TEST 3

PHYS 111, FALL 2008, SECTION 1

Total Score: ________

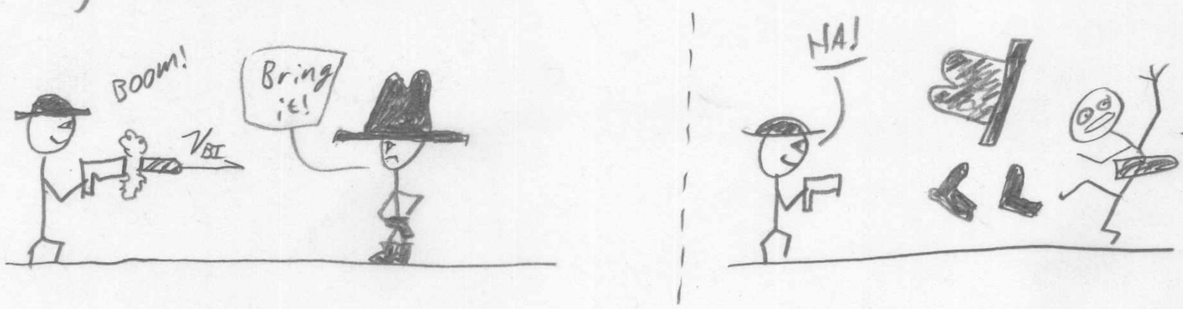
2 20 pts)

In many classic westerns, gunfighters fly backwards several meters after being shot, often crashing through windows or saloon doors. Assume that a typical bullet weights 2 g and that a typical cowboy weights 80 kg.

a) If the bullet leaves the gun at 200 m/s, what is the velocity of the cowboy/bullet system after the impact?

b) What velocity does the bullet need for the cowboy to slide 3 meters across the floor after being shot (assuming $U_k = 0.5$)?

a) Inelastic collision. conserve momentum



$$m_B V_{BI} = (m_B + m_c) V_F$$

$$\boxed{V_F = \frac{m_B}{(m_B + m_c)} V_{BI}}$$

$$V_F = \frac{2\times10^{-3}\,kg}{80.002\,kg} 200\,m/s = 5\times10^{-3}\,m/s$$

Very Slow.

b) Two stage problem:

① collision from part a

$$\boxed{V_F = \frac{m_B}{m_B + m_c} V_{BI}}$$

② WE - Cowboy/Bullet system slides to a stop

$$U_I^{\,0} + K_I + W_F = U_F^{\,0} + K_F^{\,0}$$

$$\tfrac{1}{2} m V_F^2 - \mu_k m g d = 0$$

Continued ↓

$$\boxed{V_F^2 = 2\mu_k g d}$$

① and ② connect through the velocity

$$\frac{m_B}{m_B + m_C} v_{BI} = \sqrt{2\mu_K g d}$$

$$\boxed{v_{BI} = \frac{m_B + m_C}{m_B} \sqrt{2\mu_K g d}}$$

$$= \frac{2\times10^{-3} + 80}{2\times10^{-3}} \left(2(0.5)(9.8)(3)\right)^{1/2}$$

$$\boxed{= 2.2\times10^{5}\ m/s}$$

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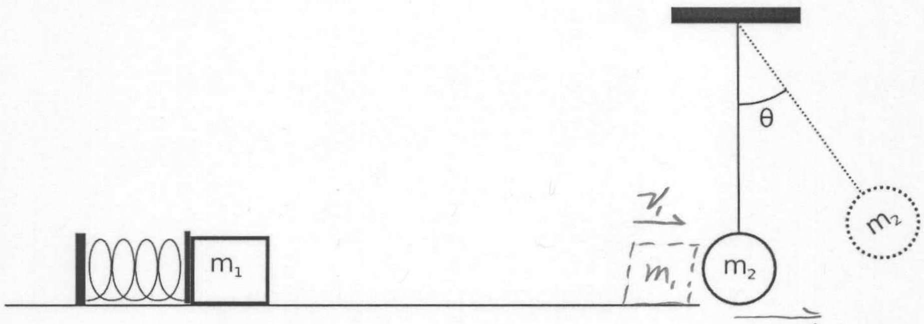
3 20pts)

A block with a mass of $m_1 = 3.5$ kg is placed in front of a spring that has been compressed $d = 0.050$ m on a rough surface with $\mu_k = 0.30$. After the spring is released, the block travels $s = 0.75$ m to a hanging pendulum. The block collides elastically with a pendulum that has mass $m_2 = 10$ kg. It is hanging from a string with length $l = 1.4$m. It takes 30 J of work to compress the spring.

