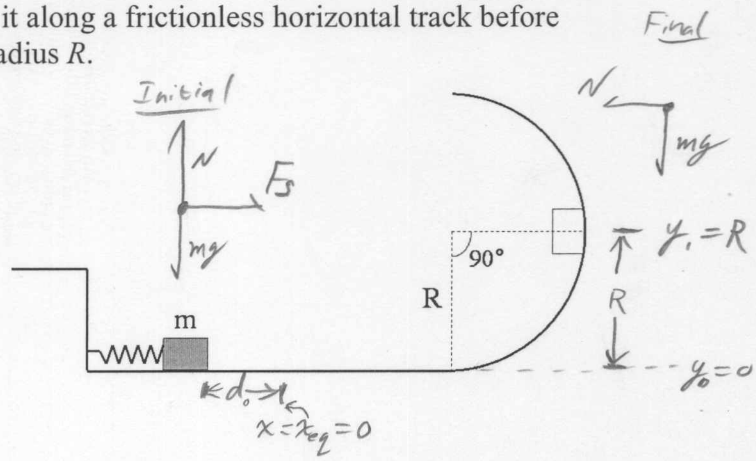
Work-Energy

A mass m rests on a frictionless horizontal track while compressing a horizontal spring of spring constant k. The mass is released and it along a frictionless horizontal track before sliding up a frictionless circular surface of radius R.



a. Find an expression for the compression d such that the mass just comes to rest at a radius position of $\theta = 90°$ as shown in the picture?

b. Now include friction in the problem. If the block stops at a radius position of $\theta = 35°$, how much work was done by the frictional force acting on the block?

a) $U_I = \cancel{mgy_0}^{0} + \frac{1}{2}kd_0^2$ $\qquad U_F = mgy_1 + \cancel{\frac{1}{2}kd_1^2}^{0}$

$K_I = \cancel{\frac{1}{2}mv_I^2}^{0}$ $\qquad K_F = \cancel{\frac{1}{2}mv_F^2}^{0}$

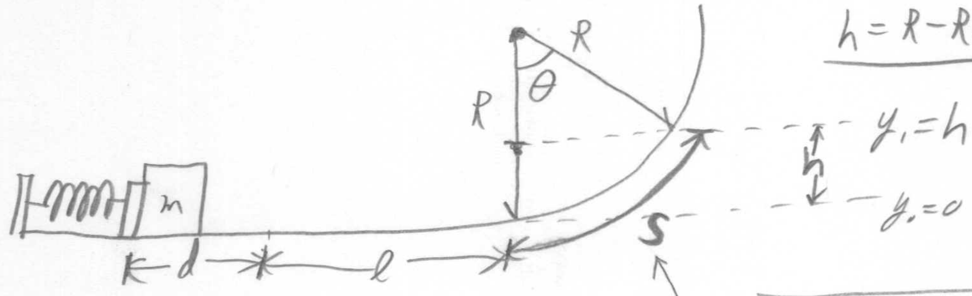
NCF

$W_N = \int \vec{N}\cdot d\vec{s} = 0, \quad \vec{N} \perp d\vec{s}$

$$\frac{1}{2}kd^2 = mgR \Rightarrow \boxed{d = \left(\frac{2mgR}{k}\right)^{1/2}}$$

continued ↓

b)



$\underline{h = R - R\cos\theta}$

This integral is hard...
How can I *not* do it?

$U_I = \tfrac{1}{2}kd^2$ $\qquad$ $U_F = mg(R - R\cos\theta)$

$K_I = 0$ $\qquad$ $K_F = 0$

$\underline{NCF}$

$$W_F = \int_0^{d+\ell+s} \vec{F}_f \cdot d\vec{s} = ??$$

Well... Conservation of Energy is:

$$U_I + \cancelto{0}{K_I} + W_F = U_F + \cancelto{0}{K_F}$$

Know this $\qquad$ Need this $\qquad$ Know this

$$\Rightarrow \boxed{W_F = U_F - U_I = mg(R - R\cos\theta) - \tfrac{1}{2}kd^2}$$

and d from part (a)

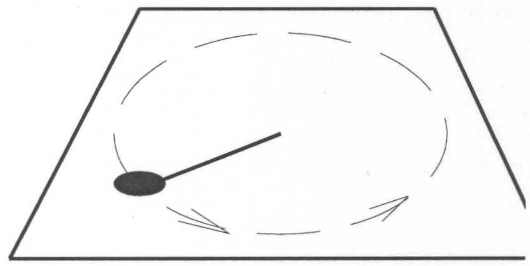
$$W_F = mg(R - R\cos\theta) - \cancel{\tfrac{1}{2}}\cancel{k}\,\frac{\cancel{2}mgR}{\cancel{k}}$$

$$W_F = mgR(\cancel{1} - \cos\theta - \cancel{1})$$

$$\boxed{W_F = -mgR\cos\theta}$$

Work-Energy

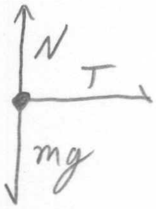
Use work-energy techniques to solve the following problem. A particle of mass m moves in a horizontal circle of radius R on a rough table. It is attached to a string fixed at the center of the circle. The initial speed of the mass is v_o. After completing one full trip around the circle, the speed is ½ v_o.



