq}$$

This force, like gravity, is conservative so we can write a *Potential Function* for it. Use the definition of potential to derive the potential function for a spring.

$$\Delta U = -W_s = -\int_{x_0}^{x_1} \vec{F}_s \cdot d\vec{s}$$



Being careful with minus signs...

$$-\int_{x_0}^{x_1} |\vec{F}_s||d\vec{s}|\cos(180), \quad |\vec{F}_s| = k\Delta x = k(x-x_{eq})$$

$$\cos(180) = -1$$

$$= \int_{x_0}^{x_1} k(x-x_{eq})\,dx = \left(\tfrac{1}{2}kx^2 - x x_{eq}\right|_{x_0}^{x_1}$$

Now let $\boxed{x_0 = x_{eq} = 0}$, then $U_{eq} = 0$ and

$$\Delta U_s = U_F - \cancelto{0}{U_{eq}} = \tfrac{1}{2}kx^2$$

$$\boxed{U_s = \tfrac{1}{2}kx^2}$$

Work-Energy

A block of mass m is pushed against a spring of spring constant k and the spring is compressed a distance l. The block is released and slides across a frictionless surface for a short distance before encountering a surface with a coefficient of friction μk.

a. Use conservation of energy to find an expression for the velocity of the block after it leaves the spring.

b. Use conservation of energy to find an expression for how far it slides on the surface with friction before coming to a stop.

a) $U_I = \frac{1}{2}k\ell^2$ $\qquad U_F = 0$

$K_I = 0$ $\qquad K_F = \frac{1}{2}m V_F^2$

$$\cancel{\tfrac{1}{2}}k\ell^2 = \cancel{\tfrac{1}{2}}m V_F^2 \Rightarrow \boxed{V_F^2 = \frac{k\ell^2}{m}}$$

b) $U_I = \frac{1}{2}k\ell^2$ $\qquad U_F = 0$ $\qquad W_F = \int_0^d \vec{F}_F \cdot d\vec{s}$

$K_I = 0$ $\qquad K_F = 0$ $\qquad = -\int_0^d \mu_k N\, dx$

$U_I + W_F = 0$

$\frac{1}{2}k\ell^2 - \mu_k m g d = 0$ $\qquad = -\int_0^d \mu_k m g\, dx$

$$* \boxed{d = \frac{k\ell^2}{2\mu_k m g}} *$$

$W_F = -\mu_k m g d$