SAMPLE TEST 5

PHYS 111, FALL 2008, SECTION 1

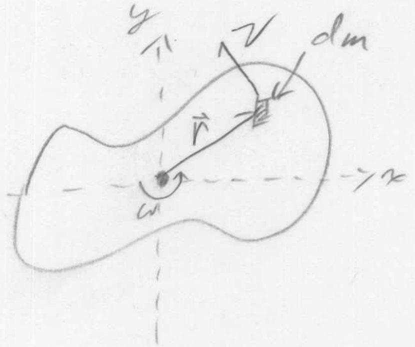
Name: ______________________

By writing my name above, I affirm that this test represents my work only, without aid from outside sources. In all aspects of this course I perform with honor and integrity.

SHOW YOUR WORK ON ALL OF THE PROBLEMS. YOUR APPROACH TO THE PROBLEM IS AS IMPORTANT AS YOUR ANSWER. **EXPLICITLY** SHOW THE **BASIC FORMULAS** YOU ARE USING. BE NEAT AND THOROUGH. THE EASIER IT IS FOR ME TO UNDERSTAND WHAT YOU ARE DOING, THE BETTER YOUR GRADE WILL BE.

1) Derivations

a) (10pts) Starting with the definition of linear Kinetic energy ($K=\frac{1}{2}mV^2$), show that rotational kinetic energy of a rigid body is $K=\frac{1}{2}I\omega^2$ where $I=\int r^2 dm$.



$dK = \frac{1}{2}V^2 dm$ $\qquad V = r\omega$

$dK = \frac{1}{2}r^2\omega^2 dm$

Integrate

$\int dK = \int \frac{1}{2} r^2 \omega^2 dm \Rightarrow$ $\quad \omega$ is constant

$\boxed{K = \frac{1}{2}\left[\int r^2 dm\right]\omega^2 = \frac{1}{2}I\omega^2}$

b) (10pts) Starting with the definition of angular momentum ($L=m(\vec{r}\times\vec{V})$), show that the angular momentum of a rigid body is $L=I\omega$ where $I=\int r^2 dm$.

using the picture above:

$dL = (\vec{r}\times\vec{V})dm = rV\text{SIN}\theta dm$, $\quad \theta = 90$, $\text{SIN}\theta = 1$

$V = r\omega$

$dL = r^2\omega dm$

Integrate

$\int dL = \int r^2 \omega dm$, $\quad \omega$ is constant

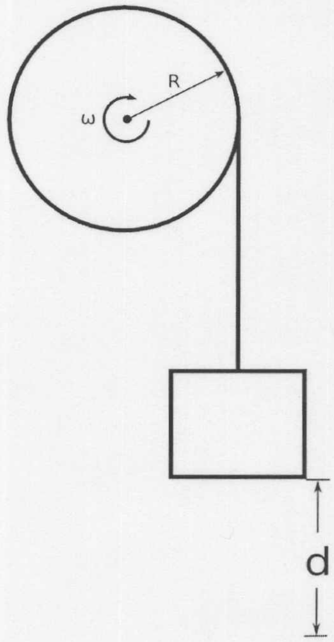
$L = \left[\int r^2 dm\right]\omega \Rightarrow \boxed{L = I\omega}$

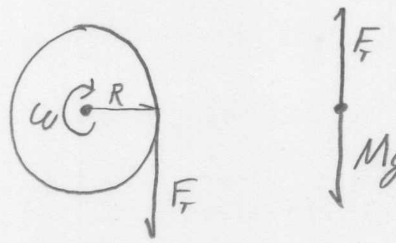
Sample Test 5
Phys 111, Fall 2008, Section 1

A weight of mass M is attached to a string that is wrapped around a pulley, also of mass M, and radius R. The moment of inertia of the pulley is $I = \frac{1}{2}MR^2$.

Use Torque/Kinematics to answer the following questions.

a) What is the angular acceleration, α, of the pulley?
b) What is the angular velocity, ω, of the pulley after the weight has dropped a distance d?
c) What is the velocity of the hanging mass after it drops a distance d?





a) Find α

Torque

$F_T R = I\alpha$

$F_T \cancel{R} = \frac{1}{2} M R^{\cancel{2}} \alpha$

$F_T = \frac{1}{2} M R \alpha$

Force

$Mg - F_T = Ma$ $\qquad a = R\alpha$

$\cancel{M}g - \frac{1}{2}\cancel{M}R\alpha = \cancel{M}R\alpha$

$g = \frac{3}{2} R\alpha$

$$\boxed{\alpha = \frac{2}{3}\frac{g}{R}}$$

continued ↓

b) use kinematics

$$\theta = \cancel{\theta_0}^{\,0} + \cancel{\omega_0 t}^{\,0} + \tfrac{1}{2}\alpha t^2 \qquad \omega = \cancel{\omega_0}^{\,0} + \alpha t$$

$$d = R\theta \Rightarrow \theta = \frac{d}{R} \qquad t = \frac{\omega}{\alpha}$$