What is the maximum angle θ that the pendulum string will make with the vertical?



① Block leaves spring and travels to the pendulum:

$U_I = W_s$ $\quad U_F = 0 \quad W_f = -\mu_k m_1 g(d+s)$

$K_I = 0 \quad K_F = \frac{1}{2} m_1 V_1^2$

$$W_s - \mu_k m_1 g(d+s) = \frac{1}{2} m V_1^2$$

$$V_1 = \left[\frac{2(30-(0.3)(3.5)(9.8)(0.755))}{3.5}\right]^{1/2}$$

$$V_1 = 3.6 \text{ m/s}$$

② m_1 and m_2 collide elastically

$$V_2 = \frac{m_2 - m_1}{m_2 + m_1} V_{2I} + \frac{2m_1}{m_2 + m_1} V_1 \quad (V_{2I} \to 0)$$

$$V_2 = \frac{2m_1}{m_2 + m_1} V_1$$

$$V_2 = \frac{2(3.5)}{(10+3.5)} 3.6 = 1.9 \text{ m/s}$$

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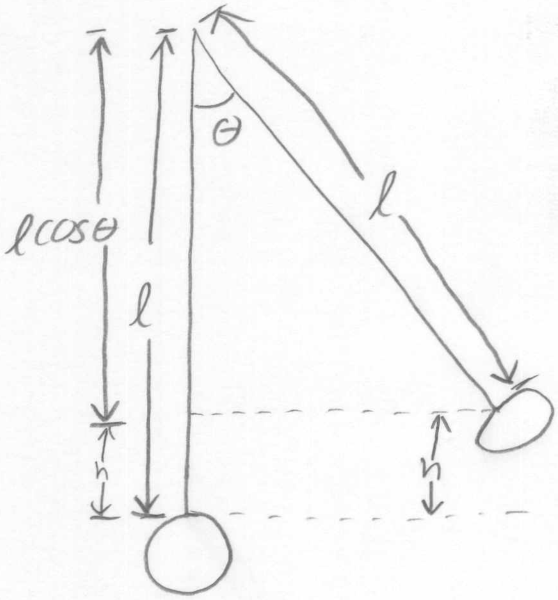
③ Pendulum swings up



$$h = l - l\cos\theta$$

$$U_I = 0 \qquad U_F = mg(l - l\cos\theta)$$

$$K_I = \tfrac{1}{2} m v_2^2 \qquad K_F = 0$$

$$\tfrac{1}{2} m v_2^2 = mg(l - l\cos\theta)$$

$$\frac{v_2^2}{2gl} = 1 - \cos\theta \Rightarrow \cos\theta = 1 - \frac{v_2^2}{2gl}$$

$$\boxed{\theta = \cos^{-1}\left(1 - \frac{v_2^2}{2gl}\right)}$$

$$\theta = \cos^{-1}\left(1 - \frac{1.9^2}{2(9.8)(1.4)}\right)$$

$$\boxed{\theta = 29^\circ}$$

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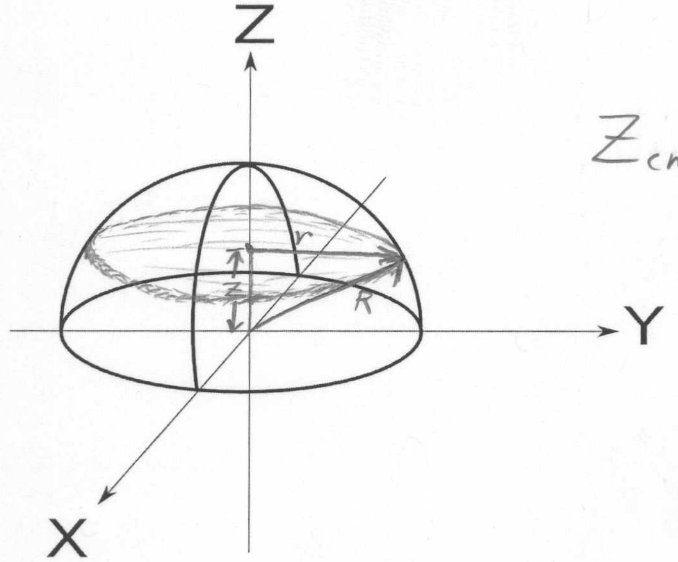
4 20pts) Find the center of mass on the Z axis of the solid hemisphere of radius R pictured below.

