

Force Problems – Set 3

For each of the following questions, pick the correct response AND explain your reasoning.

A car is driving around a circular track at a constant speed. The net force on the car:

- a) Is zero because the car is not accelerating
- b) Is directed away from the center of the circle
- c) is directed forward, in the direction of the car's travel.
- d) is directed towards the center of the circle.

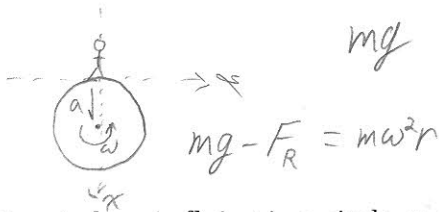
Circular motion requires a centrally directed force

If the Earth rotated faster on its axis, your weight (as measured by a scale) would:

- a) increase
- b) stay the same
- c) decrease

As the Earth spins faster, the required central force increases.

mg is constant so, as $\omega \uparrow$, $F_c \downarrow$



An airplane is flying in a circle at a constant speed. If the pilot turns so that the radius of the circle gets larger but his speed stays the same, the centripetal force: *(let radius double)*

- a) stays the same
- b) doubles
- c) quadruples
- d) halves

$a_c = \frac{v^2}{r}$, If $r \uparrow$ by 2, $a \downarrow$ by 2

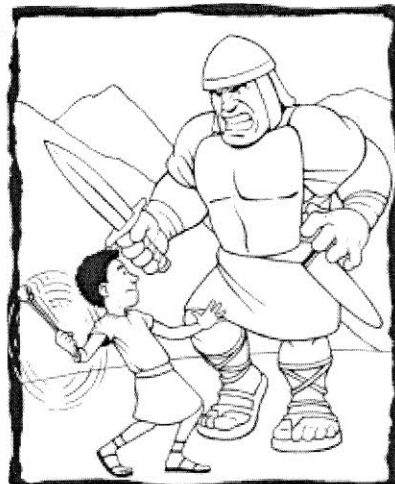
If the pilot doubles his speed but the radius of the circle stays the same, the centripetal force:

- a) stays the same
- b) doubles
- c) quadruples
- d) halves

$a_c = \frac{v^2}{r}$ so $2 \times v \Rightarrow 4 \times a$

Force Problems – Set 3

David is fighting Goliath. To defeat him, David's rock must leave his 0.5 m sling at 20 m/s. Unfortunately, his sling was damaged and the maximum tension the rope can withstand is 500 N. What is the mass of largest rock that David can sling? Ignore gravity.



- Draw a free body diagram of the rock (Ignore Gravity)
- Without writing down Newton's Second Law, write an expression for the acceleration of the rock.



$$a = r\omega^2 = \frac{v^2}{r}$$

← centripetal acc.

- Write Newton's Second Law for the rock based on your answer to parts a and b. Solve it for the mass of the rock.

$$\Sigma F = ma \Rightarrow F_T = m \frac{v^2}{r}$$

$$= \left[m = F_T \frac{r}{v^2} \right]$$

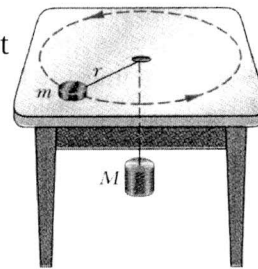
← does this give mass?
 $F_T \left(\frac{\text{kg} \cdot \text{m}}{\text{s}^2} \right) \left(\frac{\text{m}}{(\text{m}/\text{s})^2} \right) = \text{kg. okay!}$

- Calculate the mass of the rock.

$$m = (500 \text{ N}) \cdot \left(\frac{0.5 \text{ m}}{(20 \text{ m/s})^2} \right) = \left[0.625 \text{ kg} \right]$$

