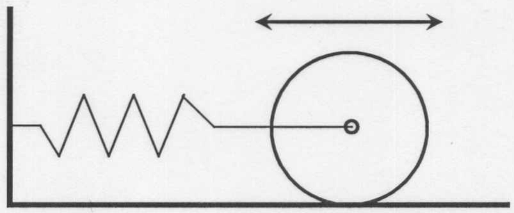
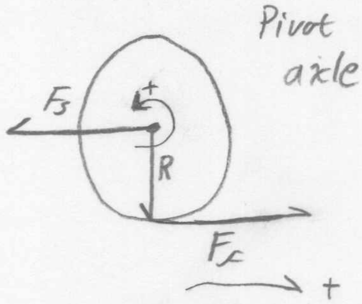
Oscillation

A solid cylinder of mass *M* is attached to a horizontal spring with force constant *k*. The cylinder can roll without slipping along the horizontal plane. When the system is displaced from the equilibrium position, it executes simple harmonic motion. Derive an expression for the period of the oscillations in terms of *M, k,* and

Let $I_{cm} = \frac{1}{2}MR^2$



Solution 1



Pivot at axle

Torque

$F_f \times R = I\alpha$

$+F_f \cancel{R} = \frac{1}{2}M R^{\cancel{2}} \frac{a}{\cancel{R}}$

$+F_f = \frac{1}{2}Ma$

Force

$F_s - F_f = Ma$

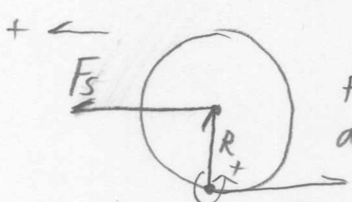
$-kx - F_f = Ma$

$-kx - \frac{1}{2}Ma = Ma$

$\boxed{-kx = \frac{3}{2}Ma}$

$\boxed{\frac{d^2x}{dt^2} = -\frac{2}{3}\frac{k}{m}x} \Rightarrow \boxed{\omega = \left(\frac{2}{3}\frac{k}{m}\right)^{1/2}}$ *

Solution 2



Pivot at ground

Torque

$F_s R = I\alpha \quad , \quad F_s = -kx$

$-kxR = I\alpha \quad , \quad I = I_{cm} + Md^2$

$I = \frac{1}{2}MR^2 + MR^2 = \frac{3}{2}MR^2$

$-kx\cancel{R} = \frac{3}{2}MR^{\cancel{2}}\frac{a}{\cancel{R}} \Rightarrow \boxed{\frac{d^2x}{dt^2} = -\frac{2}{3}\frac{k}{m}x}$

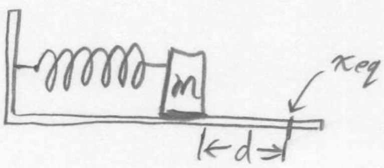
$\omega = \left(\frac{2}{3}\frac{k}{m}\right)^{1/2}$

Oscillation

A block of mass m_b = 5.0 kg is attached to a spring of spring constant k = 4 N/m where it is allowed to oscillate horizontally on a frictionless surface. The spring is compressed a distance d = 0.5 m from equilibrium and released. After ~~4~~ 5 seconds, a wad of clay, m_c = 3.0 kg, falls from directly above and sticks to the block.

a) What is the period of oscillation of the block/clay system?
b) What is the amplitude of the block/clay system?

Before collision



$$\left.\begin{aligned} x(0) &= -d \\ v(0) &= 0 \end{aligned}\right] \rightarrow \text{Initial conditions}$$

$$F = m_b a$$

$$-kx = m_b \frac{d^2x}{dt^2} \Rightarrow \frac{d^2x}{dt^2} = -\frac{k}{m}x, \quad \omega = \sqrt{\frac{k}{m}}$$

generally, $x(t) = A\,\mathrm{SIN}(\omega t + \phi)$

$$v(t) = \omega A \cos(\omega t + \phi)$$

at $t=0$

$$-d = A\,\mathrm{SIN}(\phi)$$

$$0 = \omega A\,\mathrm{COS}(\phi) \Rightarrow \phi = \pi/2,\ 3\pi/2$$

$$A = \frac{-d}{\mathrm{SIN}(3\pi/2)} = d \qquad \mathrm{SIN}\left(\frac{3\pi}{2}\right) = -1$$

continued ↓

SO: $x(t) = d\,\sin\left(\sqrt{\frac{k}{m}}\,t + \frac{3\pi}{2}\right)$

$v(t) = \sqrt{\frac{k}{m}}\,d\cos\left(\sqrt{\frac{k}{m}}\,t + \frac{3\pi}{2}\right)$

After 5 seconds,

$$x(5) = 0.5\,\sin\left(\left(\frac{4}{5}\right)^{1/2} 5 + \frac{3\pi}{2}\right) = 3.27\times10^{-1}\ m$$

$$v(5) = 0.5\left(\frac{4}{5}\right)^{1/2}\left(\left(\frac{4}{5}\right)^{1/2}\cdot 5 + \frac{3\pi}{2}\right) = -4.34\times10^{-1}\ m/s$$

collide and conserve momentum in x

$$m_B v_5 = (m_B + m_c)\,v_0 \Rightarrow$$

$$v_0 = \frac{m_B}{m_B + m_c}\,v_5 = \frac{-5}{15}\,4.34\times10^{-1} = -.145\ m/s$$

