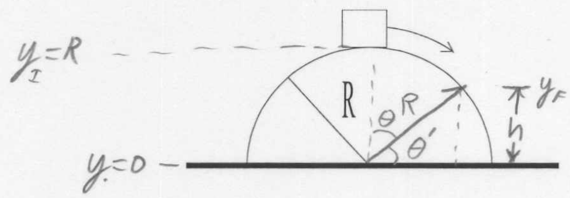
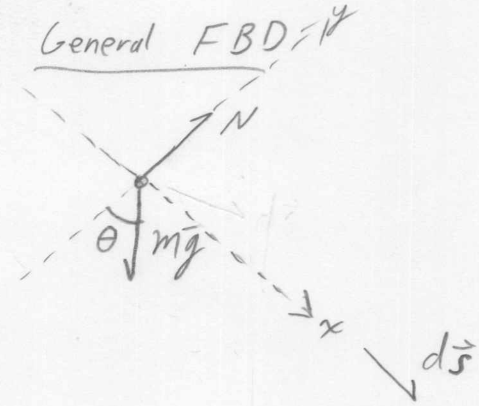
Sample Test 3
Phys 111, Fall 2009, Section 1

2. *Use work-energy techniques to solve the following problem.*

A block is seated on the top of a hemispherical mound of ice. The block is given a *slight* push (assume $V_0 = 0$) and starts sliding down the ice. If the ice is frictionless, show that the block will lose contact with the ice at a height of $\frac{2}{3}R$, where R is the radius of the ice mound.





$h = R\,SIN\,\theta'$

$\theta' = 90 - \theta \Rightarrow \boxed{h = R\cos\theta}$

according to conservation of Energy:

$U_I = mgR \qquad U_F = mgR\cos\theta$

$K_I = 0 \qquad K_F = \frac{1}{2}mV^2$

We need to work out this V. We have uniform circular motion, so let's write Newton's 2^{nd} law.

$y:\ N - mg\cos\theta = ma_c\,, \quad a_c = +\frac{V^2}{R}$ ← This is the V^2...

lose contact when $N \to 0$

So: $-\cancel{m}g\cos\theta = +\cancel{m}\frac{V^2}{R}$

$V^2 = -Rg\cos\theta$

continued ↓

conserve

$$\cancel{mg}R = \cancel{mg}R\cos\theta + \tfrac{1}{2}\cancel{m}R\cancel{g}\cos\theta$$

$$1 = (1 + \tfrac{1}{2})\cos\theta \Rightarrow \boxed{\cos\theta = \tfrac{2}{3}}$$

and since $h = R\cos\theta$

$$\Rightarrow \boxed{h = \tfrac{2}{3}R}$$

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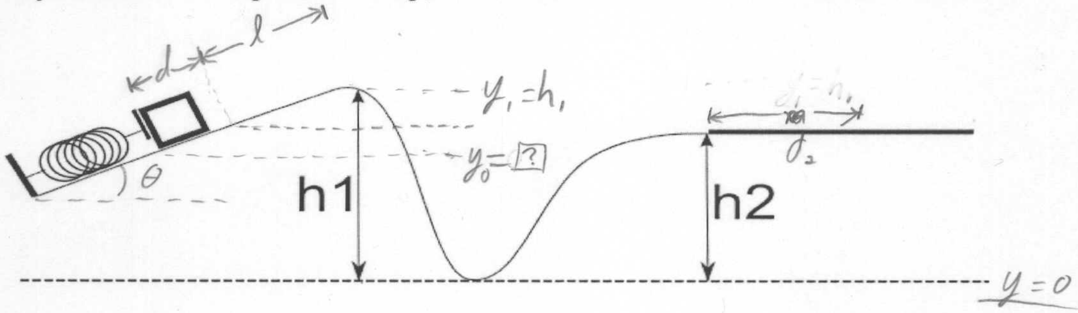
3 20pts)

In the system below, the spring is compressed and a 3 kg block resting against it is released from rest. The surface is frictionless except for the dark horizontal surface at the end where the coefficient of friction is $\mu_k = 0.15$. The distance between the top of the *uncompressed* spring and the top of the ramp is 2 m. The spring constant is k = 200 N/m. The ramp makes a 30 degree angle with the horizontal, h1 = 3 m, and h2 = 2 m.

a) How much does the spring have to be compressed for the block to **just** clear the top of the ramp.

b) If the block **just** clears the top of the ramp, how far across the frictional surface will it slide.

