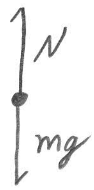
Force Problems – Set 1

You toss a rock straight up into the air by placing it on the palm of your hand (you're not gripping it) then pushing your hand up very rapidly. Draw free body diagrams for the following:

a) As you hold the rock at rest on your palm, before moving your hand.



$N = mg$, no accel.

b) As your hand is moving up but before the rock leaves your hand.

$N > mg$

c) One-tenth of a second after the rock leaves your hand.

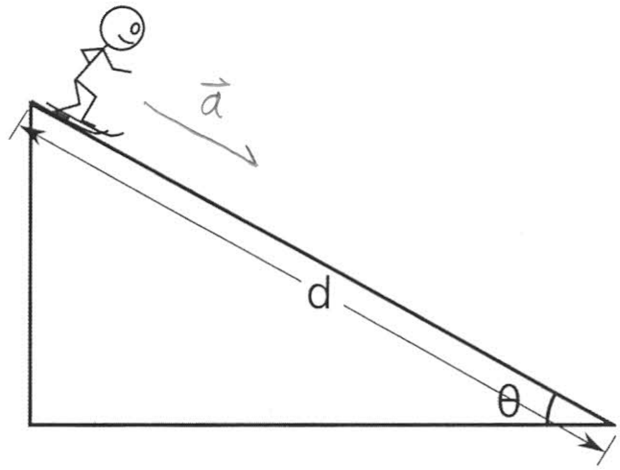
No $F_{Aristotle}$

d) After the rock has reached its highest point and is now falling straight down.

①

Force Problems – Set 1

A very talented stick skier is accelerating down a VERY slippery slope. (there's no friction, that's next period).



a) List all of the forces on the skier.

b) In what direction will the skier accelerate? Draw the acceleration vector in the picture at the right.

c) Draw a *Free Body Diagram* for the skier.

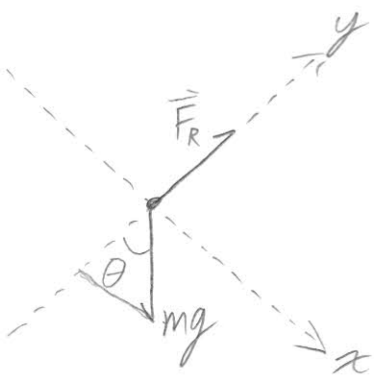
d) Choose a coordinate system and superimpose it on your free body diagram.

e) Write the x and y versions of Newton's Second Law for the skier based on your coordinate system. Solve these equations for acceleration.

g) What's the skier's velocity after accelerating a distance d?

a) Gravity, Reaction Force from Slope

c, d)



Rotate coordinate system to align x with the $\vec{a}$ vec.

e)

$\underline{x}$

$$\Sigma F_x = ma_x$$

$$\cancel{m}g\sin\theta = \cancel{m}a_x$$

$$\boxed{a_x = g\sin\theta}$$

$\underline{y}$ no accel in y

$$\Sigma F_y = ma_y$$

$\underline{a_y = 0}$

$$F_R - mg\cos\theta = 0$$

$$\boxed{F_R = mg\cos\theta}$$

continued ↓

Force Problems: Set 1, P2 continued

g) Now use kinematics. But, only x since nothing is happening in y.

$$x = \cancel{x_0}^{0} + \cancel{v_{0x}t}^{0} + \frac{1}{2}a_x t^2 \qquad v_x = \cancel{v_0}^{0} + a_x t$$

$$d = \frac{1}{2}a_x t^2 \qquad v_x = at \Rightarrow t = \frac{v_x}{a}$$

$$d = \frac{1}{2}\cancel{a_x}\frac{v_x^2}{a_x^{\cancel{2}}} \Rightarrow d = \frac{v_x^2}{2a_x}$$

$$\Rightarrow v_x = (2da_x)^{1/2}$$

$$\Rightarrow \boxed{v_x = (2dg\sin\theta)^{1/2}}$$

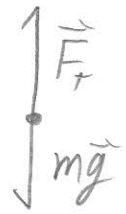
Force Problems – Set 1

For the following situations, draw free-body diagrams to indicate all forces acting on the object(s) in question. *Indicate relative magnitudes of forces by drawing long, short, or equal-length vectors.*

a) An elevator suspended by a cable is descending at a constant velocity.

const $v \Rightarrow a = 0 \Rightarrow F_{net} = 0$

So:

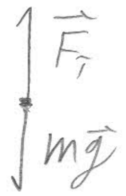


where $|\vec{F}_T| = |m\vec{g}|$

b) An elevator suspended by a cable is ascending at a constant velocity.