$$\frac{d}{R} = \tfrac{1}{2}\alpha t^2$$

$$\frac{d}{R} = \tfrac{1}{2}\cancel{\alpha}\frac{\omega^2}{\alpha^{\cancel{2}}}$$

$$\omega^2 = \frac{2d\alpha}{R} = \frac{4}{3}\frac{d}{R^2}g$$

$$\boxed{\omega = \left(\frac{4}{3}\frac{d}{R^2}g\right)^{1/2}}$$

c) $v^2 = R^2\omega^2 = \cancel{R^2}\left(\frac{4}{3}\frac{d}{\cancel{R^2}}g\right)$

$$\boxed{v = \left(\frac{4}{3}dg\right)^{1/2}}$$

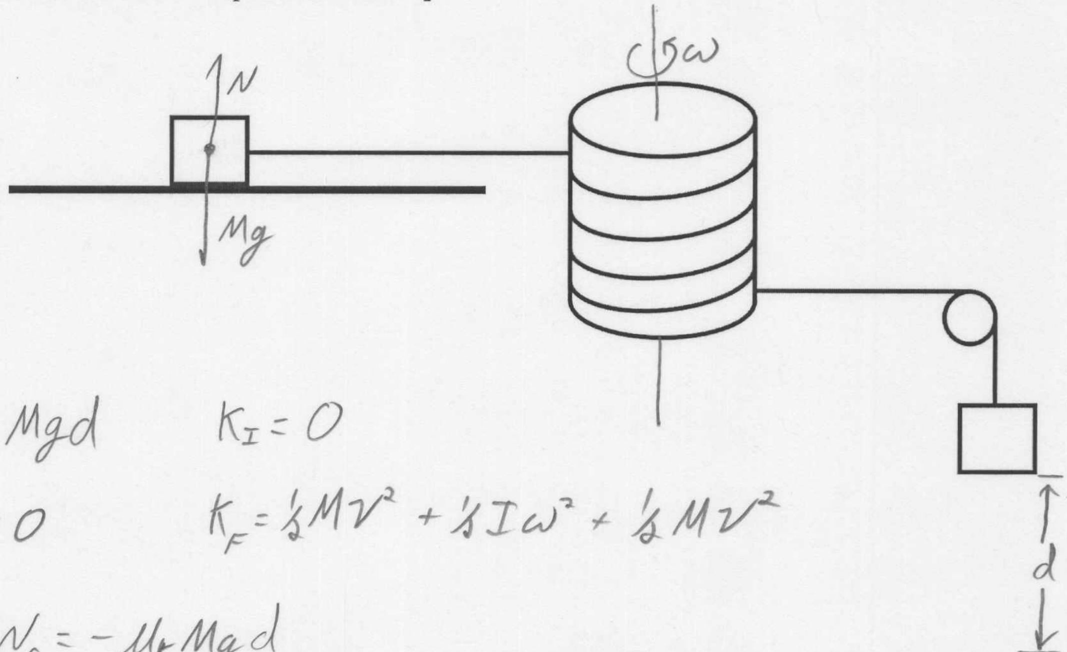
Sample Test 5
Phys 111, Fall 2008, Section 1

A block of mass M rests on a rough table with $\mu_k = 0.3$. A massless string is attached to the block, wrapped around a solid cylinder having a mass M and a radius R, runs over a massless frictionless pulley, and is attached to a second block of mass M that is hanging freely.

Using work/energy techniques, calculate the velocity of the blocks after they have moved a distance d.

$$I_{cylinder} = \frac{1}{2} MR^2$$

[NOTE: Do NOT use torque/kinematics]



$U_I = Mgd \qquad K_I = 0$

$U_F = 0 \qquad K_F = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 + \frac{1}{2}Mv^2$

$W_F = -\mu_k Mgd$

$$\cancel{M}gd - \mu_k \cancel{M}gd = \cancel{M}v^2 + \frac{1}{2}\left(\frac{1}{2}\cancel{M}R^2\right)\omega^2 \quad , \quad \omega = \frac{v}{R}$$

$$gd(1-\mu_k) = v^2 + \frac{1}{4}\cancel{R^2}\frac{v^2}{\cancel{R^2}}$$

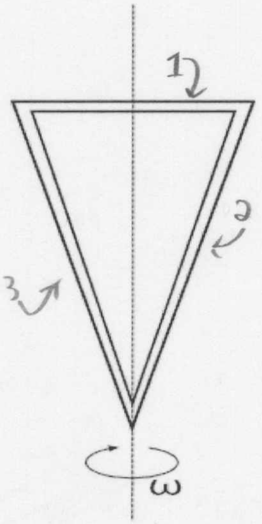
$$gd(1-\mu_k) = \frac{5}{4}v^2$$

$$\boxed{v = \left[\frac{4}{5}gd(1-\mu_k)\right]^{1/2}}$$

Sample Test 5
Phys 111, Fall 2008, Section 1

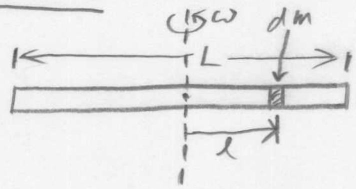
The triangular object below is constructed from three thin rods of uniform density. The total mass of the object is M. The sides each have length 2L and the top has length L. Calculate the moment of inertia of this object about the axis shown.

