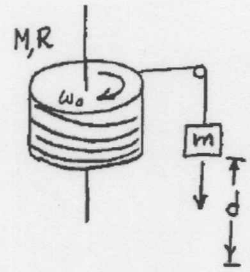
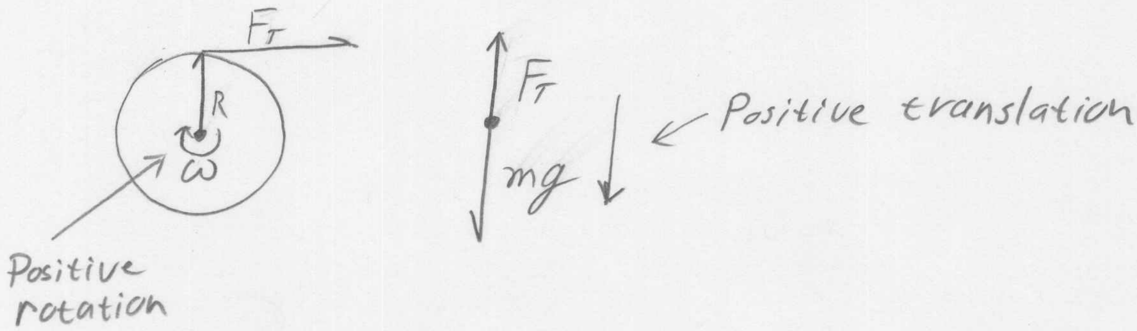
Rotation, moment of inertia

Consider a solid disk of mass M and radius R that rotates without friction about its vertical axis of symmetry. A massless string wrapped around the periphery of the disk passes over a small, massless, frictionless pulley and hangs down vertically, attached to a small mass m. If the mass and disk start from rest, What is ω after m drops a distance d? No slipping of the string on the disk occurs, and no energy is lost to friction.



USE TORQUE AND KINIMATICS to solve this problem.



$$\tau = I\alpha$$

$$F_T \not{R} = \tfrac{1}{2} M R^{\not{2}} \alpha$$

$$F_T = \tfrac{1}{2} M \not{R} \frac{a}{\not{R}}$$

$$F_T = \tfrac{1}{2} M a$$

$$F = ma$$

$$mg - F_T = ma$$

$$mg - \tfrac{1}{2} M a = ma$$

$$mg = (\tfrac{1}{2} M + m) a$$

$$\boxed{a = \frac{m}{\tfrac{1}{2} M + m} g}$$

continued

↓

Now, use kinematics to find v of ~~of~~ Block

$$y = y_0^{\,0} + v_0^{\,0} t + \frac{1}{2}at^2 \qquad v = v_0^{\,0} + at$$

$$d = \frac{1}{2}at^2$$

$$t = \left(\frac{2d}{a}\right)^{1/2} \longrightarrow v = at$$

$$v = a\left(\frac{2d}{a}\right)^{1/2}$$

$$v = (2da)^{1/2}$$

$$\boxed{v = \left(2d\,\frac{m}{\frac{1}{2}M + m}\,g\right)^{1/2}}$$

Oh wait... we wanted ω, not v

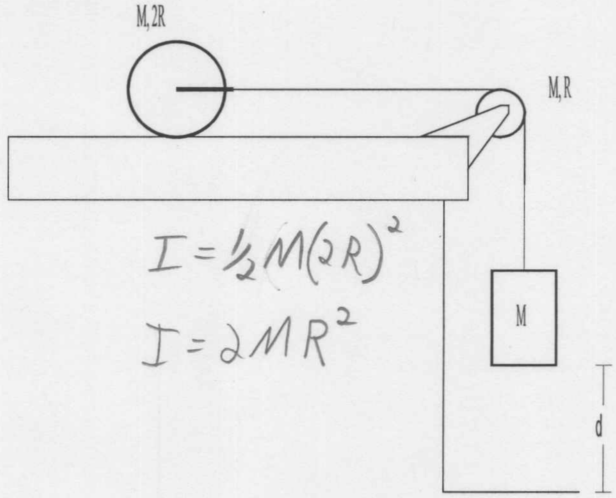
$$\omega = \frac{v}{R} \Rightarrow v = \omega R$$

$$\boxed{\omega = \frac{1}{R}\left(2d\,\frac{m}{\frac{1}{2}M + m}\,g\right)^{1/2}}$$

Rotation, moment of inertia

A solid cylinder (radius = 2R, mass = M) rolls without slipping as it is pulled by a massless yoke attached to a string. The string goes over a frictionless pulley shaped as a solid disk (radius = R, mass = M) and is attached to a mass (m = M). What is the velocity of the system after the hanging mass has fallen a distance d?

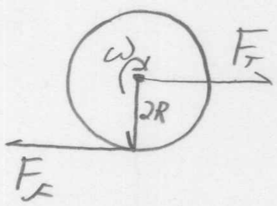
USE TORQUE AND KINIMATICS to solve this problem.



Skipping the second pulley

$I = \frac{1}{2}M(2R)^2$

$I = 2MR^2$



$\tau_{net} = I\alpha$

$F_f 2R = 2MR^2\alpha$

$F_f = MR\alpha = MR\frac{a}{2R}$

$F_f = \frac{1}{2}Ma$

$\vec{F}_{net} = Ma$

$F_T - F_f = Ma$

$F_T - \frac{1}{2}Ma = Ma$

$F_T = \frac{3}{2}Ma$

$F_{net} = Ma$

$mg - F_T = Ma$

$Mg - \frac{3}{2}Ma = Ma$

$g = \frac{5}{2}a$

$a = \frac{2}{5}g$

continued ↓

(c)

Kinematics

$$y = \cancelto{0}{y_0} + \cancelto{0}{v_0 t} + \tfrac{1}{2}at^2 \qquad v = \cancelto{0}{v_0} + at$$

$$d = \tfrac{1}{2}at^2$$

$$t = \left(\frac{2d}{a}\right)^{1/2} \longrightarrow v = a\left(\frac{2d}{a}\right)^{1/2}$$

$$v = (2da)^{1/2}$$

$$v = \left(2d\,\tfrac{2}{5}g\right)^{1/2}$$

$$\boxed{v = \left(\tfrac{4}{5}dg\right)^{1/2}}$$