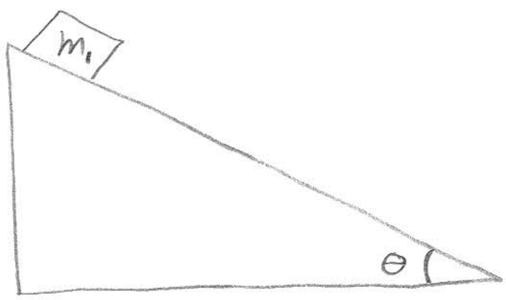
Force Problems – Set 2

A block rests on an incline plane that makes an angle θ with the horizontal. The coefficient of static friction between the block and the plane is μ_s and the coefficient of kinetic friction is μ_k.

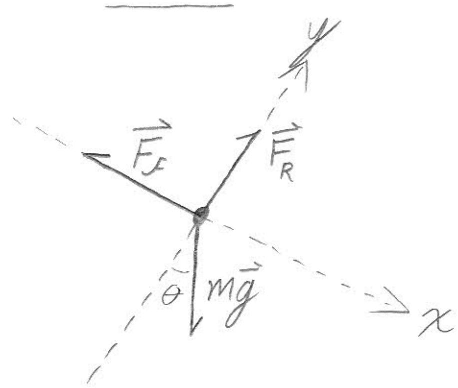
a) Find an expression for the maximum angle of the incline before the block slips.

HINT: The equation for the force of static friction ($F = \mu_s N$) represents the ***maximum*** *force that friction can provide.*

b) Assuming that the block is in motion, find an expression for the ~~velocity~~ acceleration of the block in terms of θ and μ_k.



FBD



a) The block isn't moving $\Rightarrow a_x = a_y = 0$. When the force in the positive x exceeds the max frictional force, the block slips.

$\boxed{\text{Write NSL}}$

x

$$\sum F_x = m a_x$$

$\boxed{\text{no slipping}}$

$$mg\sin\theta - F_f = 0$$

$$\boxed{mg\sin\theta = F_f} \quad (1)$$

y

$$\sum F_y = m a_y$$

$$F_R - mg\cos\theta = 0$$

$$\Rightarrow \boxed{F_R = mg\cos\theta} \quad (2)$$

continued ↓

Force problems - Set 2, P1 continued

When eq 1 is true, the block will not slip.

We know that $\underline{F_f \le \mu_s F_R}$

So, as long as:

$$\boxed{mg\sin\theta \le \mu_s F_R} \quad ③$$

The block won't slip

Now, we need to plug in ② into ③ to eliminate the unknown F_R.

$$\Rightarrow \cancel{mg}\sin\theta \le \mu_s \cancel{mg}\cos\theta$$

$$\Rightarrow \sin\theta \le \mu_s \cos\theta$$

$$\Rightarrow \tan\theta \le \mu_s$$

$$\Rightarrow \boxed{\theta \le \tan^{-1}(\mu_s)}$$

If θ exceeds this value, the block slips.

continued
↓

Force Problems - Set 2, P1 continued

b) This one we set up normally because the force from kinetic friction is constant.

$\boxed{\text{Write NSL}}$

$\underline{x}$

$\sum F_x = ma_x$

$mg\sin\theta - F_f = ma_x$

1 $\boxed{mg\sin\theta - \mu_k F_R = ma_x}$

$\underline{y}$

$\sum F_y = ma_y$

$F_R - mg\cos\theta = 0$

$\Rightarrow \boxed{F_R = mg\cos\theta}$ 2

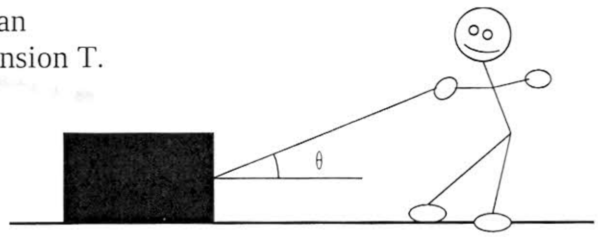
- Combine 1 and 2 to eliminate F_R

$\Rightarrow \cancel{m}g\sin\theta - \mu_k \cancel{m}g\cos\theta = \cancel{m}a_x$

