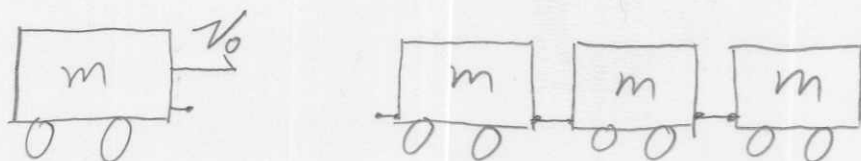


Momentum, Impulse, and Collisions

A 4000 kg railroad car inelastically collides with three other 4000 kg cars sitting at rest on a rough track. The four cars travel together down the rough track for 1.5 m before they stop. Assuming $\mu_k = 0.10$, what is the velocity of the first car at impact?



Find velocity after collision

$$m v_0 = 4m v_f \quad : \text{ conserve momentum}$$

$$v_f = \frac{v_0}{4}$$

Find distance to stop

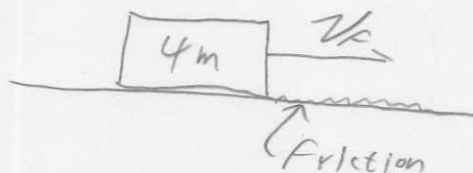
$$K_i = \frac{1}{2} (4m) v_f^2 \quad U_i = 0$$

$$K_f = 0 \quad U_f = 0$$

$$E_f = N \mu_k d = 4mg \mu_k d$$

$$\frac{1}{2} (4m) \frac{v_0^2}{4^2} = 4mg \mu_k d$$

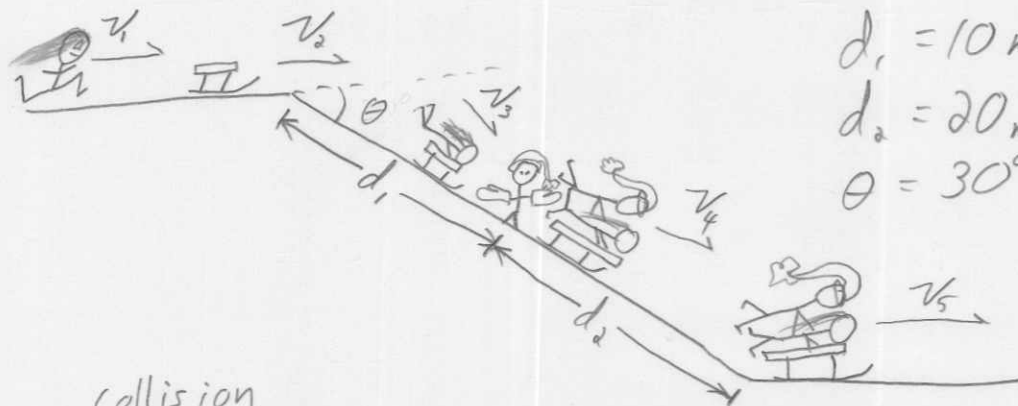
$$\boxed{v_0 = \sqrt{32g \mu_k d}} = \boxed{6.9 \text{ m/s}}$$



Momentum, Impulse, and Collisions

Gayle runs at a speed of 4.0 m/s and dives onto a sled that is initially at rest on top of a frictionless snow covered hill sloping down at 30° . After she has traveled 10 m down the slope, her brother Billy hops on the sled; and together they travel on down the slope another 20 m. (Billy was at rest initially) What is their speed at the bottom of the hill?

$m_{\text{Gayle}} = 50.0 \text{ kg}$, $m_{\text{Billy}} = 30.0 \text{ kg}$, $m_{\text{sled}} = 5.0 \text{ kg}$



$v_1 = 4.0 \text{ m/s}$

$d_1 = 10 \text{ m}$

$d_2 = 20 \text{ m}$

$\theta = 30^\circ$

collision

① $m_g v_1 = (m_g + m_s) v_2$: conserve momentum

$v_2 = \frac{m_g}{(m_g + m_s)} v_1 = \frac{50}{50 + 5} (4) = \boxed{3.63 \text{ m/s}}$

② work-energy

$K_i = \frac{1}{2} (m_g + m_s) v_2^2$

$K_f = \frac{1}{2} (m_g + m_s) v_3^2$

$U_i = (m_g + m_s) g h = (m_g + m_s) g d_1 \sin \theta$. $U_f = 0$

$\frac{1}{2} (m_g + m_s) v_2^2 + (m_g + m_s) g d_1 \sin \theta = \frac{1}{2} (m_g + m_s) v_3^2$

$v_3 = (v_2^2 + 2 g d_1 \sin \theta)^{\frac{1}{2}} = (3.63^2 + 2(9.8)(10) \sin(30))^{1/2}$

$\boxed{= 10.5 \text{ m/s}}$

③ collision #2

$$(m_s + m_g) v_3 = (m_s + m_g + m_B) v_4 \quad ; \text{ conserve momentum}$$

$$v_4 = \frac{m_s + m_g}{m_s + m_g + m_B} v_3 = \frac{50 + 5}{50 + 5 + 30} \cdot 10.5$$
$$= \underline{6.79 \text{ m/s}}$$

