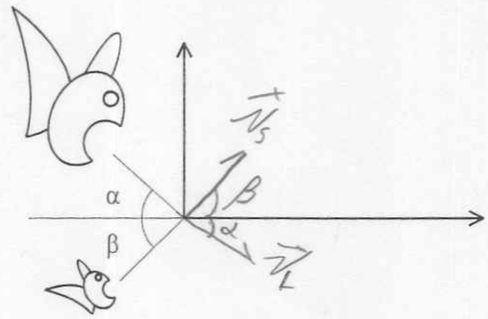
1. A large fish will soon make a dish of a smaller fish. What is the velocity of the large fish and his dinner immediately after he eats? Give both the magnitude and direction of the final velocity with respect to the x-axis.

$m_{\text{large fish}} = 4.0$ kg
$v_{o\,\text{large fish}} = 1.0$ m/s
$\alpha_{\text{large fish}} = 25.0^\circ$

$m_{\text{small fish}} = 0.20$ kg
$v_{o\,\text{small fish}} = 5.0$ m/s
$\beta_{\text{small fish}} = 50.0^\circ$



Conserve momentum in both axis

$$x: m_L v_L \cos\alpha + m_s v_s \cos\beta = (m_L + m_s) v_F \cos\theta$$

$$y: -m_L v_L \sin\alpha + m_s v_s \sin\beta = (m_L + m_s) v_F \sin\theta$$

Divide y by x to eliminate v_F

$$\frac{-m_L v_L \sin\alpha + m_s v_s \sin\beta}{m_L v_L \cos\alpha + m_s v_s \cos\beta} = \frac{\cancel{(m_L+m_s)}\cancel{v_F}\sin\theta}{\cancel{(m_L+m_s)}\cancel{v_F}\cos\theta}$$

$$\tan\theta = \frac{-m_L v_L \sin\alpha + m_s v_s \sin\beta}{m_L v_L \cos\alpha + m_s v_s \cos\beta} \Rightarrow$$

$$\theta = \tan^{-1}\left[\frac{-(4.0)(1.0)\sin(25) + (0.2)(5)\sin(50)}{(4.0)(1.0)\cos(25) + (0.2)(5)\cos(50)}\right] = \boxed{-12^\circ}$$

Plug back into x (or y) to get v_F

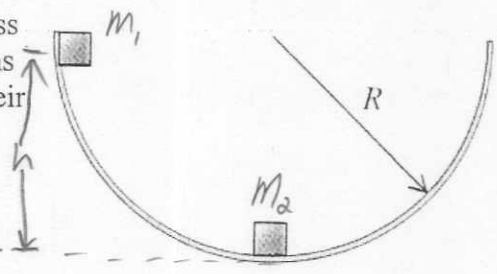
$$v_F = \frac{m_L v_L \cos\alpha + m_s v_s \cos\beta}{(m_L + m_s)\cos\theta} = \frac{(4)(1)\cos 25 + (0.2)(5)\cos(50)}{(4 + 0.2)\cos(-12)} = \boxed{1.0\,\text{m/s}}$$

Momentum, Impulse, and Collisions

Two masses are released from rest in a frictionless hemispherical bowl or radius R from the positions shown in the figure. Derive an expression for their final height in the case of :

a) An elastic collision

b) An inelastic collision



$h=R$

c) How much bigger than the second mass does the first mass have to be so that the second mass gets out of the bowl.

a) Find Velocity of m_1 before collision

$U_i = mgh \quad K_i = 0$

$U_f = 0 \quad K_f = \frac{1}{2} m_1 V_{1i}^2 \Rightarrow V_{1i} = \sqrt{2gh}$

Find both velocities post collision

① $\frac{1}{2} m_1 V_{1i}^2 = \frac{1}{2} m_1 V_{1f}^2 + \frac{1}{2} m_2 V_{2f}^2$: conserve energy