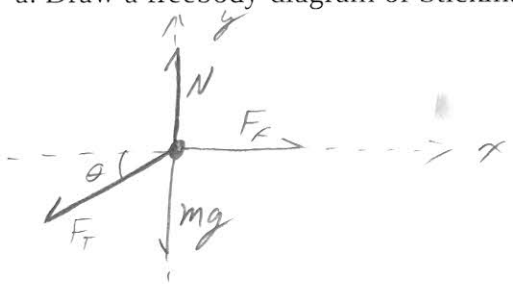
$$\boxed{a_x = g(\sin\theta - \mu_k\cos\theta)}$$

Force Problems – Set 2

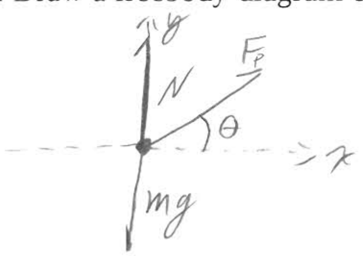
While standing on a rough surface, Stickman is pulling an ice block to the right with a tension T.



a. Draw a freebody diagram of Stickman. (He does not slide)



b. Draw a freebody diagram of the ice block. (It's frictionless)



c. Assume that the Box is frictionless and calculate the velocity of the box after it has traveled a distance d starting from rest. Your velocity should be in terms of m,T, θ, and d.

NSL

$x:\ F_P\cos\theta = ma \Rightarrow \boxed{a = \frac{F_P}{m}\cos\theta}$

$y:\ N + F_P\sin\theta - mg = 0$ Okay... Boring.

Kinematics

$x = x_0 + V_0 t + \frac{1}{2}at^2$

$d = 0 + 0 + \frac{1}{2}at^2$

$t = \left(\frac{2d}{a}\right)^{1/2}$

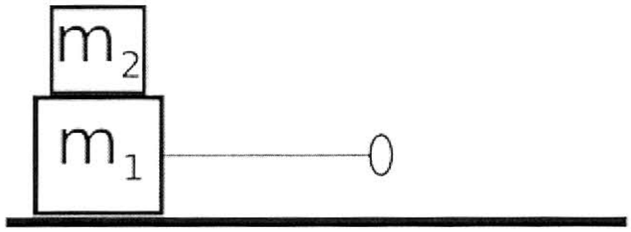
$V = V_0 + at$

$V = 0 + at$

$V = a\left(\frac{2d}{a}\right)^{1/2} \Rightarrow \boxed{V = \left(2d\frac{F_P}{m}\cos\theta\right)}$

Force Problems – Set 2

Two blocks with masses m_1 and m_2 are stacked up as shown in the picture below. A rope with a handle is attached to m_1 as shown. There is no friction between m_1 and the table. The coefficient of static friction between m_2 and m1 is μ_s.



a) Draw free body diagrams for m_1 and m_2.
b) Find an expression for the maximum force that can be applied to to the rope on m_1 without m_2 slipping.

a)

Two blocks stacked together exert Forces on each other (Newton's 3^{rd} law). They are equal in magnitude and opposite in direction.

$\Rightarrow \vec{F}_{21} = -\vec{F}_{12}$

continued

↓

Force Problems - Set 2 P3 continued

b) Now write <u>NSL</u> for each block

<u>m_1</u>

<u>x</u>

$\sum F_x = m_1 a_x$

$\boxed{F_P = m_1 a_{1x}}$ ①

<u>y</u>

$\sum F_y = ma_y$

$\boxed{F_R - F_{21} - m_1 g = 0}$ ②

<u>m_2</u>

<u>x</u>

$\sum F_x = ma_x$

$F_f = m_2 a_{2x}$

$\boxed{\mu_s F_{12} \le m_2 a_{2x}}$ ③

If $m_2 a_{2x}$ exceeds $\mu_s F_{12}$, the block slips...

<u>y</u>

$\sum F_y = ma_y$

$\boxed{F_{12} - m_2 g = 0}$ ④

F_{12} is the "squishing" force that drives the friction.

Now, $a_{1x} = a_{2x}$ since the blocks are mated.

So, subst ④ → ③ : $\boxed{\mu_s m_2 g \le m_2 a_{1x}}$ ⑤

and subst ① → ⑤ : $\mu_s \cancel{m_2} g \le \cancel{m_2} \frac{F_P}{m_1}$

So if $F_P \ge \mu_s m_1 g$, $\boxed{\text{the block slips}}$

Force Problems – Set 2 P3 continued

c) What's the total reaction force from the floor?