④ Work - Energy

$$K_i = \frac{1}{2} (m_s + m_g + m_B) v_4^2$$

$$K_f = \frac{1}{2} (m_s + m_g + m_B) v_5^2$$

$$U_i = (m_s + m_g + m_B) g d_2 \sin \theta$$

$$U_f = 0$$

$$\frac{1}{2} (m_s + m_g + m_B) v_4^2 + (m_s + m_g + m_B) g d_2 \sin \theta = \frac{1}{2} (m_s + m_g + m_B) v_5^2$$

$$v_5 = [v_4^2 + 2 g d_2 \sin \theta]^{\frac{1}{2}}$$

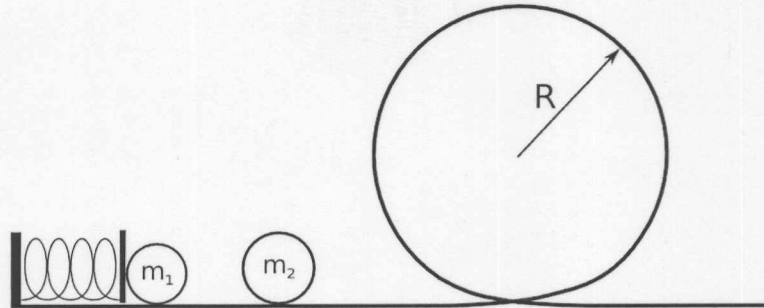
$$= [(6.79)^2 + (2)(9.8)(20) \sin(30)]^{\frac{1}{2}}$$

$$\boxed{= 15.6 \text{ m/s}}$$

Momentum, Impulse, and Collisions

In the system below, a ball of mass m_1 is placed against a spring with spring constant k that has been compressed a distance d . It is released from rest and collides with a second ball of mass m_2 which then goes around the loop the loop of radius R .

What is the minimum spring compression d such that m_2 makes it around the loop?



Stage 1 - Get m_1 off of the spring

Work - Energy

$$U_I = \frac{1}{2}kd^2 \quad U_F = 0$$

$$K_I = 0 \quad K_F = \frac{1}{2}m_1v_1^2$$

$$\frac{1}{2}kd^2 = \frac{1}{2}m_1v_1^2 \Rightarrow v_1 = \sqrt{\frac{kd^2}{m_1}}$$

Stage 2 - Collision, Elastic

$$m_1v_1 = m_1v_{1F} + m_2v_2 \quad \text{conserve momentum}$$

$$\frac{1}{2}m_1v_1^2 = \frac{1}{2}m_1v_{1F}^2 + \frac{1}{2}m_2v_2^2 \quad \text{conserve kinetic energy}$$

Gather terms and divide

$$\frac{m_1(v_1^2 - v_{1F}^2)}{m_1(v_1 - v_{1F})} = \frac{m_2v_2^2}{m_2v_2}$$

continued



Remember: $(a+b)(a-b) = a^2 - b^2$

$$\frac{(v_i + v_{if})(-v_i - v_{if})}{(v_i - v_{if})} = v_o \Rightarrow \underline{v_{if} = v_o - v_i}$$

Plug back into momentum eq.

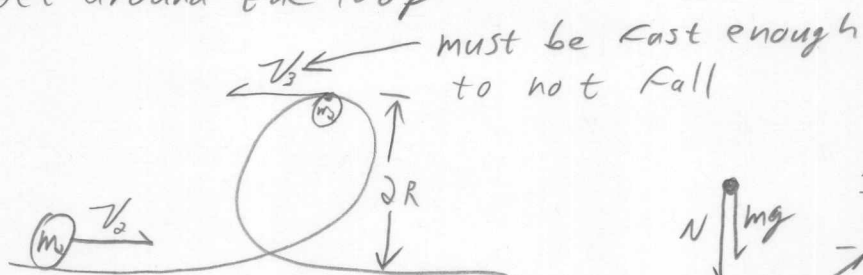
$$m_1 v_i = m_1 (v_o - v_i) + m_2 v_2$$

$$m_1 v_i = m_1 v_o - m_1 v_i + m_2 v_2$$

$$2m_1 v_i = (m_1 + m_2) v_2$$

$$\boxed{v_2 = \frac{2m_1}{m_1 + m_2} v_i}$$

Stage 3 - Get around the loop



$$U_i = 0 \quad U_f = 2mgR$$

$$K_i = \frac{1}{2} m v_o^2 \quad K_f = \frac{1}{2} m v_3^2 = \frac{1}{2} mgR$$

$$\frac{1}{2} m v_o^2 = 2mgR + \frac{1}{2} mgR$$

$$\boxed{v_o^2 = \frac{5}{2} gR}$$

continued



$$N \downarrow mg \quad \frac{NSL}{-N + mg} = +m \frac{v_3^2}{R}$$

↑
goes to zero when it just loses contact

$$\underline{v_3^2 = gR}$$

Put all three stages together.

$$v_1 = \sqrt{\frac{kd^2}{m}}, \quad v_2 = \frac{2m_1}{m_1 + m_2} v_1, \quad v_2 = \sqrt{5gR}$$

$$\sqrt{\frac{kd^2}{m_1}} \frac{2m_1}{m_1 + m_2} = \sqrt{5gR}$$

$$d = \frac{(m_1 + m_2)}{2} \cdot \left(\frac{5gR}{m_1 k} \right)^{1/2}$$