SAMPLE TEST 6

PHYS 111, FALL 2008, SECTION 1

Name: ______________________

By writing my name above, I affirm that this test represents my work only, without aid from outside sources. In all aspects of this course I perform with honor and integrity.

SHOW YOUR WORK ON ALL OF THE PROBLEMS. YOUR APPROACH TO THE PROBLEM IS AS IMPORTANT AS YOUR ANSWER. **EXPLICITLY** SHOW THE **BASIC FORMULAS** YOU ARE USING. BE NEAT AND THOROUGH. THE EASIER IT IS FOR ME TO UNDERSTAND WHAT YOU ARE DOING, THE BETTER YOUR GRADE WILL BE.

1) Derivations

a) (6pts) Given a differential equation of the form $\frac{d^2x(t)}{dt^2}=-\omega^2 x(t)$, write the general solution for $x(t)$, $v(t)$, and $a(t)$ in terms of the angular frequency ω, the amplitude A, and the phase angle φ.

$$x(t) = A\,SIN(\omega t + \phi)$$
$$v(t) = \omega A\cos(\omega t + \phi)$$
$$a(t) = -\omega^2 A\,SIN(\omega t + \phi)$$

b) (7pts) Given the boundary conditions $x(t_0)=x_0$ and $v(t_0)=v_0$, derive an expression for the phase angle φ and the amplitude A.

$$\left[x_0^2 + \frac{v_0^2}{\omega^2}\right]^{1/2} = \left[A^2 SIN^2(\omega t + \phi) + A^2\cos^2(\omega t + \phi)\right]^{1/2} = A$$

$$\Rightarrow \boxed{A = \left[x_0^2 + \frac{v_0^2}{\omega^2}\right]^{1/2}}$$

$$\frac{x_0}{v_0} = \frac{A\,SIN(\omega t_0 + \phi)}{A\omega\cos(\omega t_0 + \phi)} \Rightarrow \tan(\omega t_0 + \phi) = \omega\frac{x_0}{v_0} \Rightarrow \boxed{\phi = \tan^{-1}\left(\omega\frac{x_0}{v_0}\right) - \omega t_0}$$

c) (7pts) Show that, because sin and cos are $\frac{\pi}{2}$ radians out of phase, $v_{max}=\omega A$.

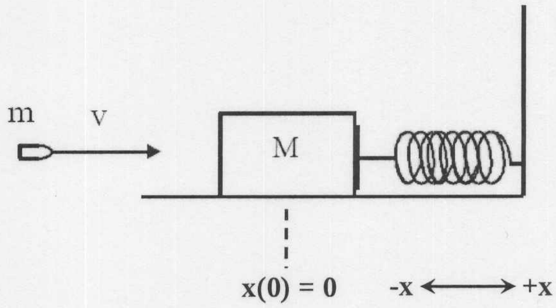
$$x(t_m) = 0 = A\,SIN(\omega t_m + \phi) \Rightarrow SIN^{-1}(0) = \omega t_m + \phi$$
$$\Rightarrow \omega t_m = -\phi \Rightarrow t_m = -\frac{\phi}{\omega}$$

$$v(t_m) = \omega A\cos\left(-\cancel{\omega}\frac{\phi}{\cancel{\omega}} + \phi\right) = \omega A\cos(0) = \omega A$$

$$\boxed{v(t_m) = v_{max} = \omega A}$$

Sample Test 6
Phys 111, Fall 2008, Section 1

A 1.0-kg block is attached to a horizontal spring with spring constant 2500 N/m. The block is at rest on a frictionless surface. A 0.01-kg bullet is fired into the block, and it sticks. The subsequent oscillations have an amplitude of 0.10 m. (Assume that the oscillations begin at $t = 0$ and that $x(0) = 0$.)



(a) What was the bullet's speed?
(b) What is the angular frequency of oscillation?
(c) What is the maximum speed and maximum acceleration of the oscillator?

b) $F = ma = -kx = (m+M)\frac{d^2x}{dt^2} \Rightarrow \frac{d^2x}{dt^2} = -\frac{k}{m+M}x$

$\Rightarrow \boxed{\omega = \left(\frac{k}{m+M}\right)^{1/2}} \Rightarrow \boxed{\omega = \left(\frac{2500}{1+0.01}\right)^{1/2} = 49.8 \ \text{rad/s}}$

a) $x(0) = 0$

$v(0) = v_F \rightarrow$ Post collision velocity.

$P_i = P_F \rightarrow$ conserve momentum

$mv_i = (m+M)v_F \Rightarrow v_F = \frac{m}{m+M}v_i$

<u>In general</u>

$x(t) = A\sin(\omega t + \phi)$
$v(t) = \omega A\cos(\omega t + \phi)$

at $t=0$, $x(0) = A\sin(\phi) = 0$
$v(0) = \omega A\cos(\phi) = v_F$

$0 = A\sin(\phi) \Rightarrow \boxed{\phi = 0 \text{ or } \pi}$,

Continued ↓

$$v_f = \omega A \cos(0) \Rightarrow v_f = \omega A$$

$$\frac{m}{m+M} v_i = \frac{k^{1/2}}{(m+M)^{1/2}} A \Rightarrow v_i = \frac{m+M}{(m+M)^{1/2}} \frac{k^{1/2}}{m} A$$

