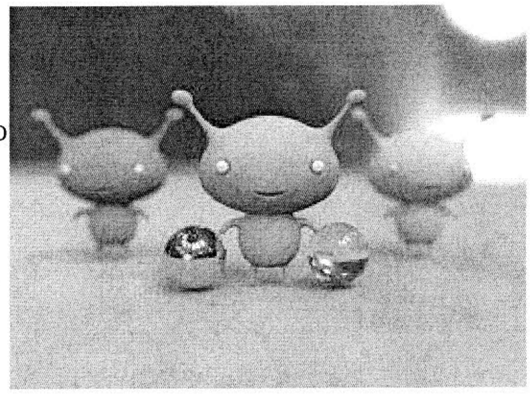
Energy Problems – Set ~~4~~ 5

The Zeronians live on a planet with a mass of 5.0×10^{23} kg, a radius of 3.0×10^{6} m. Their planet is rapidly running out of atmosphere (so there's no wind resistance) and, because they enjoy skydiving AND breathing, they have built a space-craft to leave.

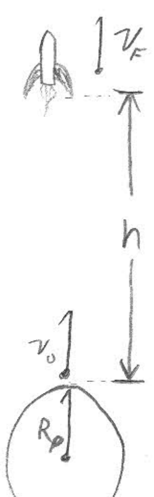


a) Their space-craft, which weighs 10 kg (the zeronians are tiny), launched with an initial velocity of 3000 m/s. What will be its velocity at when it is 4.0×10^{6} m from the center of the planet?

b) What maximum altitude will it achieve?

~~b~~ c) What initial velocity does it need to get to escape the planet's gravity completely?
HINT: To "escape" means to get to a distance of infinity from the center of the planet.

a)



$M_p = 5\times10^{23}$ kg

$R_p = 3\times10^{6}$ m

$m_s = 10$ kg

$V_0 = 3000$ m/s

$h + R_p = 4\times10^{6}$ m

$U_I = -\dfrac{G M_p m_s}{R_p}$ $\qquad K_I = \frac{1}{2} m_s V_0^2$

$U_F = -\dfrac{G M_p m_s}{(R_p + h)}$ $\qquad K_F = \frac{1}{2} m_s V_F^2$

$$-\frac{G M_p \cancel{m_s}}{R_p} + \frac{1}{2}\cancel{m_s} V_0^2 = -\frac{G M_p \cancel{m_s}}{(R_p + h)} + \frac{1}{2}\cancel{m_s} V_F^2$$

$$\Rightarrow \boxed{V_F = \left[V_0^2 + 2GM_p\left(\frac{1}{R_p + h} - \frac{1}{R_p}\right)\right]^{1/2}}$$

$$V_F = \left[(3\times10^3 \text{ m/s})^2 + (2)(6.67\times10^{-11} \text{ m}^3/\text{kg}\cdot\text{s}^2)(5\times10^{23} \text{ kg})\left(\frac{1}{4\times10^6 \text{ m}} - \frac{1}{3\times10^6 \text{ m}}\right)\right]^{1/2}$$

$$\boxed{V_F = 1.86\times10^{3} \text{ m/s}}$$

Energy Problems Set 5, P1 continued.

b) $U_I = -\frac{GM_p m_s}{R_p}$ $K_I = \frac{1}{2} m_s v^2$

$U_F = -\frac{GM_p m_s}{R_p + h_{max}}$ $K_F = 0$

$$-\frac{GM_p \cancel{m_s}}{R_p} + \frac{1}{2}\cancel{m_s} v^2 = -\frac{GM_p \cancel{m_s}}{R_p + h_{max}}$$; multiply by $\frac{R_p}{GM_p}$

$$\Rightarrow -1 + \frac{R_p v^2}{GM_p} = -\frac{R_p}{R_p + h_{max}}$$

$$\Rightarrow R_p + h_{max} = -\frac{R_p}{\frac{R_p v^2}{GM_p} - 1}$$

$$\Rightarrow h_{max} = -\frac{R_p}{\frac{R_p v^2}{GM_p} - 1} - R_p$$

$$\Rightarrow h_{max} = -R_p\left[\frac{1}{\frac{R_p v^2}{GM_p} - 1} - 1\right]$$

