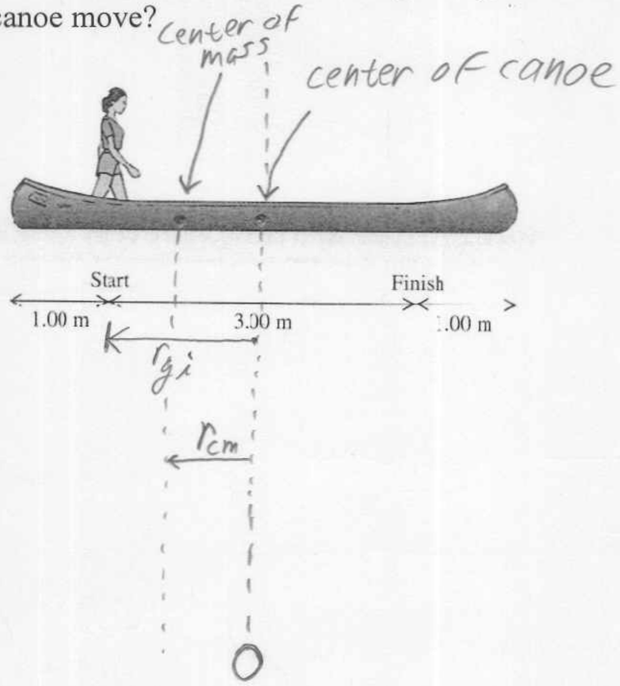
MOMENTUM, IMPULSE, AND COLLISIONS

Use conservation of momentum to solve the following problem.
A 45 kg woman stands up in a 60 kg canoe of length 5.0 m. She walks from a point 1 m from one end to a point one meter from the other end. Ignoring resistance due to the water, how far does the canoe move?



center of mass doesn't move

$$r_{cmi} = r_{cmf} \Rightarrow \frac{m_g r_{gi} + m_c r_{ci}}{(m_g + m_c)} = \frac{m_g r_{gf} + m_c r_{cf}}{(m_g + m_c)}$$

Both canoe and girl move but girl's position relative to the center of the canoe is known.

$r_{gf} = r_{cf} + 1.5\,m$

So: $m_g r_{gi} + m_c r_{ci} = m_g (r_{cf} + 1.5) + m_c r_{cf}$

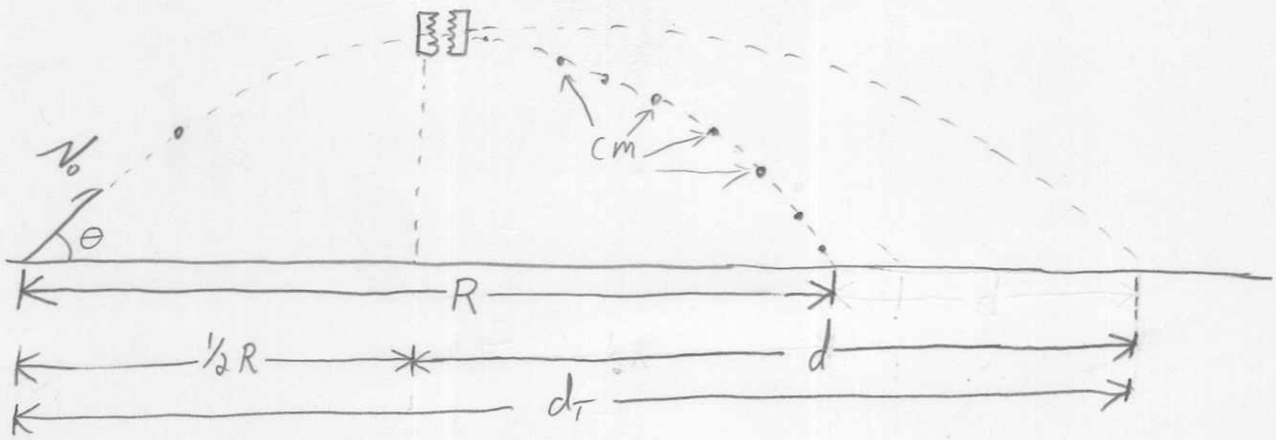
$= m_g r_{cf} + m_g \cdot 1.5 + m_c r_{cf}$

$m_g (r_{gi} - 1.5) + m_c r_{ci} = (m_g + m_c) r_{cf}$

$$r_{cf} = \frac{m_g (r_{gi} - 1.5) + m_c r_{ci}}{(m_g + m_c)} = \frac{45(-1.5-1.5)}{(45+60)} = 1.28\,m$$

Momentum, Impulse, and Collisions

A 20.0 kg projectile is fired at an angle of 60.0 degrees above the horizontal with a speed of 80.0 m/s. At the hightest point of its trajectory, it explodes into two fragments with equal mass. One falls vertically to the ground. Where does the second fragment land? How much energy was released in the explosion?



Find R with kinematics eq.s

x: $R = V_0 \cos\theta\, t$

y: $0 = V_0 \sin\theta\, t - \frac{1}{2} g t^2$

$$t = \frac{2 V_0 \sin\theta}{g}$$

$$\boxed{R = \frac{2 V_0^2 \cos\theta \sin\theta}{g}}$$

Find Velocity of second fragment

inelastic "collision", so conserve momentum in x

$$2m V_0 \cos\theta = m V_1 + m V_2 \quad (V_1 = 0)$$

↑ This one drops vertically.

$$\boxed{V_2 = 2 V_0 \cos\theta}$$

The two fragments (since their velocity in the y is unchanged) strike the ground at the same time, which is also the same time that the CM reaches the ground.