From eq ②: $F_R = F_{21} + m_1 g$

and from eq ④ and the fact that $|\vec{F}_{21}| = |\vec{F}_{12}|$

$$\Rightarrow F_R = m_2 g + m_1 g$$

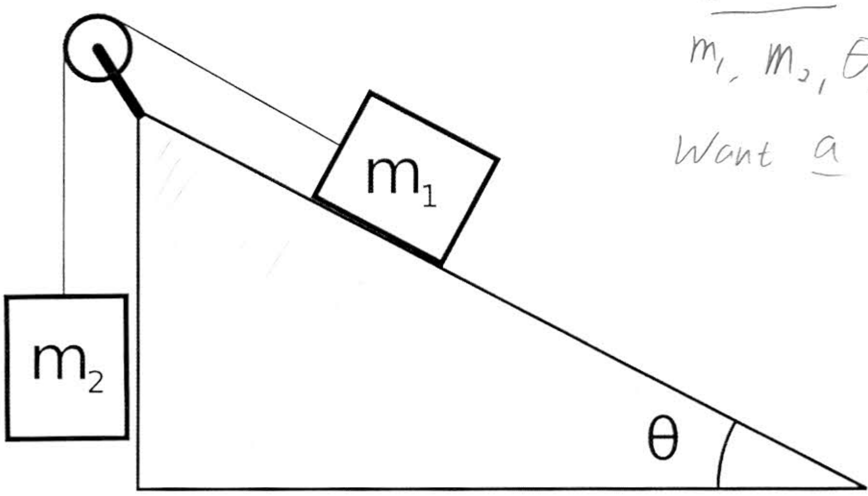
$$\Rightarrow \boxed{F_R = (m_1 + m_2) g}$$

So the floor must hold up the combined weight of both blocks.

Force Problems – Set 2

In the picture below, the coefficient of kinetic friction between the ramp and m1 is $\cancel{\mu_s}$ μ_k. A rope connecting m1 and m2 passes over a massless frictionless pulley.

Calculate the acceleration of the system.

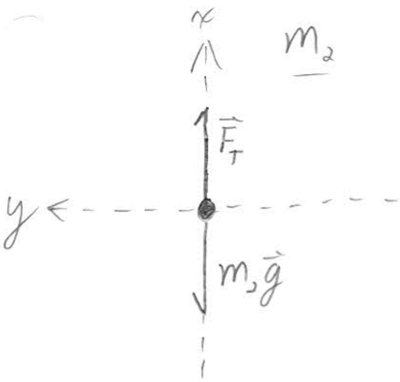


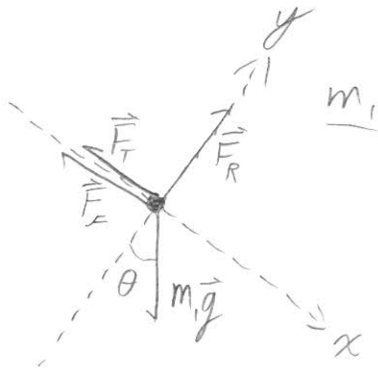
Given

m_1, m_2, θ, μ_k

Want a

Draw FBDs





The coordinate systems here are aligned so that the positive x-axes are in the direction of the acceleration of the block.

continued

↓

Force Problems - Set 2 P4 continued

Write $\boxed{NSL}$

m_1

x

$\sum F_x = ma_x$

$m_1 g \sin\theta - F_T - F_f = m_1 a_{1x}$

and $F_f = \mu_k F_R$ so...

$\Rightarrow \boxed{m_1 g \sin\theta - F_T - \mu_k F_R = m_1 a_{1x}}$ ①

y

$\sum F_y = ma_y$

$\boxed{F_R - m_1 g \cos\theta = 0}$ ②

m_2

x

$\sum F_x = ma_x$

$\boxed{F_T - m_2 g = m_2 a_{2x}}$ ③

y

$\sum F_y = ma_y$

$0 = 0$

$a_{1x} = a_{2x}$ because we aligned the coordinate systems and they are connected by a rope.