$$\boxed{h_{max} = 1.9 \times 10^7 m}$$

c) $U_I = -\frac{GM_p m_s}{R_p}$ $K_I = \frac{1}{2} m_s v_{esc}^2$

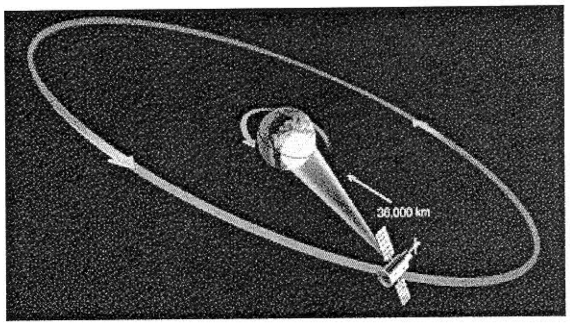
$U_F = 0$ $K_F = 0$

$$\Rightarrow v_{esc} = \left[\frac{2GM_p}{R_p}\right]^{1/2}$$

$$\boxed{v_{esc} = 4.7 \times 10^3 m/s}$$

Energy Problems – Set 4

Satellites in geosynchronous orbit always remain above the same geographic spot on the Earth's surface, making them extremely handy for communications.
Using what you know about Newton's Universal Law of Gravity and the rotation of the Earth:



a) calculate the radius of the geosynchronous orbit.
b) calculate how much energy is required to put a 1000kg communications satellite into geosynchronous orbit.

The radius of the Earth is: $R_{Earth} = 6.37\text{x}10^6$ m.
The gravitaional constant: $G = 6.67\text{x}10^{-11}\ m^3/(kg\ s^2)$ $M_\oplus$

a) Assume geosynchronous orbit is circular.

To stay above the same spot on the Earth, it must have a period of 24 hours.

To be going in a circle, it must have a centrally directed force providing an acceleration $\underline{\vec{a}_c = -r\omega^2\hat{r}}$

$$F_G = m a_c \rightarrow NSL$$

$$-\frac{G M_\oplus \cancel{m}}{r^2} = -\cancel{m} r \omega^2 \Rightarrow r^3 = \frac{G M_\oplus}{\omega^2}, \quad \boxed{\omega = \frac{2\pi}{P}} \leftarrow 24\ hrs$$

$$\Rightarrow r^3 = \frac{G M_\oplus P^2}{4\pi^2}$$

$$\boxed{r = \left[\frac{G M_\oplus}{4\pi^2} P^2\right]^{1/3}} = \left[\frac{(6.37\times10^{-11})\cdot(5.97\times10^{24})}{4\pi^2}\left[(24\ hrs)(3600\ s/hr)\right]^2\right]^{1/3}$$

$$= 4.1\times10^7\ m \cong \boxed{41{,}000\ km}$$

EP4,3

b) Energy on the surface is all potential (ignoring the small amount from Earth's rotation)

$$E_I = U_I = -\frac{GM_\oplus m}{R_\oplus}$$

In orbit, total energy is potential plus kinetic.

$$E_F = U_F + K_F$$

$$= -\frac{GM_\oplus m}{r} + \frac{1}{2}m\underline{v}^2 \leftarrow \text{What's this?}$$

$$\overset{F}{\frac{GM_\oplus \cancel{m}}{r^{\cancel{2}}}} \overset{= ma}{=} \cancel{m}\frac{v^2}{\cancel{r}} \Rightarrow \underline{v^2 = \frac{GM_\oplus}{r}}$$

$$\underline{E_F} = -\frac{GM_\oplus m}{r} + \frac{1}{2}\frac{GM_\oplus m}{r} = \boxed{-\frac{1}{2}\frac{GM_\oplus m}{r}}$$

$$\Delta E = E_F - E_I$$

$$= -\frac{1}{2}\frac{GM_\oplus m}{r} + \frac{GM_\oplus m}{R_\oplus} = GM_\oplus m\left[\frac{1}{R_\oplus} - \frac{1}{2r}\right]$$

$$\Delta E = (6.67\times10^{-11}\,\tfrac{m^3}{kg\cdot s^2})(5.97\times10^{24}\,kg)(1\times10^3\,kg)\left[\frac{1}{6.37\times10^6\,m} - \frac{1}{(2)(4.1\times10^7\,m)}\right]$$

$$\boxed{\Delta E = 5.8\times10^{10}\,J}$$