Initial conditions for post-collision oscillator:

$$\boxed{\begin{aligned} x_0 &= 3.27\times10^{-1}\ m \\ v_0 &= -.145\ m/s \end{aligned}}$$

a) New oscillator (Post - collision)

$$\omega_1 = \left(\frac{k}{m_B + m_c}\right)^{1/2} = \boxed{.516} \Rightarrow P = 2\pi/\omega = 12.2\ \text{seconds}$$

b) New oscillator starts at $t = 0$

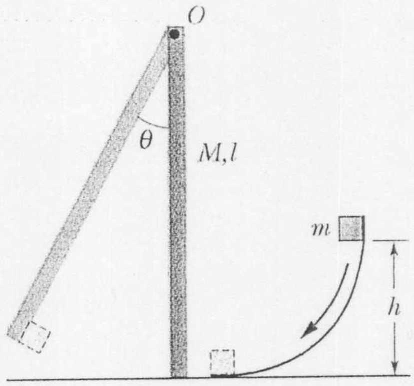
$$\begin{aligned} x_0 &= A_1 \sin(\phi_1) \\ v_0 &= \omega_1 A_1 \cos(\phi_1) \end{aligned} \Rightarrow \phi_1 = \tan^{-1}\left(\omega_1 \frac{x_0}{v_0}\right) = -0.861$$

$$A_1 = \frac{x_0}{\sin(\phi_1)} = -0.431$$

Oscillation

A particle of mass m slides down a frictionless surface, collides with a uniform vertical rod of mass M and length l, and sticks. Let m = M.

The moment of inertia of a rod about its center of mass is $I = 1/12\ Ml^2$



a) Find the moment of inertia of the Rod about its end.

$$I_R = I_{cm} + M\left(\frac{l}{2}\right)^2 = \frac{1}{12}Ml^2 + \frac{1}{4}Ml^2 = \frac{1}{3}Ml^2$$

b) Find the moment of inertia of the Rod/Block combo

$$I_T = I_R + I_B = \frac{1}{3}Ml^2 + Ml^2 = \frac{4}{3}Ml^2$$

c) Find the center of mass of the Rod/Block combo

$$l_{cm} = \frac{1}{M_T}\sum r_i m_i = \frac{1}{M_R + M_B}\left(lM_B + \frac{l}{2}M_R\right)$$

$$= \frac{1}{2\cancel{M}}\cdot\frac{3}{2}l\cancel{M} = \boxed{\frac{3}{4}l}$$

d) Find the frequency of the resulting pendulum.

$$\vec{l}_{cm} \times Mg = I\alpha \Rightarrow l_{cm}Mg\,SIN(-\theta) = I\alpha$$

$$\alpha = -\left(\frac{l_{cm}Mg}{I}\right)\theta \Rightarrow \omega = \left(\frac{\frac{3}{4}\cancel{l}\cancel{M}g}{\frac{4}{3}\cancel{M}l^{\cancel{2}}}\right)^{1/2} = \left(\frac{9}{16}\frac{g}{l}\right)^{1/2}$$

e) Find the amplitude of the resulting pendulum in terms of θ.

① Find ω_F post inelastic collision:

$$M_B(\vec{l} \times \vec{v}) = I_T\omega_F, \qquad \cancel{M_B}gh = \frac{1}{2}\cancel{M_B}v^2 \Rightarrow \underline{v = \sqrt{2gh}}$$

$$+\cancel{l}\cancel{M}(2gh)^{1/2} = \frac{4}{3}\cancel{M}l^{\cancel{2}}\omega_F$$

$$\boxed{\omega_F = \frac{3}{4}\frac{\sqrt{2gh}}{l}}$$

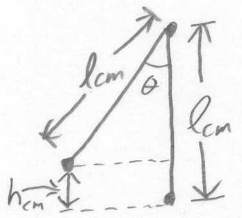
2) Find h_{cm} using <u>energy</u>

<u>Note</u>: We <u>can't</u> use the kinematics equations because α is <u>not</u> constant.

$$\frac{1}{2} I_f \omega_f^2 = 2Mgh_{cm} \Rightarrow \cancel{\frac{1}{2}}\,\cancel{\frac{4}{3}}\,\cancel{M}\cancel{l^2}\,\frac{\overset{3}{\cancel{9}}}{\underset{4}{\cancel{16}}}\,\frac{\cancel{2g}h}{\cancel{l^2}} = 2\cancel{M}\cancel{g}h_{cm}$$

$$\boxed{h_{cm} = \frac{3}{8}h}$$

3) Find θ



$$h_{cm} = l_{cm} - l_{cm}(\cos\theta)$$

$$= l_{cm}(1 - \cos\theta)$$

$$\frac{\cancel{3}}{\underset{2}{\cancel{8}}}h = \frac{\cancel{3}}{\cancel{4}}l(1 - \cos\theta)$$

$$\frac{1}{2}\frac{h}{l} = 1 - \cos\theta$$

$$\boxed{\cos\theta = 1 - \frac{h}{2l}}$$