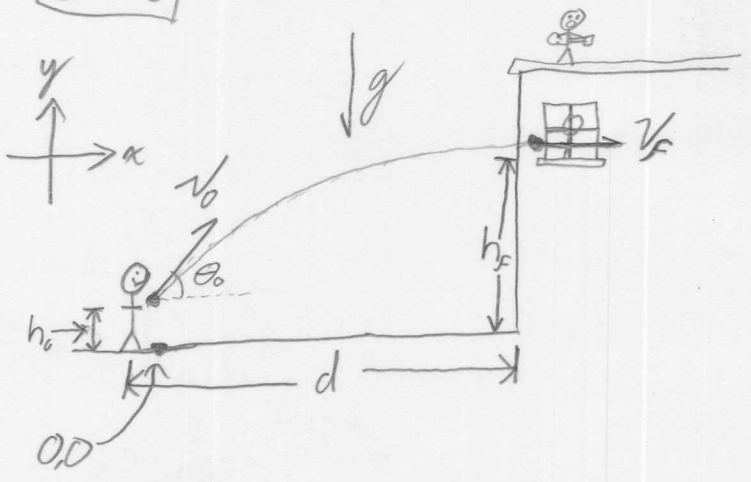
Physics 111
Homework
Probems 3-63, 59, 61

3-63



GIVEN
$d = 3.0\,m$
$h_0 = 1.5\,m$
$h_f = 4.2\,m$

WANT
$|\vec{v}_0| = ?$
$\theta_0 = ?$

2D Problem, trajectory

x

$x_f = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$

① $d = 0 + v_{0x}t$

y

$y_f = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$

② $h_f = h_0 + v_{0y}t - \frac{1}{2}gt^2$

3 unknown Variables, 2 eq.

* Problem asks For $|\vec{v}_0|$ and θ_0, But we'll solve for v_{0x} and v_{0y} and then convert because it's much easier.

Velocity equations

$v_{fx} = v_{0x} + a_x t$ ($a_x = 0$)

$v_{fx} = v_{0x}$

$v_{fy} = v_{0y} + a_y t$

$v_{fy} = v_{0y} - gt$

continued ↓

We interpret this to mean that it's at the top of the trajectory. At this point, $\underline{v_{fy} = 0}$.

So when $y = h_f$, $v_{fy} = 0$

③ $v_{fx} = v_{0x}$

$$v_{fy} = v_{oy} - gt$$

$$0 = v_{oy} - g \cdot t$$

④ $\underline{v_{oy} = gt}$

Now we have 4 eqs and 4 unknowns

But, we don't need v_{fx}, so we'll ignore eq ③

Eliminate t by solving ④ for t and plugging into ① and ②

$$t = \frac{v_{oy}}{g} \xrightarrow{\text{into ②}} h_f - h_o = v_{oy}\frac{v_{oy}}{g} - \frac{1}{2}g\frac{v_{oy}^2}{g^2}$$

$$h_f - h_o = \frac{v_{oy}^2}{g} - \frac{1}{2}\frac{v_{oy}^2}{g} = \frac{1}{2}\frac{v_{oy}^2}{g}$$

$$\boxed{v_{oy} = \left[2g(h_f - h_o)\right]^{1/2}}$$

continued ↓

3-63 continued

Plug time eq into ①

$$d = v_{ox}\frac{v_{oy}}{g} \leftarrow \text{we have this now!}$$

$$d = \frac{v_{ox}}{g}\left[2g(h_f - h_o)\right]^{1/2}$$

$$= v_{ox}\left[\frac{2}{g}(h_f - h_o)\right]^{1/2}$$

$$\boxed{v_{ox} = \frac{d}{\left[\frac{2}{g}(h_f - h_o)\right]^{1/2}}}$$

$$v_{ox} = \frac{3.0}{\left[\frac{2}{9.8}(4.2 - 1.5)\right]^{1/2}} = \underline{4.04\ m/s}$$

