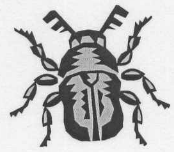
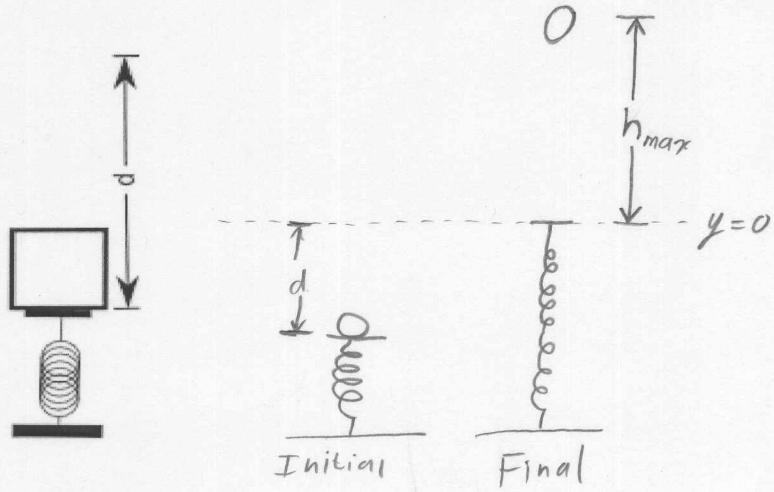
Plug The Bug Lab – The Physics

Problem: We are starting an exterminating service that removes pests using non-explosive mortar fire. In order to acquire capital for our new business, we need to derive and verify the physics of bug mortars in the laboratory.



1. The mortar consists of a spring and a steel ball bearing. Our first task is to determine the spring constant of the spring. Consider setting the mortar so that it points straight up, as in the picture below. A mass (the steel ball) is placed in on the mortar and the spring compressed a distance d. Derive an expression for the spring constant in terms of *g*, *m* (the mass of the ball), *d* (the spring compression), and h_{max} (the height of the ball above the spring equilibrium point).



$U_I = \frac{1}{2}kd^2 - mgd$ $\quad$ $U_F = mgh_{max}$

$K_I = 0$ $\quad$ $K_F = 0$

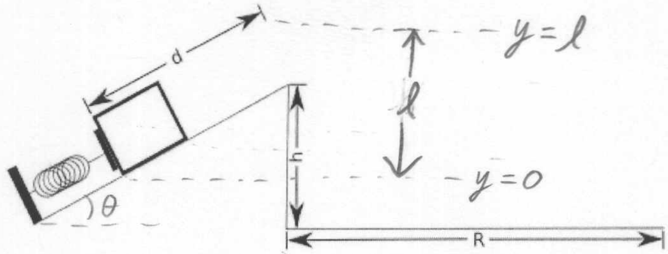
$$\frac{1}{2}kd^2 - mgd = mgh_{max}$$

$$\frac{1}{2}kd^2 = mg(d+h)$$

$$\boxed{k = \frac{2mg(d+h)}{d^2}}$$

Plug The Bug Lab – The Physics

2. Consider the picture below. Now the mortar makes an angle, θ, with the horizontal. Using the spring constant that you derived in part 1, derive an expression for the velocity of the mass at the top of the ramp in terms of g, ~~k~~ h_{max}, d, and θ.



$$U_I = \tfrac{1}{2}kd^2 \qquad U_F = mgd\sin\theta$$

$$K_I = 0 \qquad K_F = \tfrac{1}{2}mV_F^2$$

$$\tfrac{1}{2}kd^2 = mgd\sin\theta - \tfrac{1}{2}mV_F^2$$

$$\tfrac{1}{2}mV_F^2 = mgd\sin\theta - \tfrac{1}{2}kd^2$$

$$\boxed{V_F^2 = 2gd\sin\theta - \frac{kd^2}{m}}$$

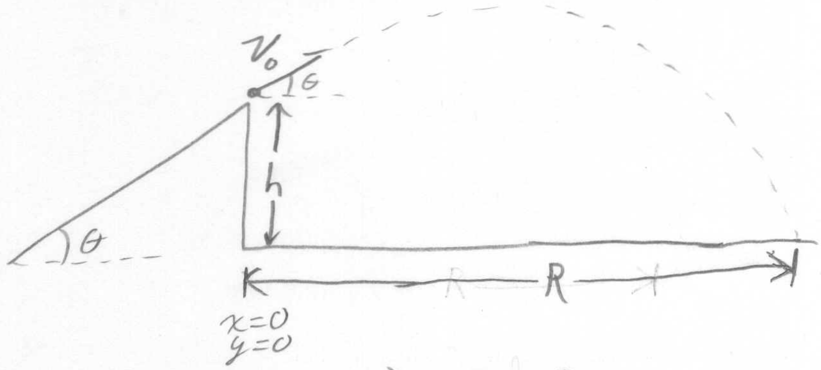
$$V_F^2 = 2gd\sin\theta - \frac{2\cancel{m}g(d+h_{max})}{\cancel{d^2}}\frac{\cancel{d^2}}{\cancel{m}}$$

$$V_F^2 = 2gd\sin\theta - 2g(d+h_{max})$$

$$\boxed{V_F = \left[2g(d\sin\theta - d - h_{max})\right]^{1/2}}$$

Plug The Bug Lab – The Physics

3. Finally, we have to solve the classic trajectory problem. Assuming that you've calculated the velocity at the top of the ramp, derive an expression for the the range R of ball in terms of V_0, g, h, and θ. We can't use Work Energy to solve this problem, so we have to resort Kinematics.



$$mg = ma \Rightarrow a = g$$

$$x_F = \cancelto{0}{x_0} + V_{0x}t + \cancelto{0}{\tfrac{1}{2}a_x t^2}$$

$$\cancelto{0}{y_F} = y_0 + V_{0y}t + \tfrac{1}{2}a_y t^2$$

$$R = V_0 \cos\theta \, t$$

$$0 = h + V_0 \sin\theta \, t - \tfrac{1}{2}g t^2$$

Positive root

$$t = \frac{+V_0 \sin\theta \pm \left(V_0^2 \sin^2\theta + 2gh\right)^{1/2}}{+g}$$

$$\boxed{R = \frac{1}{g} V_0 \cos\theta \left[V_0 \sin\theta + \left(V_0^2 \sin^2\theta + 2gh\right)^{1/2}\right]}$$