$$\Rightarrow \boxed{v_i = \frac{A}{m}\sqrt{k(m+M)}}$$

$$v_i = (0.10\,m)\left((2500)(1.01)\right)^{1/2} \frac{1}{0.01}$$

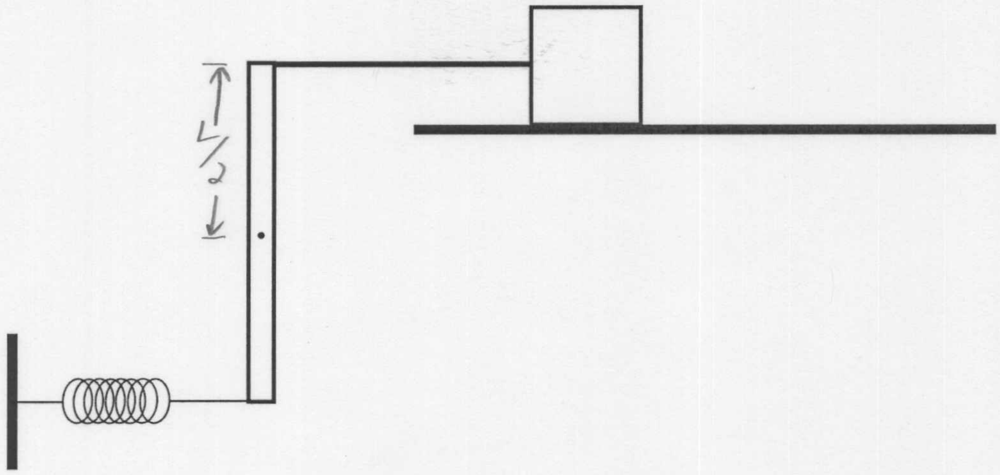
$$\boxed{v_i = 5.02 \times 10^2 \text{ m/s}}$$

c) $\boxed{v_{max} = \omega A = (49.8)(0.1) = 4.98 \text{ m/s}}$

SAMPLE TEST 6

PHYS 111, FALL 2008, SECTION 1

A block of mass *M* resting on a frictionless surface is attached to a stiff rod of negligible mass. The other end of the rod is attached to the top of a thin bar of length *l* mass *M* that is allowed to rotate about its center. The bottom of a bar is attached to a light spring of spring constant k. The spring is relaxed when the bar is vertical. Find the frequency of small oscillations.



I like energy for this one!

$$U = \tfrac{1}{2}kx^2$$

$$K = \tfrac{1}{2}I\omega^2 + \tfrac{1}{2}mv^2 \quad , \quad v = \tfrac{L}{2}\omega \Rightarrow \omega = 2\frac{v}{L}$$

$$I = \tfrac{1}{12}ML^2$$

$$E_T = \tfrac{1}{2}kx^2 + \tfrac{1}{2}\,\frac{1}{\cancel{12}_3}\,m\cancel{L^2}\cdot\cancel{4}\,\frac{v^2}{\cancel{L^2}} + \tfrac{1}{2}mv^2$$

$$\frac{dE_T}{dt} = kx\cancel{v} + \frac{1}{\cancel{6}_3}2m\cancel{v}a + m\cancel{v}a = 0$$

$$kx + \tfrac{4}{3}ma = 0 \Rightarrow \boxed{a = -\tfrac{3}{4}\tfrac{k}{m}x}$$

$$\boxed{\omega = \sqrt{\tfrac{3}{4}\tfrac{k}{m}}}$$

Sample Test 6

Phys 111, Fall 2008, Section 1

The fact that g varies from place to place over Earth's surface drew attention when Jean Richer in 1672 took a pendulum clock from Paris to Cayenne, French Guiana, and found that it lost 2.5 minutes/day.

If $g = 9.81\ \text{m/s}^2$ in Paris, what is g in Cayenne?

$$\omega_P = \sqrt{\frac{g_P}{L}}, \quad \omega_C = \sqrt{\frac{g_C}{L}}$$ For a simple pendulum.

$$\frac{\omega_P}{\omega_C} = \left(\frac{g_P}{\cancel{L}} \cdot \frac{\cancel{L}}{g_C}\right)^{1/2} = \left(\frac{g_P}{g_C}\right)^{1/2}$$

So, what's $\frac{\omega_P}{\omega_C}$?

In Paris, the period is $T_P = \frac{2\pi}{\omega_P}$

In Cayenne, the period is $T_C = \frac{2\pi}{\omega_C}$

In Paris, $nT_P = 24\ \text{hrs}$ (n = number of periods)

In Cayenne $nT_C = 24\ \text{hrs} - 2.5\ \text{min}$

or: $n\frac{2\pi}{\omega_P} = (24\ \text{hrs})(60\ \text{min/hr})$

$n\frac{2\pi}{\omega_C} = (24\ \text{hrs})(60\ \text{min/hr}) - 2.5\ \text{min}$

$$\Rightarrow \boxed{\frac{\cancel{n}\,\cancel{2\pi}\,\omega_C}{\cancel{n}\,\omega_P\,\cancel{2\pi}} = \frac{(24)(60)}{(24)(60)-2.5} = 1.00174}$$

$$\boxed{\frac{\omega_P}{\omega_C} = 9.98 \times 10^{-1}}$$

continued ↓

so: $\frac{\omega_p}{\omega_c} = \left(\frac{g_p}{g_c}\right)^{1/2} \Rightarrow g_c = \left(\frac{\omega_c}{\omega_p}\right)^2 \cdot g_p$

$$g_c = (1.00174)^2 (9.81)$$

$$\boxed{g_c = 9.84 \text{ m/s}^2}$$