Oscillation

A simple harmonic oscillator consists of a block of mass 2.00 kg attached to a spring of spring constant 100 N/m. When t = 1.00 s, the position and velocity of the block are x(1s) = 0.129 m and v(1s) = 3.415 m/s.

What is the frequency of the oscillator?

$$\omega = \sqrt{\frac{k}{m}} = \left(\frac{100}{2}\right)^{1/2} = 7.1 \ \text{rad/sec}$$

Use the **Boundary Condition** technique to find:

a) the phase constant (as a multiple of π)

b) the amplitude of the oscillations

c) the position of the block at t = 0.00 s?

Choose a solution: (Pick either SIN or cos)

$$x(t) = A\sin(\omega t + \phi)$$

$$v(t) = \omega A\cos(\omega t + \phi)$$

Apply the boundary conditions:

$$① \ x(1) = A\sin(\omega + \phi) = x_1$$

$$② \ v(1) = \omega A\cos(\omega + \phi) = v_1$$

a) solve for ϕ by dividing $\frac{①}{②}$

$$\frac{\cancel{A}\sin(\omega+\phi)}{\omega\cancel{A}\cos(\omega+\phi)} = \frac{x_1}{v_1} \Rightarrow \tan(\omega+\phi) = \frac{\omega x_1}{v_1}$$

$$\phi = \tan^{-1}\left(\frac{\omega x_1}{v_1}\right) - \omega = \tan^{-1}\left(\frac{(7.1)(0.129)}{3.415}\right) - 7.1 = \boxed{-6.838}$$

$$-6.8 = -2.2\pi = -0.2\pi = \boxed{1.8\pi} \qquad 2.177$$

more than 1 cycle

b) Plug ϕ back into ① to solve for A

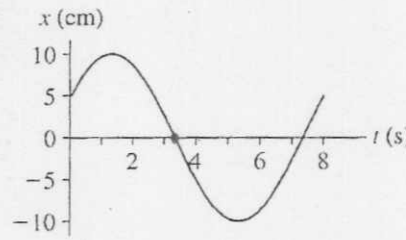
$$A\sin(\omega+\phi)=x_1 \Rightarrow A=\frac{x_1}{\sin(\omega+\phi)}$$

$$\boxed{\underline{A}=\frac{0.129}{\sin(7.1+1.82\pi)}=\underline{0.5\,m}}$$

c) $x(0)=A\sin(\phi) = 0.5\sin(1.82\pi)=\boxed{-.03}$

Oscillation

The figure below is a position -vs- time graph of a 2.5 kg particle in simple harmonic motion.



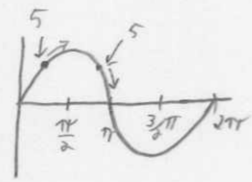
What is the amplitude of the oscillation?

10cm

What is the angular frequency?

$$\omega = \frac{2\pi}{P} = \frac{2\pi}{8} = \frac{\pi}{4}$$

What is the phase constant?



IF $x(t) = A\,SIN(\omega t + \phi)$

then: $x(0) = 10 \cdot SIN(\phi) = 5 \Rightarrow SIN(\phi) = \frac{1}{2}$

$$\phi = SIN^{-1}\left(\tfrac{1}{2}\right), \boxed{\phi = 0.17\pi}$$

What is v_{max}?

$$V_{max} = \omega A,\ \frac{\pi}{4} \cdot 10 = \boxed{\frac{5}{2}\pi}$$

What is v(t) in terms of the numbers you determined above?

$$V(t) = \frac{5}{2}\pi\,COS\left(\frac{\pi}{4}t + 0.17\pi\right)$$

What is v(0)?

$$V(0) = \frac{5}{2}\pi\,COS(0.17\pi) = 6.8\ cm/s$$

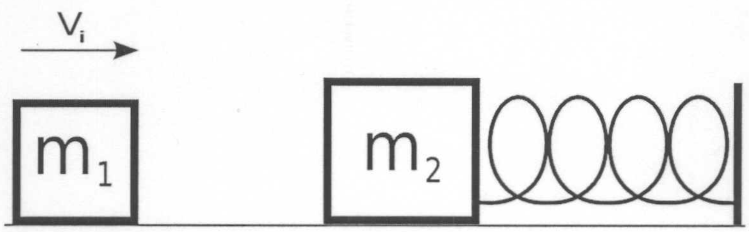
What is the total mechanical energy of the oscillator?

$$E_T = \frac{1}{2} m V_{max}^2 = \frac{1}{2}(2.5)\left(\frac{5}{4}\pi \frac{cm}{s} \cdot 1\times10^{-2}\frac{m}{cm}\right)^2$$

$$= 1.9\times10^{-3}\,J$$

Oscillation

A block with a mass of m_1 = 10 kg is moving to the right with a velocity V_i. It collides and sticks to a block with a mass of m_2 = 15 kg. The second mass is attached to a spring with spring constant k=3 N/m. Before the collision, the spring is at rest in it's equilibrium position.



a) What is the frequency, ν, of the resulting oscillator after the collision?
b) What is the phase constant?
c) If the amplitude is A = 3 m, what was the initial velocity of m_1?

a) After the collision, we have

$$F = (m_1 + m_2)a, \qquad F = -kx$$

$$\Rightarrow \quad -kx = (m_1 + m_2)\frac{d^2x}{dt^2}$$

$$\Rightarrow \quad \frac{d^2x}{dt^2} = -\frac{k}{m_1 + m_2}x \quad \Rightarrow \quad \boxed{\omega = \left(\frac{k}{m_1+m_2}\right)^{1/2}}$$

$$\omega = \left(\frac{3}{10+15}\right)^{1/2} = \boxed{\frac{\sqrt{3}}{5}}$$

b) Initial conditions: Let $t_0 = 0$

Just after the collision: $x(0) = 0$, $v(0) = v_F$

conserve momentum

$$P_I = P_F \Rightarrow m_1 v_i = (m_1 + m_2)v_F \Rightarrow \boxed{v_F = \frac{m_1}{m_1 + m_2}v_i}$$

continued ↓

In general,

$$x(t) = A\sin(\omega t + \phi) \Rightarrow x(0) = A\sin(\phi)$$

$$v(t) = \omega A\cos(\omega t + \phi) \Rightarrow v(0) = \omega A\cos(\phi)$$

so:

$$0 = A\sin(\phi) \Rightarrow \sin(\phi) = 0 \Rightarrow \boxed{\phi = 0 \text{ or } \pi}$$

$$v_F = \omega A\cos(\phi) \Rightarrow A = \frac{v_F}{\omega\cos(\phi)}$$

want this to be positive so let $\boxed{\phi = 0}$, then $\cos(\phi) = 1$

c) $$A = \frac{v_F}{\omega\cos(\phi)} = \frac{m_1 v_i}{m_1 + m_2} \cdot \frac{5}{\sqrt{3}} = \frac{10}{\cancel{25}\,5} \cdot \frac{5}{\sqrt{3}} v_i = \boxed{\frac{2}{\sqrt{3}} v_i}$$

And apparently, I didn't provide an initial velocity so... There you go.