② $m_1 V_{1i} = m_1 V_{1f} + m_2 V_{2f}$: conserve momentum

$$\frac{\cancel{m_1}(V_{1i}^2 - V_{1f}^2)}{\cancel{m_1}(V_{1i} - V_{1f})} = \frac{\cancel{m_2} V_{2f}^{\cancel{2}}}{\cancel{m_2}\cancel{V_{2f}}}$$: Divide ① by ②

$$\Rightarrow \frac{\cancel{(V_{1i} - V_{1f})}(V_{1i} + V_{1f})}{\cancel{(V_{1i} - V_{1f})}} = V_{2f} \Rightarrow V_{2f} = V_{1i} + V_{1f}$$

$V_{1f} = V_{2f} - V_{1i}$

Plug back into momentum equation

$m_1 V_{1i} = m_1 V_{1f} + m_2 V_{1i} + m_2 V_{1f}$

$$\boxed{V_{1f} = \frac{(m_1 - m_2)}{(m_1 + m_2)} V_{1i}}$$

$m_1 V_{1i} = m_1 V_{2f} - m_1 V_{1i} + m_2 V_{2f}$

$$\boxed{V_{2f} = \frac{2m_1}{(m_1 + m_2)} V_{1i}}$$

Find h_1 and h_2

$U_i = 0 \quad K_i = \frac{1}{2} m v^2$
$U_f = mgh \quad K_f = 0$ $\Rightarrow$ $\boxed{h = \frac{v^2}{2g}}$

$$h_1 = \frac{(m_1 - m_2)^2}{(m_1 + m_2)^2} \frac{v_{1i}^2}{2g} \qquad h_2 = \frac{4m_1^2}{(m_1+m_2)^2} \cdot \frac{v_{1i}^2}{2g}$$

$$= \frac{(m_1 - m_2)^2}{(m_1+m_2)^2} \cdot \frac{\cancel{2g}h}{\cancel{2g}} \qquad = \frac{4m_1^2}{(m_1+m_2)^2} \frac{\cancel{2g}h}{\cancel{2g}}$$

$$\boxed{h_1 = \frac{(m_1-m_2)^2}{(m_1+m_2)^2} R} \qquad \boxed{h_2 = \frac{4m_1^2}{(m_1+m_2)^2} R}$$

b) Use $v_{1i} = \sqrt{2gh}$ from part a

Find velocity of combined mass post collision.

$m_1 v_{1i} = (m_1 + m_2) v_f$: conserve momentum

$$v_f = \frac{m_1}{(m_1+m_2)} v_{1i}$$

Find final height

$U_i = 0 \quad K_i = \frac{1}{2} m v_f^2$
$U_f = mgh_f \quad K_i = 0$ $\Rightarrow h_f = \frac{v_f^2}{2g} = \frac{m_1^2}{(m_1+m_2)^2} \frac{v_{1i}^2}{2g}$

$$= \frac{m_1^2}{(m_1+m_2)^2} \frac{\cancel{2g}h}{\cancel{2g}}$$

$$\boxed{h_f = \frac{m_1^2}{(m_1+m_2)^2} R}$$

c) If $h_2 \geq R$, the second mass will escape.

set $h_2 = R$

$$\cancel{R} = \frac{4m_1^2}{(m_1+m_2^2)}\cancel{R} \Rightarrow m_1 + m_2 = 2m_1$$