Solve ② for F_R and subst → eq ①:

$\Rightarrow m_1 g \sin\theta - F_T - \mu_k m_1 g \cos\theta = m_1 a_{1x}$ ④

continued

↓

Force Problems - Set 2 P4 continued

Solve 3 For F_T and subst into 4

From 3: $F_T = m_2 g + m_2 a_{1x}$ ← replaced a_{2x} with a_{1x}

into 4: $\boxed{m_1 g \sin\theta - m_2 g - m_2 a_{1x} - \mu_K m_1 g \cos\theta = m_1 a_{1x}}$ 5

Eq 5 has no unknown variables. Isolate a_{1x} (which is the same as a_{2x})

From 5

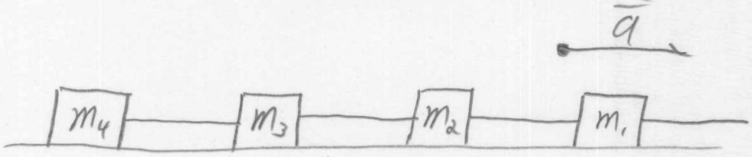
$$m_1 a_{1x} + m_2 a_{1x} = m_1 g \sin\theta - \mu_K m_1 g \cos\theta - m_2 g$$

$$\Rightarrow (m_1 + m_2) a_{1x} = g\left[m_1(\sin\theta - \mu_K \cos\theta) - m_2\right]$$

$$\Rightarrow \boxed{a_{1x} = g\left[\frac{m_1(\sin\theta - \mu_K\cos\theta) - m_2}{m_1 + m_2}\right]}$$

Force Homework Packet

Penguins



$m_1 = 20\ kg$ $\quad T_1 = 222\ N$

$m_2 = 15\ kg$ $\quad T_2 = ?$

$\boxed{m_3 = ?}$ $\quad T_3 = 111\ N$

$m_4 = 12\ kg$ $\quad T_4 = ?$

Surface is frictionless so I'll ignore N and mg.

FBD

NSL

① m_4: $T_4 = m_4 a$ $\qquad$ ③ m_2: $T_2 - T_3 = m_2 a$

② m_3: $T_3 - T_4 = m_3 a$ $\qquad$ ④ m_1: $T_1 - T_2 = m_1 a$

The system is linked by ropes so $\underline{a}$ is the same for all penguins.

We want m_3, so we'll eliminate T_2, T_4, and a

use eq ③ and ④ to eliminate T_2:

$T_2 = T_3 + m_2 a$ $\qquad$ $T_2 = T_1 - m_1 a$

⑤ $T_3 + m_2 a = T_1 - m_1 a$

continued ↓

Penguins continued

Use eq ① and ② to eliminate T_4

$$T_4 = m_4 a \qquad T_4 = T_3 - m_3 a$$

$$m_4 a = T_3 - m_3 a \quad ⑥$$

Re-arrange ⑤ and ⑥ to get a on one side

$$T_1 - T_3 = m_2 a + m_1 a \qquad T_3 = m_4 a + m_3 a$$

$$T_1 - T_3 = a(m_2 + m_1) \qquad T_3 = a(m_3 + m_4)$$

divide

$$\frac{T_1 - T_3}{T_3} = \frac{\cancel{a}(m_2 + m_1)}{\cancel{a}(m_3 + m_4)}$$

Isolate m_3

$$(T_1 - T_3)(m_3 + m_4) = T_3(m_2 + m_1)$$

$$T_1 m_3 + T_1 m_4 - T_3 m_3 - T_3 m_4 = T_3 m_2 + T_3 m_1$$

$$m_3(T_1 - T_3) = T_3 m_2 + T_3 m_1 + T_3 m_4 - T_1 m_4$$

$$\boxed{m_3 = \frac{T_3(m_1 + m_2 + m_4) - T_1 m_4}{T_1 - T_3}} \qquad m_3 = \frac{111(20 + 15 + 12) - 222 \cdot 12}{222 - 111}$$

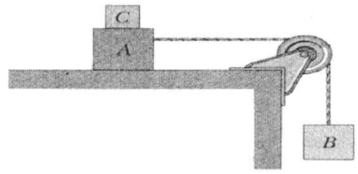
$$\boxed{= 23\,\mathrm{kg}}$$

SAMPLE TEST 2
PHYS 111 SPRING 2010

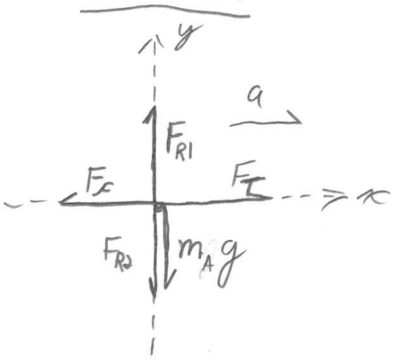
2) A and B are blocks with ~~weights~~ masses of 44 ~~N~~ kg and 22 ~~N~~ kg respectively. The coefficient of friction between the table and the block is 0.20.

a) Determine the minimum weight of block C so that block A does not slide.

