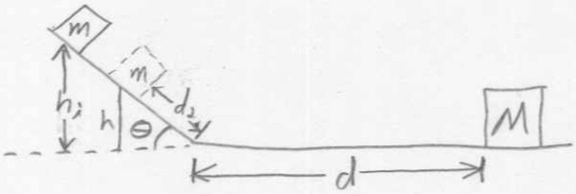
Ch9, #71




1 Find initial velocity of m at bottom of hill

$$\frac{1}{2} \cancel{m} v_{1i}^2 = \cancel{m} g h_i \Rightarrow \underline{v_{1i} = \sqrt{2gh_i}}$$

2 Find velocities of m and M post collision

$$\boxed{v_{1F} = \frac{(m-M)}{(m+M)} v_{1i}, \quad v_{2f} = \frac{2m}{m+M} v_{1i}}$$

3 m travels a distance d at $v_{1F}$ which takes time:

$$\boxed{t_1 = \frac{v_{1F}}{d}}$$

It goes back up the ramp a distance $d_2$, and slides back down. The time to go up is equal to the time to go down, so we can write:


From kinematics (time to slide down)

$$0 = d_2 + 0 - \frac{1}{2} g \sin\theta t^2$$

Then find $d_2$ from work energy

$$\frac{1}{2} m v_{1F}^2 = mgh \Rightarrow \frac{1}{2} \cancel{m} v_{1F}^2 = \cancel{m} g d_2 \sin\theta$$

$$d_2 = \frac{v_{1F}^2}{2g \sin\theta}$$

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$$\frac{v_{1f}^2}{2g\sin\theta} = \tfrac{1}{2} g \sin\theta\, t^2 \Rightarrow \boxed{t_2 = \frac{v_{1f}}{g\sin\theta}}$$

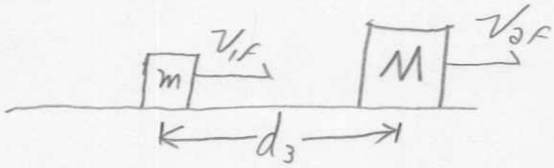
So, the time to back to the hill, up, down, and back across is:

$$\boxed{t_3 = 2(t_1 + t_2)}$$

In that time, M has moved a distance:

$$\boxed{d_3 = v_{2f} t_3}$$

So m has to catch up




$$d_3 = (v_{2f} - v_{1f}) t_4$$

$$\Rightarrow \boxed{t_4 = \frac{d_3}{v_{2f} - v_{1f}}}$$

and the <u>total</u> time is

$$t_T = t_3 + t_4 = 2(t_1 + t_2) + t_4$$

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These are the same...

$$t_T = 2\left[\frac{v_{1f}}{d} + \frac{v_{1f}}{g\sin\theta}\right] + \frac{v_{2f}\,t_3}{v_{2f} - v_{1f}}$$

so we can factor out a $t_3$

$$= v_{1f}\left[\frac{1}{d} + \frac{1}{g\sin\theta}\right]\left[2 + \frac{v_{2f}}{v_{2f} - v_{1f}}\right]$$

$$v_{2f} - v_{1f} = \frac{2m}{m+M}v_{1i} - \frac{(m-M)}{(m+M)}v_{1i}$$

$$= \frac{2m - m + M}{m+M}v_{1i} = \frac{\cancel{m+M}}{\cancel{m+M}}v_{1i} = v_{1i}$$

$$\text{so: } \frac{v_{2f}}{v_{2f} - v_{1f}} = \frac{2m}{m+M}$$

and

$$\boxed{t_T = \left[\frac{1}{d} + \frac{1}{g\sin\theta}\right]\frac{m-M}{(m+M)}\cdot\frac{2m}{(m+M)}\sqrt{2gh_i}}$$