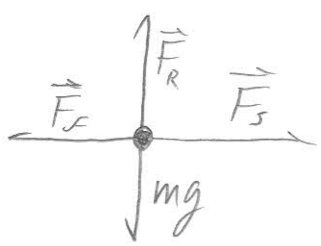
again const $v \Rightarrow a = 0 \Rightarrow F_{net} = 0$

So:



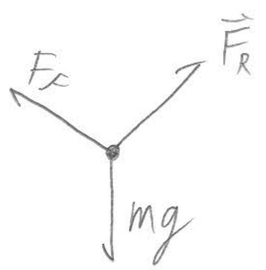
where $|\vec{F}_T| = |m\vec{g}|$

c) A compressed spring is pushing a block across a rough horizontal table (there's friction).



Friction opposes motion

d) A block is resting on a rough incline plane without sliding.



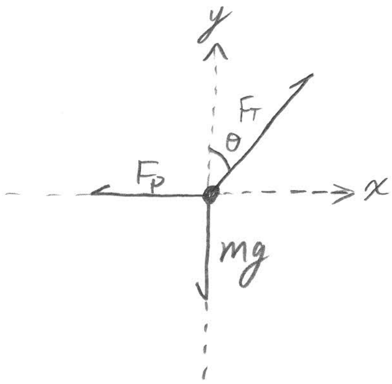
Friction opposes motion

A 30-kg child is seated in a swing of negligible mass. How much horizontal force is required to pull the child and swing aside so that the support rope makes an angle of 32° with the vertical? (The child is held fixed in that position.)

a) What is the child's acceleration?

$a = 0$, not moving

b) Draw a free body diagram of the system. Choose a coordinate system.



$m = 30\text{ kg}$

$\theta = 32°$

$a = 0$

c) Using the picture from part b, write Newton's Second Law for the x-axis and the y-axis. Solve these equations for the required force.

$$\sum \vec{F} = m\vec{a} \Rightarrow \vec{F}_P + \vec{F}_T + m\vec{g} = \cancel{m\vec{a}}^{\,0}$$

$$x:\ F_T \sin\theta - F_P = 0 \quad \Rightarrow \quad F_T \sin\theta = F_P$$

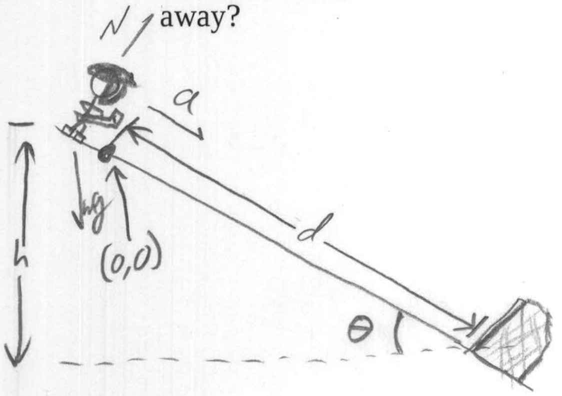
$$y:\ F_T \cos\theta - mg = 0 \quad \Rightarrow \quad F_T \cos\theta = mg$$

Divide x by y:

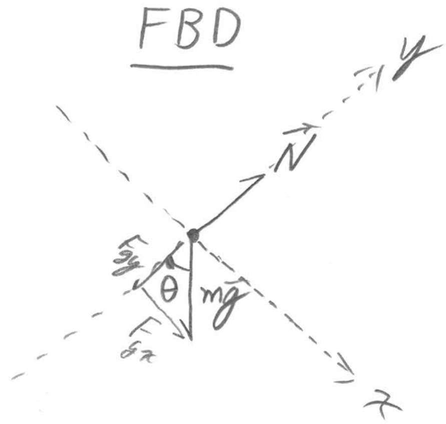
$$\frac{\cancel{F_T}\sin\theta}{\cancel{F_T}\cos\theta} = \frac{F_P}{mg} \Rightarrow \tan\theta = \frac{F_P}{mg} \Rightarrow \boxed{F_P = mg\tan\theta}$$

$$F_P = (30)(9.8)\tan(32) = \boxed{184\text{ N}}$$

A terrible earthquake has happened in San Francisco right in the middle of a critical hockey tournament. As a result of the quake, the ice rink is tilted 15° from horizontal. The 80 kg goalie begins to slide down the slope uncontrollably from his net directly into the opposing goalies net. How fast is he when he crosses the opposite goal line 53 m away?