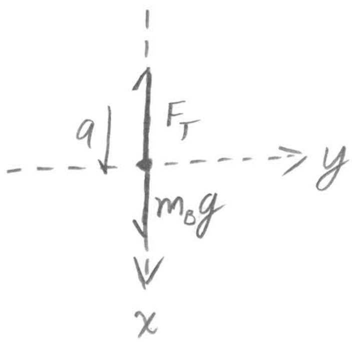
b) If block C is suddenly lifted off of A, what is the acceleration of A?



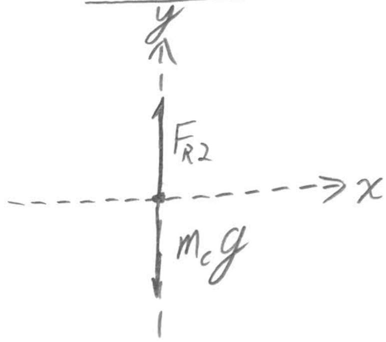
FBD A



FBD B



FBD C



NSL A

$x: \sum F_x = ma_x$

$F_T - F_f = 0$

$① \underbrace{F_T - \mu_s F_{R1} = 0}_{\text{slipping condition}}$

$y: \sum F_y = ma_y$

$② F_{R1} - F_{R2} - m_A g = 0$

NSL B

$x: \sum F_x = ma_x$

$③ m_B g - F_T = 0$ static

$y: \sum F_y = ma_y$

$0 = 0$

NSL C

$x: \sum F_x = ma_x$

$0 = 0$

$y: \sum F_y = ma_y$

$④ F_{R2} - m_c g = 0$

* Find m_c in terms of μ_s, m_A, and m_B. Need to eliminate all reaction forces

Add ② + ④ : $F_{R1} - \cancel{F_{R2}} - m_A g + \cancel{F_{R2}} - m_c g = 0 + 0$

$⑤ F_{R1} - (m_A + m_c) g = 0$

Add ① + ③ : $\cancel{F_T} - \mu_s F_{R1} + m_B g - \cancel{F_T} = 0 + 0$

$⑥ m_B g - \mu_s F_{R1} = 0$

continued ↓

continued (ST2, P2)

Solve ⑤ for F_{R1} and subst into ⑥

$$F_{R1} = (m_A + m_c)g$$

$$⑦\ m_B g - \mu_s (m_A + m_c) g = 0$$

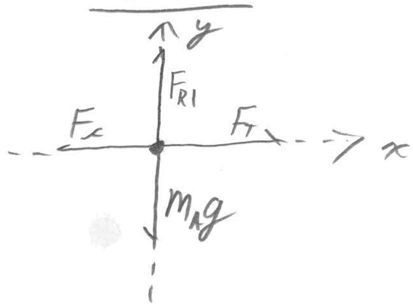
Solve ⑦ for m_c

$$m_B \cancel{g} = \mu_s m_A \cancel{g} + \mu_s m_c \cancel{g}$$

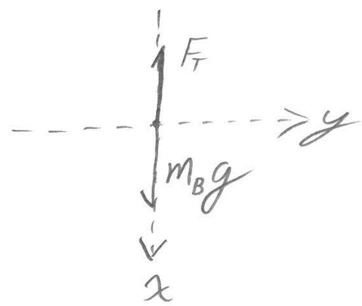
$$\boxed{m_c = \frac{1}{\mu_s}\left[m_B - \mu_s m_A\right]}$$

$$m_c = \frac{1}{0.2}\left[22 - (0.2)(44)\right] = \boxed{66\ kg}$$

b) FBD A



FBD B



NSL A

$$x: \sum F_x = m a_x$$

$$F_T - F_c = m_A a$$

$$①\ F_T - \mu_k F_{R1} = m_A a$$

$$y: \sum F_y = m a_y$$

$$②\ F_{R1} - m_A g = 0$$

NSL B

$$x: \sum F_x = m a_x$$

$$③\ m_B g - F_T = m_B a$$

$$y: \sum F_y = m a_y$$

$$0 = 0$$

continued ↓

continued (ST2, P2)

