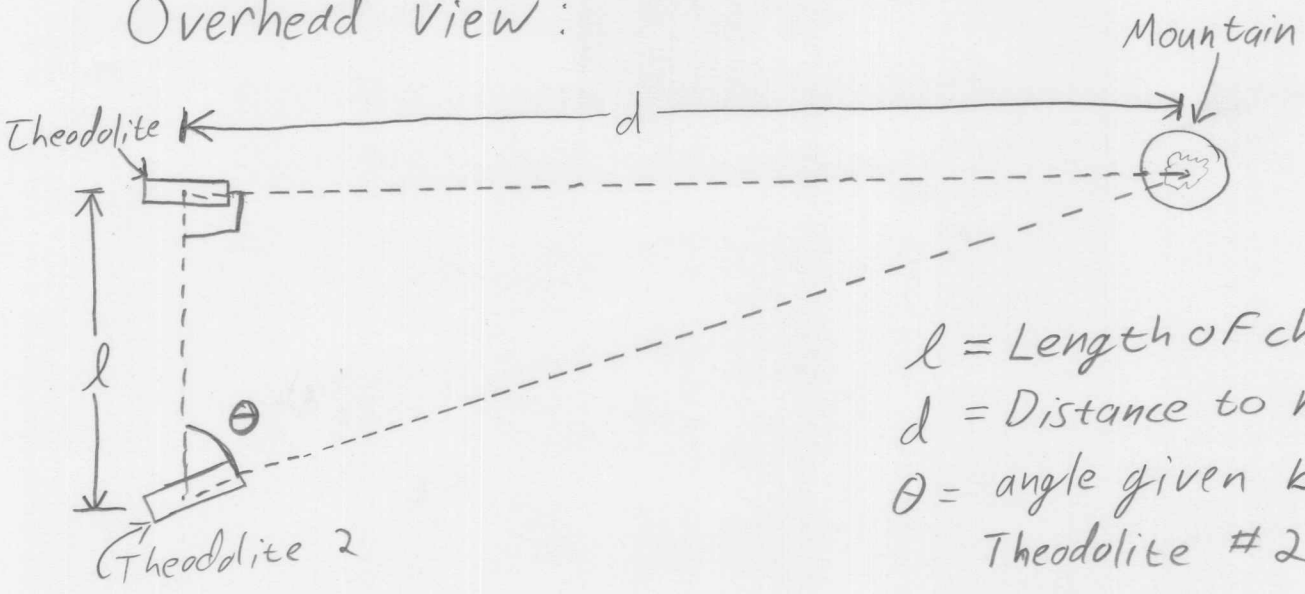
Vector Problems

1. You are given two theodolites and a 10 meter chain. You want to know how far away a distant mountain is. Draw a picture, define the relevant variables, and write an expression for the distance.

Overhedd view:



ℓ = Length of chain
d = Distance to mtn.
θ = angle given by Theodolite #2

It's just a triangle!

$$\tan\theta = \frac{d}{\ell} \Rightarrow \boxed{d = \ell \tan\theta}$$

Vector Problems

2. A particle undergoes three consecutive displacements given by

$$\vec{D}_1 = (\hat{i} + 3\hat{j} - \hat{k}) \text{ cm}, \vec{D}_2 = (2\hat{i} - \hat{j} - 3\hat{k}), \text{ and } \vec{D}_3 = (-\hat{i} + \hat{j}).$$

Find the resultant displacement vector of the particle. Write your answer in unit vector notation and calculate the magnitude of this vector.

$$\vec{D}_T = \vec{D}_1 + \vec{D}_2 + \vec{D}_3$$

$$= 1\hat{x} + 3\hat{y} - 1\hat{k} + 2\hat{x} - 1\hat{y} - 3\hat{k} - 1\hat{x} + 1\hat{y}$$

$$= (1 + 2 - 1)\hat{x} + (3 - 1 + 1)\hat{y} + (-1 - 3)\hat{k}$$

$$\boxed{\vec{D}_T = (2\hat{x} + 3\hat{y} - 4\hat{k})}$$

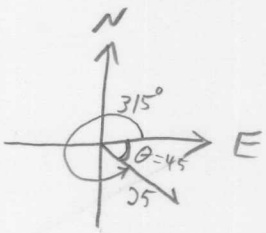
$$\boxed{|\vec{D}_T| = (2^2 + 3^2 + 4^2)^{1/2} = \sqrt{29} = 5.4}$$

Vector Problems

3. A hiker begins a trip by first walking 25 km due southeast from her base camp. On the second day, she walks 40 km in a direction 60° north of east.