FBD



$\theta = 15°$

$m = 80\ kg$

$d = 53\ m$

$V_F = ?$

Find the acceleration

$$\vec{F}_{net} = m\vec{a} \Rightarrow NSL$$

$$x: N_x + F_{gx} = ma_x$$

$$0 + mg\sin\theta = ma_x \Rightarrow \boxed{a_x = g\sin\theta}$$

$$y: N_y + F_{gy} = 0$$

$$N_y - mg\cos\theta = 0 \Rightarrow N_y = mg\cos\theta$$

okay... not useful here...

no acceleration in the y

continued ↓

Ice rink continued

We have acceleration, now find V_F using kinematics.

But there's only action in x:

$$x = x_0 + V_{0x}t + \tfrac{1}{2}a_x t^2 \qquad V = V_0 + at$$

$$① \; d = 0 + 0 + \tfrac{1}{2}g\sin\theta\, t^2 \qquad ② \; V_F = 0 + g\sin\theta\, t$$

acceleration is in positive x

Solve ① for t and plug into ②

From ①: $t = \left[\dfrac{2d}{g\sin\theta}\right]^{1/2}$

into ②: $V_F = g\sin\theta\left[\dfrac{2d}{g\sin\theta}\right]^{1/2}$

$$\boxed{V_F = (2gd\sin\theta)^{1/2}}$$

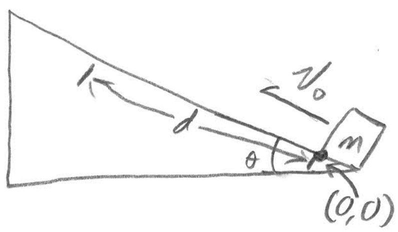
hmm... $d\sin\theta = h$

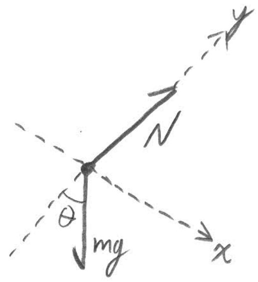
$$V_F = [(2)(9.8)(53)\sin(15)]^{1/2} = \boxed{16.4\ m/s}$$

Force Problems – Set 1

A block is given an initial velocity of 5 m/s up a frictionless 20° incline. How far up the incline does the block slide before coming to rest?

a) Draw a free body diagram of the block





$V_0 = 5\,m/s$

$\theta = 20^\circ$

$d = ?$

b) Put a coordinate system on your free body diagram and, from the resulting picture, write Newton's second law for the x axis and for the y axis. Solve these equations for the acceleration of the block.

$$x:\ \cancel{m}g\sin\theta = \cancel{m}a_x \Rightarrow a_x = g\sin\theta$$

$$y:\ N - mg\cos\theta = 0$$

c) Use the kinematics equations and the acceleration from part b to find the distance.

$$x = x_0 + V_0 t + \tfrac{1}{2}a_x t^2 \qquad V = V_0 + a_x t$$

$$-d = 0 - V_0 t + \tfrac{1}{2}g\sin\theta\, t^2 \qquad 0 = -V_0 + g\sin\theta\, t$$

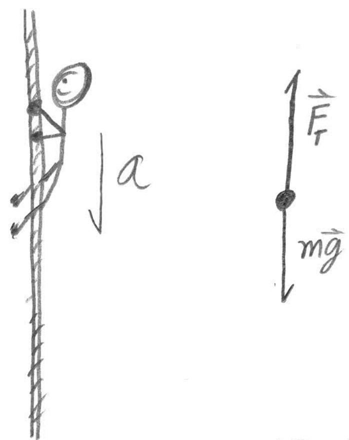
$$t = \frac{V_0}{g\sin\theta}$$

$$-d = \frac{-V_0^2}{g\sin\theta} + \frac{1}{2}\frac{V_0^2}{g\sin\theta}$$

$$\boxed{d = -\frac{V_0^2}{2g\sin\theta}} = \frac{-5^2}{(2)(9.8)\sin(20)} = \boxed{-3.7\,m}$$

A 52 kg circus performer slides down a rope that will break if the tension exceeds 425 N.

a) What happens if the performer hangs stationary from the rope?
b) At what acceleration will the performer just avoid breaking the rope?



$F_{Tmax} = 425 N$
$m = 52\ kg$

NSL: $\sum F = ma$

$F_T - mg = ma$

a) Stationary performer, (or performer at constant v):

$a = 0$

$\Rightarrow F_T - mg = 0 \Rightarrow \boxed{F_T = mg}$

$F_T = (52\ kg)(9.8\ m/s^2) = 510\ N$

Rope breaks.

b) Accelerating performer: $F_T = ma + mg < F_{Tmax}$

or: $\boxed{a < \frac{F_{Tmax}}{m} - g} \Rightarrow a < \frac{425N}{52kg} - 9.8\ m/s^2$

$a < -1.6\ m/s^2$ or less.