[HINT: Find the moment of each rod separately and use superposition to combine the three moments]



$$M = \lambda 5L \Rightarrow \boxed{\lambda = \frac{M}{5L}}$$

Rod 1

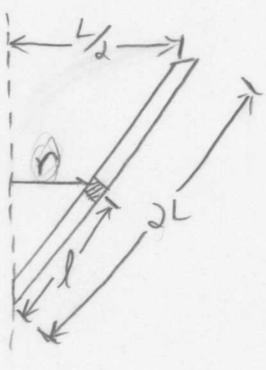


$$I_1 = \int r^2 dm, \quad dm = \lambda\, dl = \frac{M}{5L} dl$$

$$I_1 = \int_{-L/2}^{L/2} l^2 \frac{M}{5L} dl = \frac{M}{5L}\left(\tfrac{1}{3} l^3 \Big|_{-L/2}^{L/2}\right.$$

$$I_1 = \frac{M}{15\cancel{L}} 2\left(\frac{L^{\cancel{3}2}}{2^{\cancel{3}2}}\right) = \boxed{\frac{1}{60} ML^2}$$

Rods 2 and 3



$$I_{2,3} = \int r^2 dm, \quad dm = \lambda\, dl = \frac{M}{5L} dl$$

Similar triangles

$$\frac{r}{l} = \frac{\cancel{L}/2}{2\cancel{L}} = \frac{1}{4} \Rightarrow r = \frac{l}{4}$$

$$I_{2,3} = \int_0^{2L} \frac{l^2}{16} \frac{M}{5L} dl = \frac{M}{16 \cdot 5L} \frac{1}{3} (2L)^3 = \frac{M}{3 \cdot \cancel{16}_2 \cdot 5 \cancel{L}} \cancel{8} L^{\cancel{3}2}$$

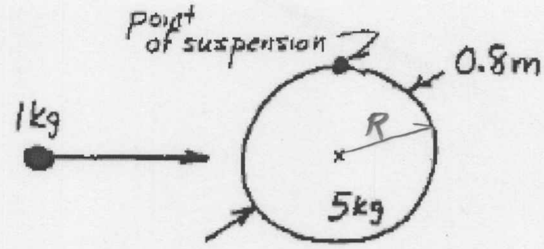
$$\boxed{I_{2,3} = \frac{1}{30} ML^2}$$

$$I_T = I_1 + 2I_{2,3} = \frac{1}{20} ML^2 + \frac{2}{30} ML^2$$

$$\boxed{I_T = \frac{7}{60} ML^2}$$

Sample Test 5
Phys 111, Fall 2008, Section 1

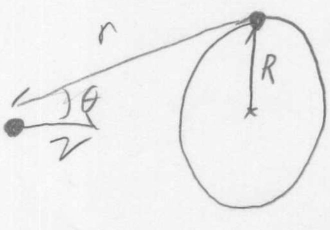
A uniform 5-kg disk has an 80-cm diameter and is suspended from its edge so that it can swing freely. A small, dense 1-kg blob of Silly Putty is thrown horizontally at the disk with a speed of 7 m/s, and it sticks to the middle of the disk's edge as shown.

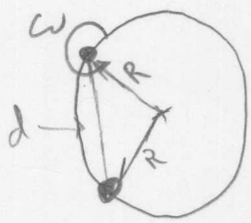


a. What is the angular speed about the point of suspension of the disk-blob combination immediately after the collision?
(For a disk, $I_{CM} = \frac{1}{2}MR^2$.)

[Think carefully about how to calculate I_{final}. What distance do you use?]

Conserve angular momentum





$$m_c(\vec{r}\times\vec{v}_i) = I_T\omega$$

Work out I_T

$I_T = I_{disk} + I_{clay}$ ⇒ by superposition principle

$I_{disk} = \frac{1}{2}m_dR^2 + m_dR^2$ by parallel axis Theorem

$I_{disk} = \frac{3}{2}m_dR^2$

$I_{clay} = m_cd^2$, $d^2 = 2R^2$

$I_{clay} = 2m_cR^2$

$$\boxed{I_T = \frac{3}{2}m_dR^2 + 2m_cR^2}$$

Continued ↓

$$m_c(\vec{r} \times \vec{v}_I) = \left(\tfrac{3}{2}m_d + 2m_c\right)R^2\omega$$

$$m_c r v_I \sin\theta = \left(\tfrac{3}{2}m_d + 2m_c\right)R^2\omega \quad , \quad r\sin\theta = R \; ,$$

$$m_c R v_I = \left(\tfrac{3}{2}m_d + 2m_c\right)R^2\omega$$

$$\boxed{\omega = \frac{m_c}{\left(\tfrac{3}{2}m_d + 2m_c\right)} \frac{v_I}{R}}$$

$$\tfrac{3}{2} + \tfrac{4}{2} = \tfrac{7}{2}$$