HINT: Consider dm to be a thin disk parallel to the XY plane with a thickness dz and integrate from Z = 0 to Z = R



$$Z_{cm} = \frac{1}{M}\int z\,dm$$

a slice of the hemisphere will be a disk with volume:

$$dV = \pi r^2 dz$$

$$dm = \rho dV = \rho \pi r^2 dz$$

ρ is the volume density: $\rho = \frac{M}{V}$, $V_{sphere} = \frac{4}{3}\pi R^3$

$$V_{hemi} = \frac{2}{3}\pi R^3$$

$$\rho = \frac{3M}{2\pi R^3}$$

r in terms of z and R is: $r^2 = R^2 - z^2$

so: $$\boxed{dm = \frac{3M}{2\pi R^3}\pi(R^2 - z^2)dz}$$

continued

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$$Z_{cm} = \frac{1}{M}\int_0^R z \, \frac{3M}{2R^3}\left(R^2 - z^2\right) dz$$

$$= \frac{3}{2R^3}\int_0^R \left(R^2 z - z^3\right) dz$$

$$= \frac{3}{2R^3}\left(\tfrac{1}{2}R^2 z^2 - \tfrac{1}{4}z^4 \right|_0^R$$

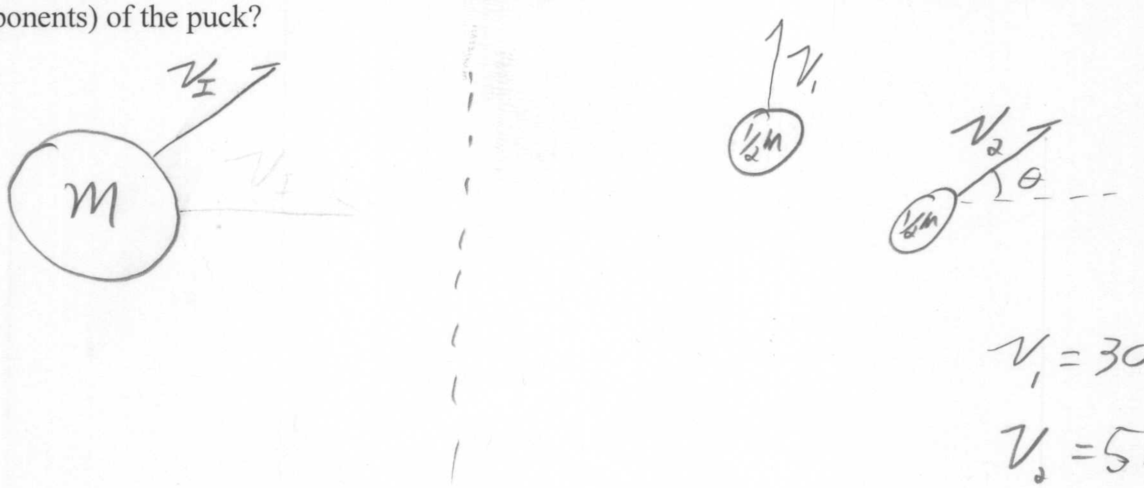
$$= \frac{3}{2R^3}\left(\tfrac{1}{2}R^4 - \tfrac{1}{4}R^4\right)$$

$$\boxed{Z_{cm} = \tfrac{3}{8}R}$$

Test 3

Phys 111, Fall 2008, Section 1

5 20pts) A 4.0 kg puck is sliding along a frictionless surface when it explodes into two equal parts, one moving 30 m/s due north and the other at 5.0 m/s 30o north of east. What was the original velocity (x and y components) of the puck?



$V_1 = 30$ m/s

$V_2 = 5$ m/s

$\theta = 30°$

Conserve momentum

x: $m V_{Ix} = \frac{1}{2} m V_2 \cos\theta$

y: $m V_{Iy} = \frac{1}{2} m V_1 + \frac{1}{2} m V_2 \sin\theta$

$V_{Ix} = \frac{1}{2}(5)\cos(30) = 2.1$ m/s

$V_{Iy} = \frac{1}{2}(30) + \frac{1}{2}(5)\sin(30) = 16$ m/s