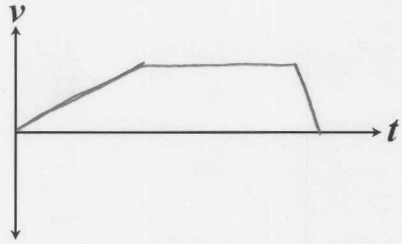
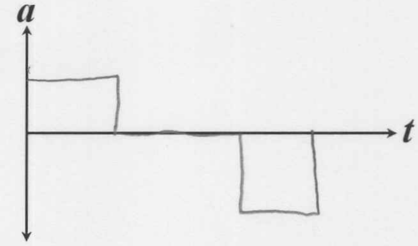
1D Kinematics, Part 2

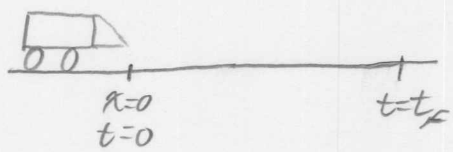
1. A subway train starts from rest at a station and accelerates at a rate of 1.60 m/s^2 for 14.0s. It runs at a constant speed for 70.0 s and slows down at a rate of 3.50 m/s^2 until it stops at the next station.

   Accurately sketch the velocity and acceleration vs. time graphs for this situation.





   During the interval of constant speed, how fast is the train going?



Given

$V_0 = 0$

$a = 1.6\ m/s^2$

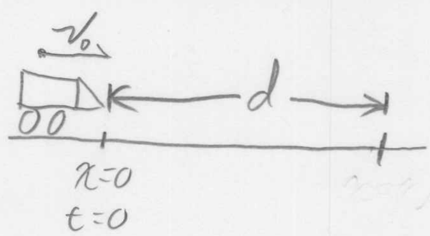
$t_f = 14.0s$

want: V_f

$v(t) = v_0 + at$

$v_f = 0 + at_f \Rightarrow \boxed{v_f = at_f} = (1.6\ m/s^2)(14.0s) = 22.4\ m/s$

   How far does the train travel as it decelerates to a stop?



Given

$V_0 = 22.4\ m/s$

$a = +3.50\ m/s$

$V_f = 0$

want

d

$x(t) = x_0 + v_0 t + \frac{1}{2}at^2$

$d = 0 + v_0 t + \frac{1}{2}at^2$

$v(t) = v_0 + at$

$0 = v_0 + at$

$\Rightarrow t = -\frac{v_0}{a}$

$d = \frac{v_0^2}{a} - \frac{1}{2}\frac{v_0^2}{a}$

$\boxed{d = \frac{1}{2}\frac{v_0^2}{a}} = \frac{1}{2}\frac{(22.4\ m/s)^2}{3.50\ m/s^2} = \boxed{71.7\ m}$

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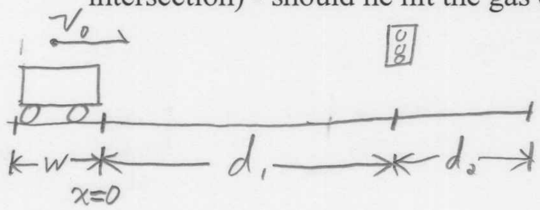
2. Wily coyote is driving his new *Acme* 0.50 m long rocket car at 20.0 m/s when he notices the light at the 20.0 m wide intersection 45.0 m ahead has just turned yellow. If he steps on the brakes his car will slow at a rate of -4.20 m/s². If he hits the gas the car will accelerate at a rate of 2.80 m/s².

a) The light will be yellow for 3.00 seconds and Wily's reaction time is 0.15 seconds. Wily needs to get home as quick as he can but he can't afford a ticket (he must clear the intersection) - should he hit the gas or the brakes? **(Do both cases.)**



$W = 0.50\,m$ $\quad d_1 = 45.0\,m$ $\quad t_y = 3.0\,s$

$V_0 = 20.0\,m/s$ $\quad a_B = -4.20\,m/s^2$ $\quad t_R = 0.15$

$d_2 = 20.0\,m$ $\quad a_G = 2.80\,m/s^2$

$\boxed{\text{Brake}}$ - Want d, stopping distance → But! have to add in reaction time.

