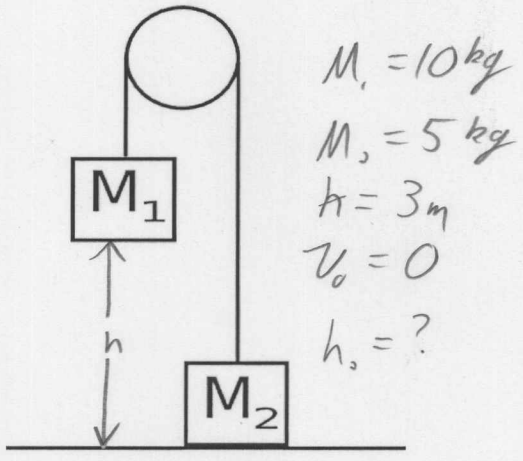
Force Problems

One end of a rope is connected to a mass M_1=10kg. The rope passes over a massless frictionless pulley and the other end is connected to a mass M_2 = 5kg. M_2 is initially resting on the ground and M_1 is suspended 3m above the ground. The system is initially at rest.

If M_1 is released and allowed to hit the ground, what is the maximum height that M_2 will reach?



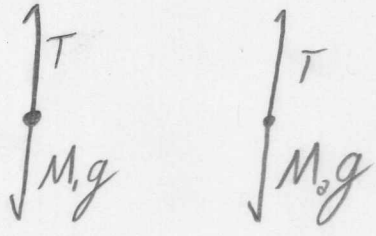
$M_1 = 10\,kg$

$M_2 = 5\,kg$

$h = 3m$

$v_0 = 0$

$h_2 = ?$



acceleration must be equal and opposite. so:

$$a_1 = -a_2$$

$$T - M_1 g = M_1 a_1 \qquad T - M_2 g = M_2 a_2$$

$$T = M_1(a_1 + g) \qquad T = M_2(a_2 + g)$$

$$T = M_1(-a_2 + g)$$

$$M_1(g - a_2) = M_2(g + a_2) \Rightarrow M_1 g - M_1 a_2 = M_2 g + M_2 a_2$$

$$g(M_1 - M_2) = a_2(M_1 + M_2)$$

$$\textcircled{1} \quad \boxed{a_2 = \frac{M_1 - M_2}{M_1 + M_2} g}$$

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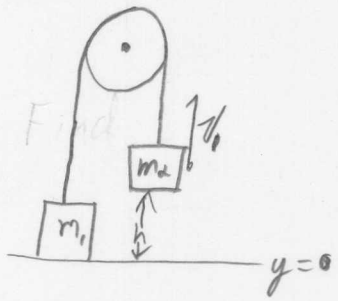
So: M_2 will accelerate upwards through a distance h. as M_1 falls.

When M_1 stops (because it hit the ground) M_2 will have an upward velocity and will continue up for an additional distance h_2.



Find v_1

$$x_f = x_0 + v_0 t + \tfrac{1}{2} a t^2 \qquad v_f = v_0 + at$$
$$h = 0 + 0 + \tfrac{1}{2} a t^2 \qquad v_1 = 0 + at$$
$$h = \tfrac{1}{2} \frac{v_1^2}{a} \qquad t = \frac{v_1}{a}$$

② $\boxed{v_1 = \sqrt{2ha}}$

Now, Find h_2 given v_1

$$x_f = x_0 + v_0 t + \tfrac{1}{2} a t^2 \qquad v_f = v_0 + at$$
$$h_2 = h + v_1 t - \tfrac{1}{2} g t^2 \qquad 0 = v_1 - gt \Rightarrow t = \frac{v_1}{g}$$

③ $\boxed{h_2 = h + \frac{1}{2} \frac{v_1^2}{g}}$

Combine eq ①, ②, and ③

$$h_2 = h + \frac{1}{2} \frac{2ha}{g} = h + \frac{h}{g} \frac{M_1 - M_2}{M_1 + M_2} g$$

continued ↓

$$h_2 = h\left(1 + \frac{m_1 - m_2}{m_1 + m_2}\right) = h\left(\frac{m_1 + m_2 + m_1 - m_2}{m_1 + m_2}\right)$$

$$* \boxed{h_2 = \frac{2m_1}{m_1 + m_2} h}$$

$$\boxed{h_2 = \frac{2 \cdot 10}{10 + 5} 3 = 4\,m}$$