$m = 3kg$
$\mu_k = 0.15$
$l = 2.0\,m$
$k = 200\,N/m$
$\theta = 30°$
$h_1 = 3m$
$h_2 = 2m$



a) $U_I = mg(h_1 - (d+l)\sin\theta) + \frac{1}{2}kd^2$ $\qquad U_F = mgh_1$

$K_I = 0$ $\qquad K_F = 0$

$$mg(h_1 - (d+l)\sin\theta) + \tfrac{1}{2}kd^2 = mgh_1$$

$$\cancel{mgh_1} + \underbrace{mgd\sin\theta}_{b} + \underbrace{mgl\sin\theta}_{c} \mp \underbrace{\tfrac{1}{2}k}_{a}d^2 = \cancel{mgh_1}$$

$$d = \frac{1}{k}\left[mg\sin\theta \pm \left((mg\sin\theta)^2 + 2kmgl\sin\theta\right)^{1/2}\right]$$

$$d = \frac{1}{200}\left[(3)(9.8)\sin 30 + \left[((3)(9.8)\sin 30)^2 + (2)(200)(3)(9.8)(2)\sin 30\right]^{1/2}\right]$$

$\boxed{d = 0.6}$

b) $U_I = mgh_1$ $U_F = mgh_2$ $W_F = \int_0^{d_2} \vec{F}_k \cdot d\vec{s} = -\mu_k mgd_2$

$K_I = 0$ $K_F = 0$

$$\cancel{mg}h_1 - \mu_k \cancel{mg}d_2 = \cancel{mg}h_2$$

$$\boxed{d_2 = \frac{h_1 - h_2}{\mu_k}} = \frac{3-2}{0.15} = \underline{6.7m}$$

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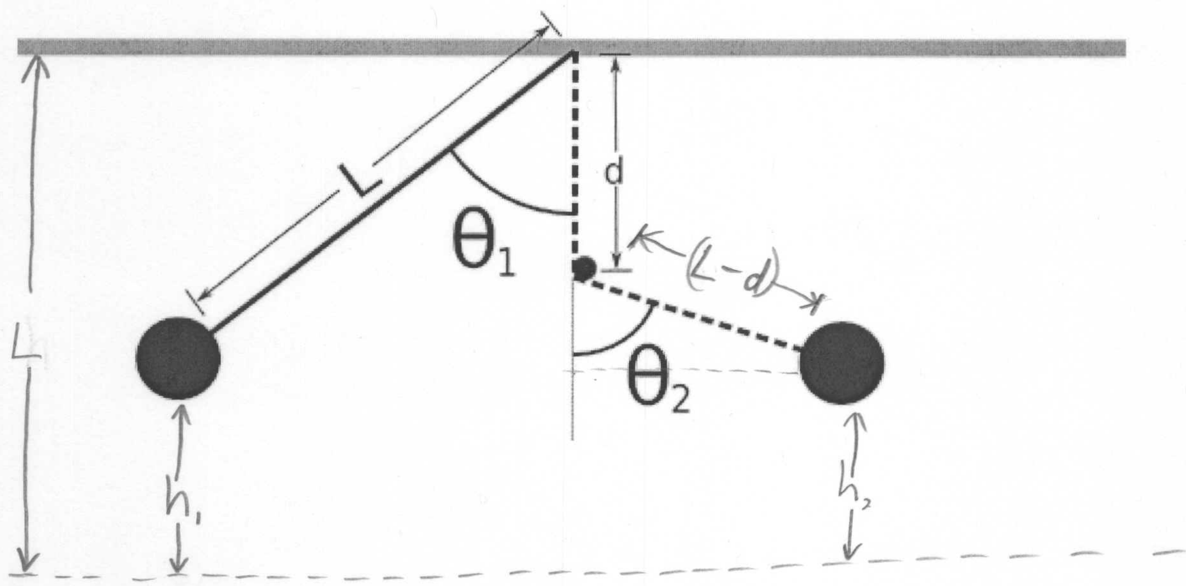
4. *Use work-energy techniques to solve the following problem.*

A pendulum consisting of a string of length L and a sphere of mass m swings in a vertical plane. The string hits a peg a distance d below the suspension point, as shown in the picture below.

a. Use work/energy to find θ_2 in terms of θ_1, L, and d.

b. If $\theta_1 = 90°$, find the minimum value of d (in terms of L) so that the pendulum will make a full circle without the string going slack.



