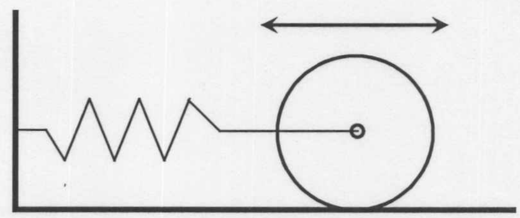
Oscillation

A solid cylinder of mass M is attached to a horizontal spring with force constant k. The cylinder can roll without slipping along the horizontal plane. When the system is displaced from the equilibrium position, it executes simple harmonic motion. Derive an expression for the period of the oscillations in terms of M and k

Use Energy and compare your answer to what you got using Force/Torque.



$$E_T = \frac{1}{2}kx^2 + \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

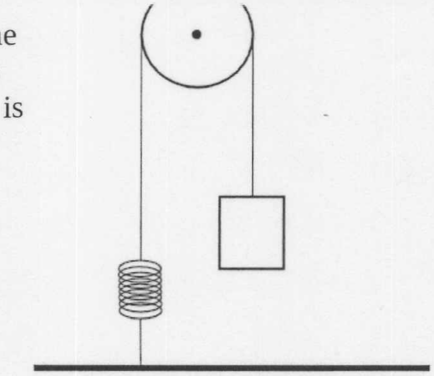
$$\frac{d}{dt}E_T = kxv + mva + I\omega\alpha = 0, \quad \alpha = \frac{a}{R}, \; I = \frac{1}{2}mR^2, \; \omega = \frac{v}{R}$$

$$kx\cancel{v} + m\cancel{v}a + \frac{1}{2}m\cancel{R^2}\frac{\cancel{v}}{\cancel{R}}\frac{a}{\cancel{R}} = 0$$

$$kx + \frac{3}{2}ma = 0$$

$$\frac{d^2x}{dt^2} = -\frac{2}{3}\frac{k}{m}x \Rightarrow \boxed{\omega = \sqrt{\frac{2}{3}\frac{k}{m}}}$$

A block of mass M is attached to one end of a massless rope that runs over a pulley of radius R and mass M. The other end of the rope is attached to the top of a massless spring with spring constant k. The bottom of the spring is fixed to the floor.



Find the frequency of this oscillator using **Energy techniques**.
Compare your expression to the one you derived using Torque and Force.

$$E_T = \frac{1}{2}k(y - y_{eq})^2 + \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2 + mgy$$

$$\frac{d}{dt}E_T = k(y - \cancel{y_{eq}})v + I\omega\alpha + mva + \cancel{mgv} = 0$$

$$\boxed{k y_{eq} = mg}$$

$$\omega = \frac{v}{R}, \quad \alpha = \frac{a}{R}$$

$$ky\cancel{v} + \frac{1}{2}M\cancel{R^2}\frac{\cancel{v}}{\cancel{R}}\frac{a}{\cancel{R}} + m\cancel{v}a = 0$$

$$ky + \left(\frac{1}{2}M + m\right)a = 0$$

$$\frac{d^2y}{dt^2} = -\frac{k}{\frac{1}{2}M + m}y \Rightarrow \boxed{\omega = \left(\frac{k}{\frac{1}{2}M + m}\right)^{1/2}}$$

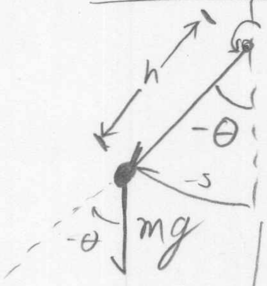
Oscillation

A meter stick of length L is suspended and allowed to swing about a point located a distance h from it's center of mass.

a) Find an expression for it's period of oscillation, T, in terms of L, g, h, and π.
b) What distance h_{max} will provide the minimum period of oscillation?

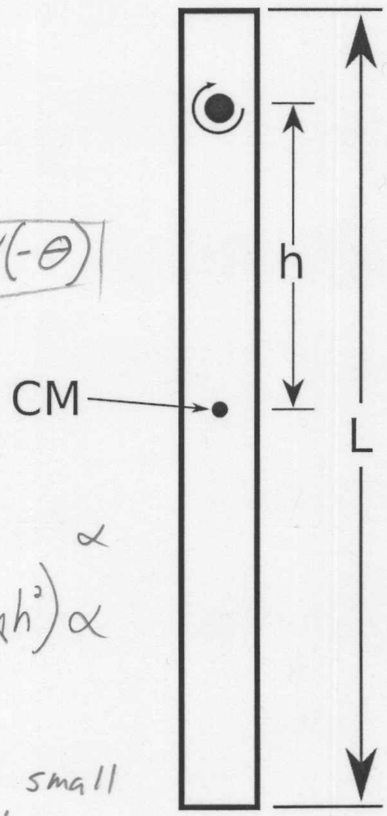
HINT: Find the minimum of T^2 (which will be at the same h as the minimum of T) to simplify the derivative.

$T = I\alpha$



$T = r \times F$
$= h \times mg$

$$\boxed{T = mgh\,\mathrm{SIN}(-\theta)}$$



$$\boxed{I = \tfrac{1}{12}mL^2 + mh^2}$$

so: $T = I \alpha$

$$-\cancel{m}gh\,\mathrm{SIN}\theta = \left(\tfrac{1}{12}\cancel{m}L^2 + \cancel{m}h^2\right)\alpha$$

$$\Rightarrow \alpha = -\frac{gh}{\frac{1}{12}L^2 + h^2}\theta,$$ for small angles, $\mathrm{SIN}\theta \cong \theta$

$$\boxed{\frac{d^2\theta}{dt^2} = -\frac{gh}{\frac{1}{12}L^2 + h^2}\theta}$$

so: $$\omega = \left(\frac{gh}{\frac{1}{12}L^2 + h^2}\right)^{1/2}$$

$$\boxed{T = \frac{2\pi}{\omega} = 2\pi\left(\frac{\frac{1}{12}L^2 + h^2}{gh}\right)^{1/2}}$$

continued ↓

If:

$$T = 2\pi\left(\frac{\frac{1}{12}L^2 + h^2}{gh}\right)^{1/2}$$

We can find the minimum by differentiating.
We'll differentiate T^2, because it's easier

$$\frac{dT^2}{dh} = \frac{d}{dh} 2\pi\left(\frac{\frac{1}{12}L^2 + h^2}{gh}\right) = 2\pi\left[-\frac{1}{12}\frac{L^2}{gh^2} + \frac{1}{g}\right]$$

$*$ min/max when $\frac{dT^2}{dh} = 0$

$$2\pi\left[-\frac{1}{12}\frac{L^2}{gh_{min}^2} + \frac{1}{g}\right] = 0$$

$$\frac{1}{12}\frac{L^2}{\cancel{g}h_{min}^2} = \frac{1}{\cancel{g}}$$

$$\boxed{h_{min} = \left[\frac{L^2}{12}\right]^{1/2} = \frac{L}{\sqrt{12}}}$$