$* x(t) = x_0 + V_0 t + \frac{1}{2} a t^2$

$d = 0 + V_0 t - \frac{1}{2} a_B t^2$ ← what t? time to stop!

$* V(t) = V_0 + at$

$0 = V_0 + a_B t \Rightarrow t = -\frac{V_0}{a_B}$

$d = \frac{V_0^2}{a} - \frac{1}{2}\frac{V_0^2}{a} = \frac{1}{2}\frac{V_0^2}{a}$

$\boxed{d = \frac{1}{2}\frac{V_0^2}{a}}$

$d_B = d + d_R$

$d_R = V_0 t_R$

$* \boxed{d_B = \frac{1}{2}\frac{V_0^2}{a_B} + V_0 t_R} *$

$d_T = \frac{1}{2}\frac{20^2}{4.2} + (20)(0.15) = \underline{51\,m}$

Too far! ↑

~~b) Where is the back of Wily's car when the light turns red?~~

$\boxed{\text{If he hits the gas}}$ - How far does the back of the car get in $t_y - t_R$ seconds?

$x(t) = x_0 + V_0 t + \frac{1}{2} a t^2$

$* \boxed{d_g = -w + V_0(t_y - t_R) + \frac{1}{2} a_g (t_y - t_R)^2} *$

$d_g = -0.5\,m + (20.0\,m/s)(2.85\,s) + \frac{1}{2}(2.80\,m/s^2)(2.85\,s)^2$

$\underline{d_g = 67.9\,m}$ $\qquad d_1 + d_2 = \underline{65\,m}$

HIT THE GAS!!

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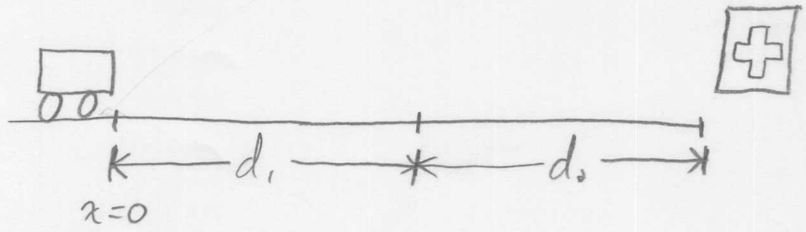
3. Pokey decides to rush off to the far side of the TV studio for some aspirin. If his cart can accelerate at 5.0 m/s^2 and decelerate at –3.0 m/s^2, and he just barely stops when he reaches the medicine cabinet (150 m away), what is the shortest time in which he can make the trip.

Hint: Think of the separate parts that make up the trip. Draw a picture and label everything you know or don't know about each segment. Then figure out how to combine the information from the two segments. Don't use more variables than you need!



$a_1 = 5.0\ m/s$

$a_2 = 3.0\ m/s$

$d = 150\ m$

Want: t_t

Pokey accelerates for d_1 at a_1, and decelerates for d_2 at a_2. Time is combined time for the trip.

Leg 1: $v_0 = 0$

$* \ x(t) = x_0 + v_0 t_1 + \frac{1}{2} a_1 t_1^2$

$d_1 = 0 + 0 + \frac{1}{2} a_1 t_1^2$

① $\boxed{d_1 = \frac{1}{2} a_1 t_1^2}$

$* \ v(t) = v_0 + at$

$v_{1f} = 0 + a_1 t_1$

② $\boxed{v_{1f} = a_1 t_1}$

Leg 2: $v_0 = v_{1f}$, $v_{2f} = 0$

$* \ x(t) = x_0 + v_0 t + \frac{1}{2} a t^2$

③ $\boxed{d = d_1 + v_{1f} t_2 - \frac{1}{2} a_2 t_2^2}$

$* \ v(t) = v_0 + at$

$0 = v_{1f} - a_2 t_2$

④ $\boxed{v_{1f} = a_2 t_2}$

4 eqns, 4 unknowns...