   (a) Determine the rectangular components of the hiker's displacements for the first and second days. (It might help to draw a picture.)

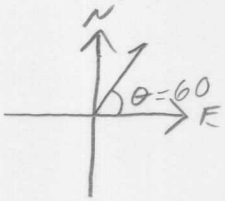
Day 1



$\vec{D}_1 = (25\cos(315)\hat{x} + 25\sin(315)\hat{y})$

$\vec{D}_1 = (17.7\hat{x} - 17.7\hat{y})$

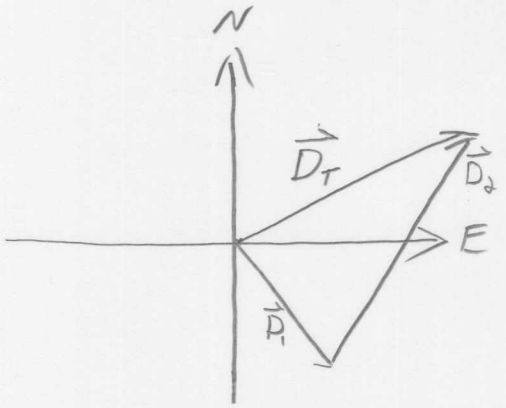
Day 2



$\vec{D}_2 = (40\cos(60)\hat{x} + 40\sin(60)\hat{y})$

$\vec{D}_2 = (20\hat{x} + 34.6\hat{y})$

   (b) Determine the rectangular components of the hiker's total displacement for the trip. Write her resultant displacement vector using unit vector notation.



$\vec{D}_T = (17.7+20)\hat{x} + (-17.7+34.6)\hat{y}$

$\vec{D}_T = 37.7\hat{x} + 16.9\hat{y}$

   (c) What is the magnitude and direction of her resultant displacement vector?

$|\vec{D}_T| = (37.7^2 + 16.9^2)^{1/2} = 41.3 \text{ km}$

$\theta = \tan^{-1}\left(\frac{16.9}{37.7}\right) = 24.1^\circ$

Vector Problems

4. After moving three times, you find yourself 5.39 m away from where you started and 21.8° below the x-axis. Your first move was 5.00 m at an angle of 53.1°. Your second move was 6.00 m along the x-axis and some unknown distance along the y-axis. Your third move was some unknown distance along the x-axis and –3.00 m along the y-axis.

(a) What were the x and y components of your first move?

$\vec{D}_1 = (5\cos(53.1)\hat{x} + 5\sin(53.1)\hat{y})$

$\boxed{\vec{D}_1 = (3.0\hat{x} + 4.0\hat{y})}$

(b) What were the unknown components of your second and third moves?

$\vec{D}_2 = (6.0\hat{x} + Y_2\hat{y})$, $\vec{D}_3 = (X_3\hat{x} + -3.0\hat{y})$

$\vec{D}_T = (5.39\cos(21.8)\hat{x} + 5.39\sin(21.8)\hat{y})$

$\vec{D}_T = (5.0\hat{x} + 2.0\hat{y})$

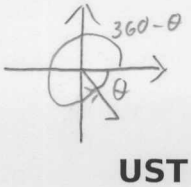
$x: 6.0 + 3.0 + X_3 = 5.0 \Rightarrow \boxed{X_3 = -4}$

$y: 4.0 + Y_2 - 3.0 = 2.0 \Rightarrow \boxed{Y_2 = 1}$

(c) What was the magnitude of your third move?

$|\vec{D}_3| = \left((-4)^2 + (-3)^2\right)^{1/2} = \boxed{5}$

(d) What was the angle of your third move with respect to the x-axis?



$\theta = 360 + \tan^{-1}\left(\frac{-3}{4}\right) = \boxed{323^\circ}$

$\boxed{(\text{or } -32^\circ)}$