$U_I = mgh_1$

$K_I = 0$

$U_F = mgh_2$

$K_F = 0$

$d-(L-d)$

$d-L+d$

$2d-L$

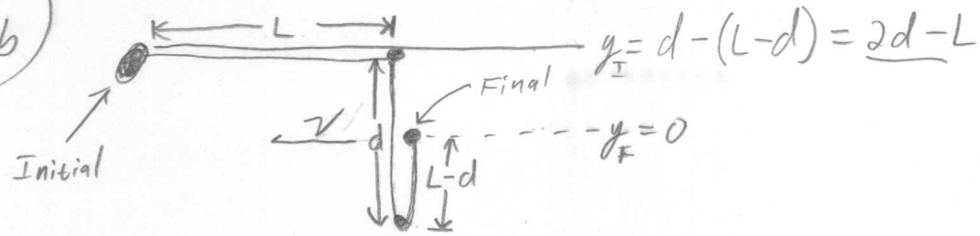
$$\cancel{m}gh_1 = \cancel{m}gh_2 \Rightarrow h_1 = h_2$$

$$h_1 = L - L\cos\theta_1 \ , \quad h_2 = L - d - (L-d)\cos\theta_2$$

$$\cancel{L} - L\cos\theta_1 = \cancel{L} - d - (L-d)\cos\theta_2$$

$$\boxed{\cos\theta_2 = \frac{d - L\cos\theta_1}{L-d}}$$

4b



$U_I = mg(2d-L)$ $\qquad$ $U_F = 0$

$K_I = 0$ $\qquad$ $K_F = \frac{1}{2} m v^2$

swings in a circle around peg

So get v^2 from uniform circular motion.

at "Final" position, we need the string to not go slack.

T mg $\quad$ when $T \to 0$, string slacks so set T to just 0

Newton's 2nd law: $-\cancel{T}^{0} - mg = ma_c, \quad a_c = -\frac{v^2}{R} = -\frac{v^2}{L-d}$

$$+\cancel{m}g = +\cancel{m}\frac{v^2}{L-d} \Rightarrow \boxed{v^2 = g(L-d)}$$

$$\cancel{m}\cancel{g}(2d-L) = \tfrac{1}{2}\cancel{m}\cancel{g}(L-d) \Rightarrow 2d - L = \tfrac{1}{2}L - \tfrac{1}{2}d$$

$$\boxed{d = \tfrac{3}{5}L}$$

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5. *Use work-energy techniques to solve the following problem.*

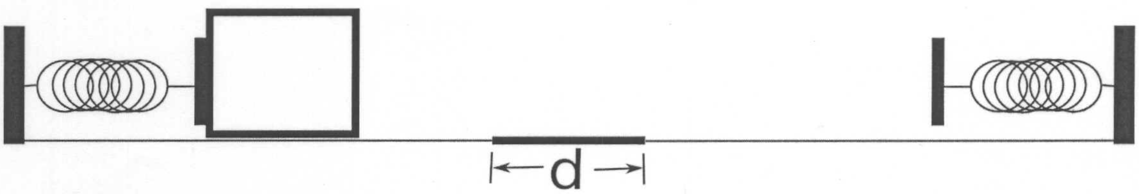
You are digging around in the physics stock room and find a box of springs. In attempting to measure the spring constants, you find one that doesn't seem to follow Hook's law. After extensive testing, you conclude that the it's force law is quadratic and is of the form:

$$\vec{F} = -(ax + bx^2)\hat{s}$$

where x is the displacement of the spring from equilibrium and $\hat{s}$ is a unit vector in the direction of the stretch.

a. Calculate a potential function for this spring.

b. A spring that follows Hook's with a spring constant k law is placed opposite the "strange" spring as in the figure below. The "strange" spring is compressed a distance l and a block of mass m is placed against it. All surfaces are frictionless except the dark patch between the two springs. How many times does the block cross the space between the springs?



a = 52.8 N/m
b = 38.4 N/m^2
l = 1.0 m
d = 0.5 m
m = 2 kg
k = 100 N/m
μ_k = 0.60

a) $\Delta U = -\int_{x_{eq}}^{x} \vec{F} \cdot d\vec{s} = \int_{x_{eq}}^{x} (ax + bx^2)\, dx$

$= \left(\frac{1}{2}ax^2 + \frac{1}{3}bx^3\right)\Big|_{x_{eq}}^{x}$

let $\underline{x_{eq}} = 0$

$$\boxed{U_s = \frac{1}{2}ax^2 + \frac{1}{3}bx^3}$$

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Extra space for problem 5

b) Want initial position compressed "strange" spring and Final position stopped on the frictiony path.

$U_I = \frac{1}{2}a\ell^2 + \frac{1}{3}b\ell^3$ $U_F = 0$

$K_I = 0$ $K_F = 0$

$$W_F = n\int_0^d \vec{F}_F \cdot d\vec{s} = -n\mu_k mgd$$

n times across

$$\frac{1}{2}a\ell^2 + \frac{1}{3}b\ell^3 - n\mu_k mgd = 0$$

$$\boxed{n = \frac{\frac{1}{2}a\ell^2 + \frac{1}{3}b\ell^3}{\mu_k mgd}} = \frac{\frac{1}{2}(52.8)(1.0)^2 + \frac{1}{3}(38.4)(1.0)^3}{(0.6)(2)(9.8)(0.5)}$$

$\boxed{n = 6.7 \text{ times}}$