So fragment 2 travels a distance d in a time ½t

$$t_2 = \frac{1}{2}t = \frac{V_0 \sin\theta}{g}, \quad d = V_2 t$$

$$\boxed{d = \frac{2V_0^2 \sin\theta\cos\theta}{g}}$$

Which is exactly R

$$\boxed{d = R}$$

The apex (where the explosion happens) is at ½R so measuring from the launch point

$$d_T = \frac{1}{2}R + d = \frac{1}{2}R + R = \boxed{\frac{3}{2}R}$$

$$\boxed{d_T = \frac{3}{2}\frac{2V_0^2\cos\theta\sin\theta}{g}}$$

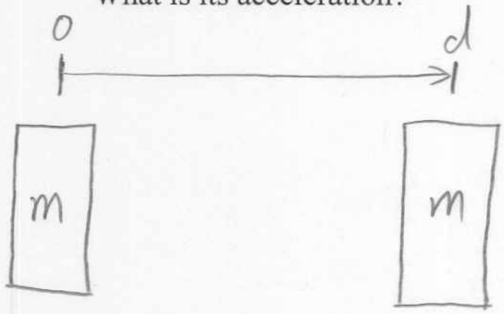
$$= \frac{(3)(80)^2\cos(60)\sin(60)}{9.8}$$

$$\boxed{= 848 \text{ m}}$$

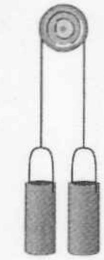
MOMENTUM, IMPULSE, AND COLLISIONS

Two identical containers of sugar are connected by a massless cord that passes over a massless frictionless pulley with a diameter of 50 mm. The two containers are at the same level. Each originally has a mass of 500 g.

a) locate the horizontal position of their center of mass.
b) If the containers are prevented from moving and 20 g of sugar is moved from one container to the other, where is the new center of mass?
c) If the containers are now released, in what direction does their center of mass move? What is its acceleration?



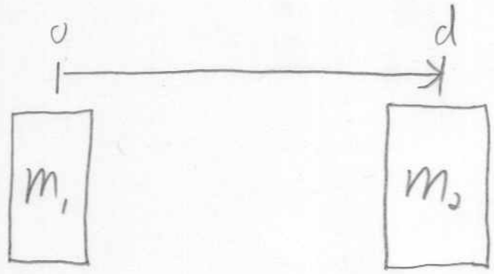
$d = 50\,mm$
$m = 500\,g$



a) $r_{cm} = \frac{m_1 r_1 + m_2 r_2}{m_1 + m_2}$, $r_1 = 0, r_2 = d$
$m_1 = m_2 = m$

$$r_{cm} = \frac{0 + md}{m + m} = \frac{\cancel{m}d}{2\cancel{m}} = \boxed{\frac{1}{2}d} = 25\,mm$$

b)



$m_1 = m - 20$, $m_2 = m + 20$

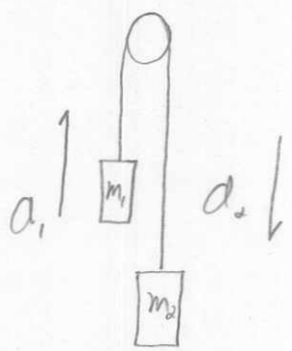
$r_1 = 0$, $r_2 = d$

$$r_{cm} = \frac{m_1 r_1 + m_2 r_2}{m_1 + m_2} = \frac{(m+20)d}{(m - \cancel{20} + m + \cancel{20})} = \frac{(m+20)d}{2m}$$

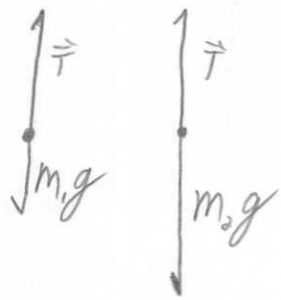
$$r_{cm} = d\left(\frac{1}{2} + \frac{10}{m}\right) = 50\,mm\left(\frac{1}{2} + \frac{1\cancel{0}}{50\cancel{0}}\right) = 50\,mm \cdot \frac{13}{25}$$

$$= \boxed{26\,mm}$$

c) Find acceleration of each mass



$$a_1 = -a_2 = a$$



$$T - m_1 g = m_1 a_1 = m_1 a$$
$$T - m_2 g = m_2 a_2 = -m_2 a$$

$$m_1 a + m_1 g = m_2 g - m_2 a$$
$$a = \frac{m_2 - m_1}{m_2 + m_1} g$$

Find a_{cm}

$$a_{cm} = \frac{m_1 a_1 + m_2 a_2}{m_1 + m_2} = \frac{m_1 a - m_2 a}{m_1 + m_2} = \boxed{\frac{m_1 - m_2}{m_1 + m_2} \cdot \frac{m_2 - m_1}{m_1 + m_2} g}$$

$$m_1 - m_2 = \cancel{m} - 20 - \cancel{m} - 20$$
$$= -40$$

$$m_2 - m_1 = \cancel{m} + 20 - \cancel{m} + 20$$
$$= 40$$

$$m_1 + m_2 = m \cancel{-20} + m \cancel{+20}$$
$$= 2m$$

$$\boxed{a_{cm} = \frac{-(40)^2}{(2 \cdot 500)^2} g = -1.6 \times 10^{-3} g}$$