$$\Rightarrow \underline{m_2 = m_1}$$

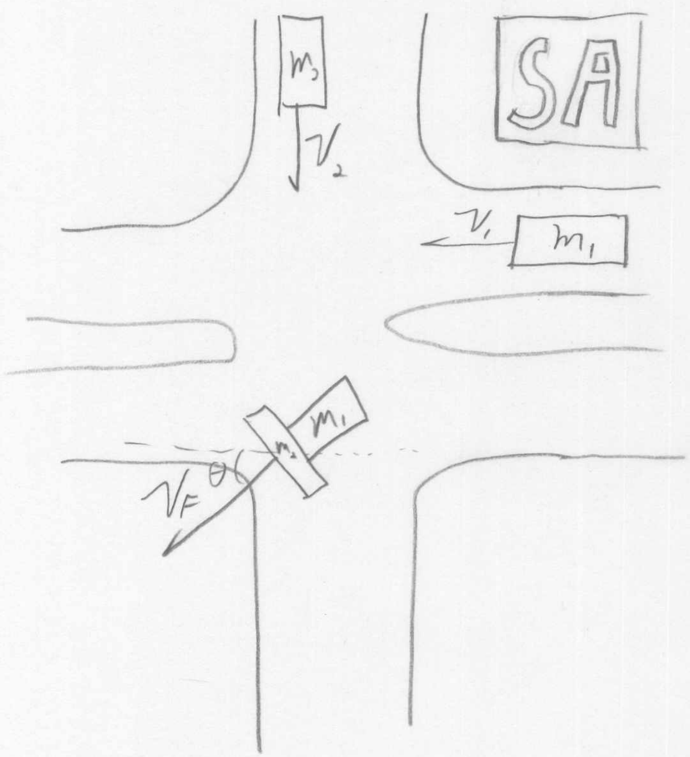
So if m_1 is greater than m_2,
m_2 will escape.

Momentum, Impulse, and Collisions

You are driving West along Summit Ave, lawfully doing the speed limit (50 km/hr) in your new car which (as you've read in the owners manual) has a mass of 1500 kg. Sleepy McSnoozer is driving South along Cleveland in his 1965 Ford pickup truck loaded with bags of cement. His truck (plus cement) weighs 2300 kg. Sleepy runs the red light and smashes into your car. The cars fuse together and skid to a stop.

Certain that Sleepy was speeding, you measure the skid mark and find that the length of the skid is L = 18 m and that it makes an angle θ = -67° with an East-West line. You look up the rubber/asphalt coefficient of friction and find that it is $\mu_k = 0.6$.

What was Sleepy's velocity? Was he speeding? The speed limit is 50 km/hr.



conserve momentum

$$x: -m_1 v_1 = -(m_1 + m_2) v_f \cos\theta$$

$$y: -m_2 v_2 = -(m_1 + m_2) v_f \sin\theta$$

The easy way to solve this: Divide y by x

$$\frac{+m_2 v_2}{+m_1 v_1} = \frac{+\cancel{(m_1+m_2)}\cancel{v_f}\sin\theta}{+\cancel{(m_1+m_2)}\cancel{v_f}\cos\theta}$$

$$\tan\theta = \frac{m_2}{m_1}\frac{v_2}{v_1}$$

$$\Rightarrow v_2 = \frac{m_1}{m_2} v_1 \tan\theta$$

$$v_2 = \frac{1500}{2300} 50 \tan(67)$$

$$\boxed{= 77\ \mathrm{km/hr} \rightarrow \text{speeding}}$$

continued ↓

How I <u>intended</u> to write the problem:

Solve the skid for V_F and plug back into y

x or y

$$U_I = 0 \qquad U_F = 0$$

$$K_I = \tfrac{1}{2}(m_1+m_2)V_F^2 \qquad K_F = 0$$

$$W_F = \int \vec{F}_F \cdot d\vec{s} = -\int_0^L (m_1+m_2)\, g\, \mu_K \, ds$$

$$= -\mu_K (m_1+m_2) g L$$

$$\tfrac{1}{2}\cancel{(m_1+m_2)}V_F^2 = \mu_K \cancel{(m_1+m_2)} g L$$

$$V_F = \left(2\mu_K g L\right)^{1/2}$$

into y:

$$V_2 = \frac{m_1+m_2}{m_2} \cdot \left(2\mu_K g L\right)^{1/2} \sin(67)$$

$$= \frac{3800}{2300}\left(2\cdot(0.6)\cdot(9.8)(18)\right)^{1/2} \sin(67)$$

$$= 22\ \mathrm{m/s} \cdot 1\times10^{-3}\ \mathrm{km/m} \cdot 3600\ \mathrm{s/hr} = \boxed{79\ \mathrm{km/hr}} \rightarrow \text{speeding}$$