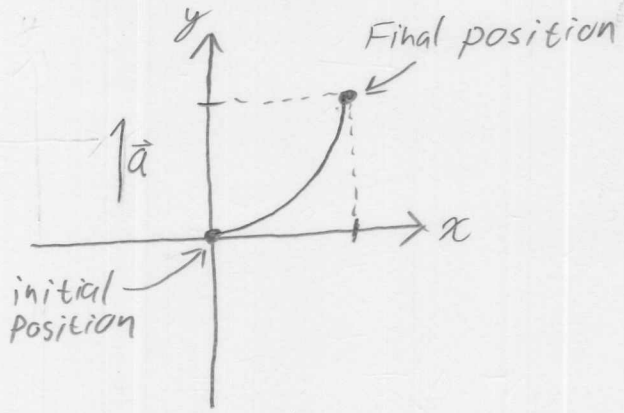
$$v_{oy} = \left[2(9.8)(4.2 - 1.5)\right]^{1/2} = \underline{7.27\ m/s}$$

$$\boxed{|\vec{v}_o| = (4.04^2 + 7.27^2)^{1/2} = 8.32\ m/s}$$

$$\boxed{\theta = \tan^{-1}\left(\frac{7.27}{4.04}\right) = \underline{61^\circ}}$$

3-59

Sketch the trajectory



Given

$v_{0x} = 4.5\ m/s$

$t = 18\ s$

$x_f = y_f$

$a_x = 0$

$v_{0y} = 0$

WANT

$a_y = ?$

Since the problem statement says $x_f = y_f$
Let's write displacement eq's for x and y and set them equal.

$$x_f = x_0 + v_{0x}t + \tfrac{1}{2}a_x t^2$$

$$x_f = 0 + v_{0x}t + 0$$

$$y_f = y_0 + v_{0y}t + \tfrac{1}{2}a_y t^2$$

$$y_f = 0 + 0 + \tfrac{1}{2}a_y t^2$$

$$x_f = y_f$$

$$v_{0x}t = \tfrac{1}{2}a_y t^2$$

$$\boxed{a_y = \frac{2v_{0x}}{t}}$$

$$a_y = \frac{(2)(4.5)}{18} = \boxed{0.5\ m/s^2}$$

3-61

Given

$v_{oy} = 0$

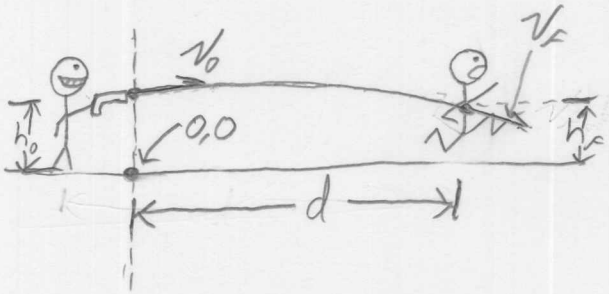
$h_o = 1.6\,m$

$h_f = 0.93\,m$

$d = 2.1\,m$

WANT

$v_{ox} = ?$



Trajectory Problem, 2D

x

$$x_f = x_o + v_{ox}t + \tfrac{1}{2}a_x t^2$$

$$d = 0 + v_{ox}t + 0$$

$$t = \frac{d}{v_{ox}}$$

y

$$y_f = y_o + v_{oy}t + \tfrac{1}{2}a_y t^2$$

$$h_f = h_o + 0 - \tfrac{1}{2}g t^2$$

$$h_f - h_o = -\frac{1}{2}g\frac{v_{ox}^2}{d^2}$$

$$v_{ox}^2 = \frac{-2(h_f - h_o)d^2}{g}$$

$$\boxed{v_{ox} = \left[\frac{2(h_o - h_f)d^2}{g}\right]^{1/2}}$$

$$v_{ox} = \left[\frac{2(1.6-0.93)(2.1)^2}{9.8}\right]^{1/2} = \boxed{0.78\ m/s}$$