combine eq. ①, ②, and ③

rewrite eq. 3: $\boxed{d = \frac{1}{2} a_1 t_1^2 + a_1 t_1 t_2 - \frac{1}{2} a_2 t_2^2}$ ⑤

continued ↓

(3.) <u>continued</u>

combine eq. (2) and (4)

$$a_1 t_1 = a_2 t_2 \Rightarrow \underline{t_1 = t_2 \frac{a_2}{a_1}}$$

replace all occurrences of t_1 in eq. (5)

$$d = \frac{1}{2}\cancel{a_1} t_2^2 \frac{a_2^2}{a_1^{\cancel{2}}} + \cancel{a_1} t_2 \frac{a_2}{\cancel{a_1}} t_2 - \frac{1}{2} a_2 t_2^2$$

$$d = \frac{1}{2}\frac{a_2^2}{a_1} t_2^2 + a_2 t_2^2 - \frac{1}{2} a_2 t_2^2$$

$$d = \frac{1}{2}\frac{a_2^2}{a_1} t_2^2 + \frac{1}{2} a_2 t_2^2$$

$$d = \frac{1}{2} a_2 \left(\frac{a_2}{a_1} + 1\right) t_2^2$$

$$d = \frac{1}{2} a_2 \left(\frac{a_1 + a_2}{a_1}\right) t_2^2$$

$$\Rightarrow \boxed{t_2 = \left[\frac{2a_1}{a_2(a_1+a_2)} d\right]^{1/2}}$$

Now get t_1:

$$t_1 = \left[\frac{2a_1}{a_2(a_1+a_2)} d\right]^{1/2} \cdot \frac{a_2}{a_1} = \left[\frac{a_2^{\cancel{2}}}{a_1^{\cancel{2}}} \cdot \frac{2\cancel{a_1}}{\cancel{a_2}(a_1+a_2)} d\right]^{1/2}$$

$$\boxed{t_1 = \left[\frac{2a_2}{a_1(a_1+a_2)} d\right]^{1/2}}$$

$$* \quad \boxed{t_T = t_1 + t_2 = \left[\frac{2a_2}{a_1(a_1+a_2)} d\right]^{1/2} + \left[\frac{2a_1}{a_2(a_1+a_2)} d\right]^{1/2}} \quad *$$

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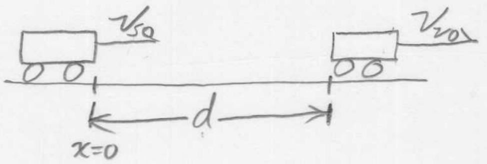
4. Speedy Sue, driving at 30.0 m/s, enters a one-lane tunnel. She then observes a slow-moving van 155 m ahead traveling at 5.00 m/s. She applies her brakes but can accelerate only at –2.00 m/s^2 because the road is wet. Will there be a collision? If so, determine how far into the tunnel and at what time the collision occurs. If not, determine the distance of closest approach between Sue's car and the van.

Make a sketch of the situation. In the sketch define your coordinate system and appropriate variables. In your solution give equations in symbols, next equation with numerical values including units, then give answer.



$v_{so} = 30.0\,\text{m/s}$ $\quad v_{sf} = 0$

$v_{vo} = 5.00\,\text{m/s}$ $\quad d = 155\,\text{m}$

$a_s = 2.00\,\text{m/s}^2$

Sue

$* \; x(t) = x_0 + v_0 t + \frac{1}{2} a t^2$

$x_{sf} = 0 + v_{so} t - \frac{1}{2} a_s t^2$

$* \; v(t) = v_0 + at$

$0 = v_{so} - a_s t$

$\Rightarrow t = \dfrac{v_{so}}{a_s}$

$x_{sf} = \dfrac{v_{so}^2}{a_s} - \dfrac{1}{2}\dfrac{v_{so}^2}{a_s}$

$x_{sf} = \dfrac{1}{2}\dfrac{v_{so}^2}{a_s}$

Van

$* \; x(t) = x_0 + v_0 t + \frac{1}{2} a t^2$

$x_{vf} = d + v_{vo} t + 0$

$x_{vf} = d + v_{vo}\dfrac{v_{so}}{a_s}$

Find Δx, distance between sue and van

$\Delta x = x_{vf} - x_{sf}$

$$\boxed{\Delta x = d + \frac{v_{vo} v_{so}}{a_s} - \frac{1}{2}\frac{v_{so}^2}{a_s}}$$

$$\Delta x = 155\,\text{m} + \frac{(5\,\text{m/s})(30\,\text{m/s})}{2.0\,\text{m/s}^2} - \frac{1}{2}\frac{(30\,\text{m/s})^2}{2.0\,\text{m/s}^2} = \boxed{5\,\text{m}}$$

No collision!