Solve ② for F_{R1} and subst. into ①

④ $F_T - \mu_K m_A g = m_A a$

add ④ + ③

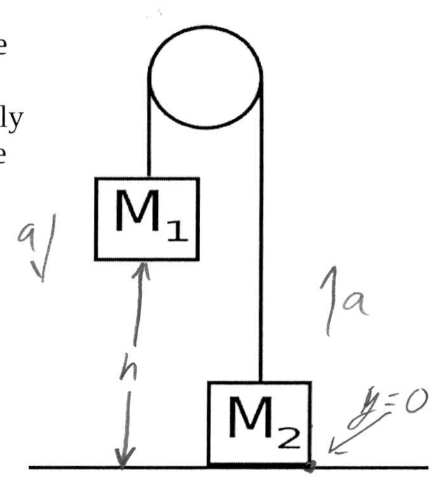
⑤ $\cancel{F_T} - \mu_K m_A g + m_B g - \cancel{F_T} = m_A a + m_B a$

solve ⑤ for a

$$\boxed{a = \frac{m_B - \mu_K m_A}{m_A + m_B} g} = \frac{22 - (0.2)(44)}{44 + 22}(9.8) = \boxed{1.96 \text{ m/s}^2}$$

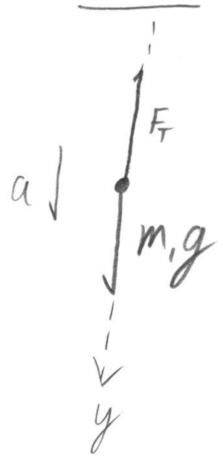
One end of a rope is connected to a mass M_1=10kg. The rope passes over a massless frictionless pulley and the other end is connected to a mass M_2 = 5kg. M_2 is initially resting on the ground and M_1 is suspended 3m above the ground. The system is initially at rest.

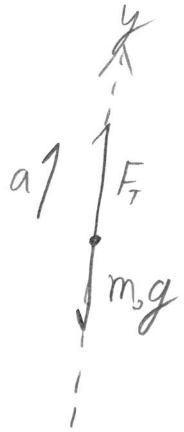
If M_1 is released and allowed to hit the ground, what is the maximum height that M_2 will reach?



*HINT: When M_1 hits the ground, M_2 will still have an upward velocity. The rope will go slack and M_2 will **continue** upward until its velocity is zero.*

FBD





NSL

① $m_1 g - F_T = m_1 a$

② $F_T - m_2 g = m_2 a$

From ②: $F_T = m_2 a + m_2 g$

Into ①: $m_1 g - m_2 a - m_2 g = m_1 a$

$$(m_1 - m_2) g = (m_1 + m_2) a$$

③ $$\boxed{a = \frac{m_1 - m_2}{m_1 + m_2} g}$$

continued

Atwood — continued

We have acceleration, now we'll do kinematics.

M_2 will accelerate through a distance h with a from the force analysis. Then, it will essentially be in "free fall" with some initial upward velocity $\underline{V_1}$.

Find V_1

$$y = y_0 + V_0 t + \tfrac{1}{2}at^2$$

$$h = 0 + 0 + \tfrac{1}{2}at^2$$

$$t = \left(\frac{2h}{a}\right)^{1/2}$$

$$V = V_0 + at$$

$$V_1 = 0 + at$$

$$V_1 = a\left(\frac{2h}{a}\right)^{1/2} = (2ha)^{1/2}$$

$$\boxed{V_1 = (2ha)^{1/2}} \quad (4)$$

Now find h_{max}

$$y = y_0 + V_0 t + \tfrac{1}{2}at^2$$

$$h_{max} = h + V_1 t - \tfrac{1}{2}gt^2$$

$$V = V_0 + at$$

$$0 = V_1 - gt$$

$$t = \frac{V_1}{g}$$

$$h_{max} = h + \frac{V_1^2}{g} - \frac{1}{2}\frac{V_1^2}{g}$$

$$\boxed{h_{max} = h + \tfrac{1}{2}\frac{V_1^2}{g}} \quad (5)$$

continued ↓

Atwood continued.

combine 3, 4, and 5

$$h_{max} = h + \frac{\cancel{2}ha}{\cancel{2}g}$$

$$h_{max} = h + \frac{h}{\cancel{g}} \frac{m_1 - m_2}{m_1 + m_2} \cancel{g}$$

$$h_{max} = h\left(1 + \frac{m_1 - m_2}{m_1 + m_2}\right) = h \frac{m_1 + \cancel{m_2} + m_1 - \cancel{m_2}}{m_1 + m_2}$$

$$\boxed{h_{max} = \frac{2m_1}{m_1 + m_2} h}$$

So... If $m_2 < m_1$, $h_{max} > h$.

$$h_{max} = \frac{(2)(10)}{10 + 5}(3) = \frac{20}{15} 3 = 4_m$$