What is μ_k in terms of v_o, π, R and g?

FBD

side view



top view

$\int d\vec{s}$

No change in y . . .

$U_I = mgy_o$ $\qquad$ $U_F = mgy_o$

$K_I = \frac{1}{2}mV_o^2$ $\qquad$ $K_F = \frac{1}{2}mV_F^2$

$$W_F = \int_0^{2\pi R} \vec{F}_F \cdot d\vec{s} = -\int_0^{2\pi R} \mu_k mg\, ds = \underline{-2\pi R \mu_k mg}$$

~~How many times around will the particle go? (You should get a number)~~ ← next page

$$U_I + K_I + W_{NCF} = U_F + K_F$$

$$\cancel{mgy_o} + \frac{1}{2}mV_o^2 - 2\pi R\mu_k mg = \cancel{mgy_o} + \frac{1}{2}mV_F^2$$

$$V_F = \frac{1}{2}V_o$$

$$-2\pi R\mu_k mg = \frac{1}{2}mV_o^2 - \frac{1}{2}m\left(\frac{1}{2}V_o\right)^2$$

$$2\pi R\mu_k \cancel{m}g = \frac{3}{8}\cancel{m}V_o^2$$

$$\boxed{\mu_k = \frac{3}{16}\frac{V_o^2}{\pi R g}}$$

continued ↓

Now, let $y_o = 0$ and $K_F = 0$

So:

$U_I = 0$ $\qquad$ $U_F = 0$

$K_I = \frac{1}{2} m V_0^2$ $\qquad$ $K_F = 0$

n times around

Once around

$$W_F = \int_0^{n 2\pi R} \vec{F}_f \cdot d\vec{s} = -\int_0^{n\boxed{2\pi R}} \mu_k mg\, ds$$

$$W_F = -n \boxed{2\pi R \mu_k mg}$$

Work by friction in one trip

times n trips

Plug in μ_k from previous part:

$$W_F = -n\cancel{2}\cancel{\pi}\cancel{R}\left[\frac{3}{\cancel{16}_8}\frac{V_0^2}{\cancel{\pi}\cancel{R}\cancel{g}}\right] m\cancel{g}$$

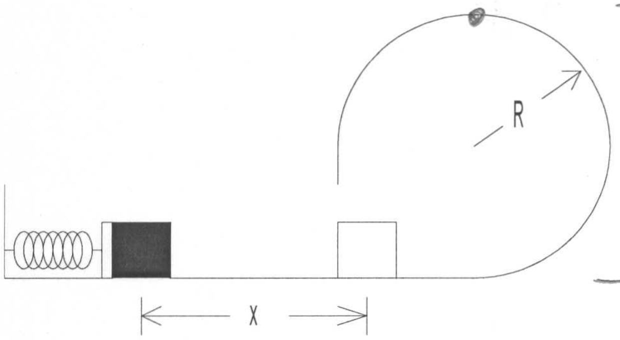
$$\underline{W_F = -\frac{3}{8} n m V_0^2}$$

$$K_I + W_F = 0 \Rightarrow \frac{1}{\cancel{2}} \cancel{m} \cancel{V_0^2} = \frac{3}{\cancel{8}_4} n \cancel{m} \cancel{V_0^2}$$

$$\boxed{n = \frac{4}{3}}$$

Work-Energy

A block of mass *m* is pressed against a spring with a spring constant of *k* a distance *x* from its starting position and then released. What is the minimum distance *x* such that the block will travel around the loop and end up back at the starting point? All surfaces are frictionless, the loop has a radius of *R*, and the size of the block is small compared to the radius of the loop.



$-y_1 = 2R$

The block is in uniform circular motion after leaving the spring.

It must be going fast enough at the top so that it stays on the loop.

$-y_0 = 0$

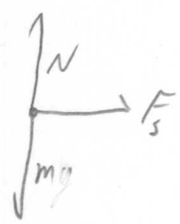
So we take the "Final" position in the energy balance to be at the top.

$U_I = \frac{1}{2}kx^2$ $\qquad U_F = 2mgR$

$K_I = 0$ $\qquad K_F = \frac{1}{2}m V_F^2$ → Required velocity at top of loop

FBD

initial



Final

When it just loses contact, $N \to 0$

Newton: $+\cancel{N}^{\,0} + mg = +ma_c$

$a_c = m\frac{V_F^2}{R}$

so: $\cancel{m}g = \cancel{m}\frac{V_F^2}{R}$

$\Rightarrow \underline{V_F^2 = gR}$

$\frac{1}{2}kx^2 = 2mgR + \frac{1}{2}mgR$

$$\boxed{x^2 = \frac{5mgR}{k}}$$