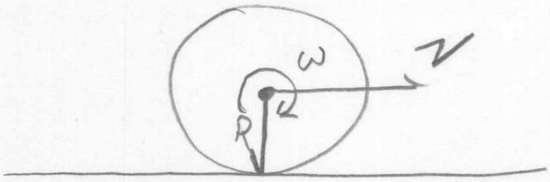
Rotation, moment of inertia

My car came from the factory with tires with a diameter of $d_1 = 60$ cm. They wore out and I replaced them with tires with a diameter of $d_2 = 80$ cm.

a) If the speed limit is 65 mph, what should my speedometer read so that I don't get a speeding ticket?

b) If I've added 10,000 miles to the odometer since putting the new tires on, how many miles have I actually driven the car?

a)



So, we know $\underline{v = R\omega}$ in general.

The speedometer is calibrated to read the correct speed when $R = \frac{1}{2}d_1$.

It measures the angular velocity of the axle and performs this calculation:

Reading on Speedometer $\rightarrow$ $v_s = \frac{1}{2}d_1\omega$

If ~~I~~ I put the wrong tire on, I'll measure an angular velocity:

$$\omega = \frac{v_R}{\frac{1}{2}d_2}$$ ← Speed a radar gun would measure

Feed that ω back into the speedometer eq:

$$v_s = \cancel{\tfrac{1}{2}}d_1 \frac{v_R}{\cancel{\tfrac{1}{2}}d_2} \Rightarrow \boxed{v_s = \frac{d_1}{d_2}v_R}$$

Continued ↓

So, if I have tire d_2 on and I want the Radar gun to read 65:

$$v_s = \frac{60}{80} 65 = \boxed{49 \ mi/hr}$$

b) For the odometer we make a similar argument.

$$l_0 = \tfrac{1}{2} d_1 \theta \quad , \quad \theta = \frac{l_R}{\tfrac{1}{2} d_2}$$

l_R ← Actual distance traveled

↑ distance read by odometer (l_0)

$$\boxed{l_0 = \frac{\cancel{\tfrac{1}{2}} d_1}{\cancel{\tfrac{1}{2}} d_2} l_R} \Rightarrow l_R = \frac{d_2}{d_1} l_0$$

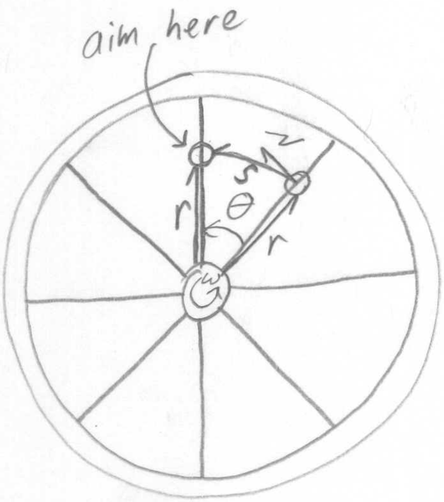
$$l_R = \frac{80}{60} 1 \times 10^4$$

$$\boxed{l_R = 1.33 \times 10^4}$$

Rotation, moment of inertia

A wagon wheel with a radius of R=30cm with 8 equally spaced spokes is spinning with an angular velocity of ω=2.5 revolutions/sec. You want to shoot a 20 cm arrow parallel to the wheel's axle between the spokes without hitting one. Assuming that the spokes and the arrow are very thin:

a) What minimum speed must the arrow have?
b) Does it matter where between the axle and the rim you aim? If so, where is the best location?



Circle is 2π radians.
There are 8 equal wedges at: $\boxed{\theta = \frac{2\pi}{8} = \frac{\pi}{4}}$ each

Let's consider that we aim the arrow a distance r above the axle. If it just misses a spoke, It has to be all the way through before the next spoke gets there.

The spoke is moving at $v = r\omega$
and has to travel a distance $s = r\theta$

$$s = vt \Rightarrow \cancel{r}\theta = \cancel{r}\omega t$$

$\underline{\theta = \omega t}$ $\Rightarrow$ time is independant of r.

continued ↓

So: $t = \frac{\theta}{\omega}$ So, the arrow travels it's length in $l = vt = v\frac{\theta}{\omega}$

So it has to go

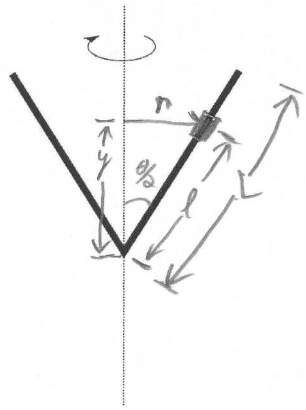
$$\boxed{v = l\frac{\omega}{\theta}}$$

$$v = 20\,cm \cdot \frac{2.5\,\cancel{rev}/s \cdot 2\cancel{\pi}\,\cancel{rad}/\cancel{rev}}{\cancel{\pi}/4\,\cancel{rad}}$$

$$v = (2)(2.5)(20)\,cm/s = 100\,cm/s$$

Rotation, moment of inertia

Calculate the moment of inertia of the bent rod of mass **M** shown in the figure below. The rotation axis is in the plane of the "V" bisecting it at the vertex. The rod is bent at an angle **θ** and each leg has a length **L**.



$$I = \int r^2 \lambda \, dl$$

$$\lambda = \frac{M}{2L}, \quad r = l \underbrace{\sin\left(\frac{\theta}{2}\right)}_{\rightarrow \text{constant}}$$

$$dr = dl \sin\left(\frac{\theta}{2}\right)$$

$$I = \cancel{2}\int_0^L l^2 \sin^2\left(\frac{\theta}{2}\right) \frac{M}{\cancel{2}L} \sin\left(\frac{\theta}{2}\right) dl$$

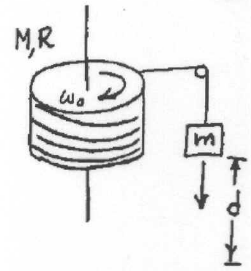
$$= \sin^3\left(\frac{\theta}{2}\right) \frac{M}{\cancel{L}} \frac{1}{3} L^{\cancel{3}2}$$

$$\boxed{I = \frac{1}{3} \sin^3\left(\frac{\theta}{2}\right) M L^2}$$

Rotation, moment of inertia

$I = \frac{1}{2}MR^2$

Consider a solid cylinder of mass *M* and radius *R* that rotates without friction about its vertical axis of symmetry. A massless string is wrapped around the cylinder, passes over a small massless frictionless pulley and is attached to a small mass *m*. The string does not slip on the disk and no energy is lost to friction.



If the mass and disk start from rest, find an expression for d in terms of ω_f, M, m, and R.

HINT: USE CONSERVATION OF ENERGY to solve this problem. What is the initial Potential Energy? What is the final Kinetic Energy of the cylinder? What is the final Kinetic Energy of the small mass? How is the velocity of the small mass related to the angular velocity of the cylinder?

$U_I = mgd \qquad K_I = 0$

$U_F = 0 \qquad K_F = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

$$mgd = \frac{1}{2}mv^2 + \frac{1}{2}\frac{1}{2}MR^2\omega^2$$

$$v = R\omega$$

$$mgd = \frac{1}{2}mR^2\omega^2 + \frac{1}{4}MR^2\omega^2$$

$$\boxed{d = \frac{1}{2}R^2\omega^2\left(m + \frac{1}{2}M\right)}$$