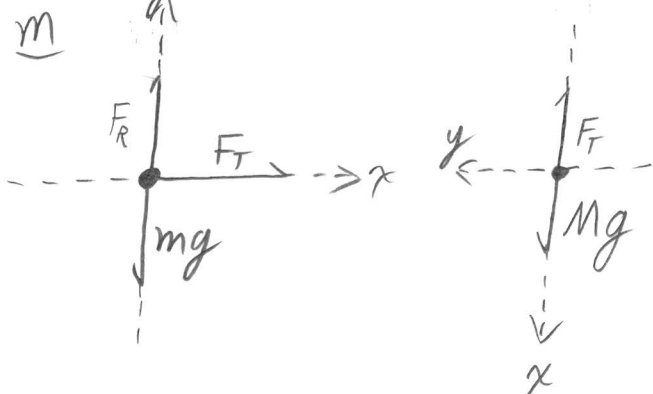
Force Problems – Set 3

A mass m on a frictionless table is attached to a hanging mass M by a cord through a hole in the table. Find the speed with which m must move in order for M to stay at rest.



FBD

side view



NSL

m

$$x: \sum F_x = ma_x$$

$$\textcircled{1} F_T = m \frac{v^2}{r} \Rightarrow \text{UCM}$$

$$y: \sum F_y = md_y$$

$$\textcircled{2} F_R - mg = 0 \Rightarrow \text{No motion in } y$$

NSL

M

$$x: \sum F_x = Ma_x$$

$$\textcircled{3} Mg - F_T = 0$$

$$y: 0 = 0$$

So, from $\textcircled{3}$: $F_T = Mg$

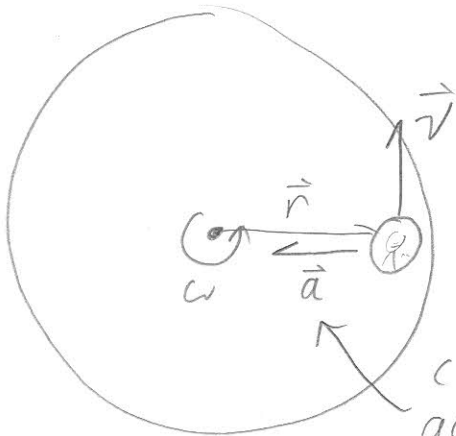
then into $\textcircled{1}$: $Mg = m \frac{v^2}{r}$

$$\Rightarrow v = \left[\frac{M}{m} gr \right]^{1/2}$$

Force Problems – Set 3

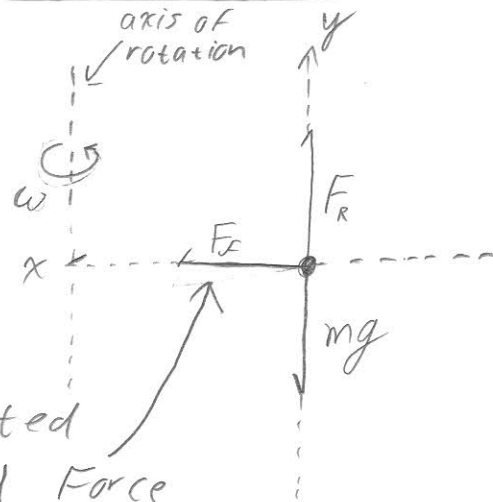
A penny of mass m is placed on a turntable (remember those?) and the turntable begins spinning. The coefficient of static friction between the penny and the turntable is μ_s . What is the fastest angular velocity ω that the turntable can spin before the penny slips.

Top view



centrally directed acceleration and Force

FBD - Side view



NSL

x

$$\sum F_x = ma_x$$

$$F_f = ma_x \leftarrow \text{centripetal acceleration}$$

$$\boxed{\mu_s F_R \geq mr\omega^2} \quad (1)$$

Because $\boxed{F_f \leq \mu_s F_R}$

Subst (2) \rightarrow (1): $\mu_s mg \geq mr\omega^2$

$$\Rightarrow \boxed{\omega \leq \left(\frac{\mu_s g}{r}\right)^{1/2}}$$

y

$$\sum F_y = ma_y$$

$$F_R - mg = 0$$

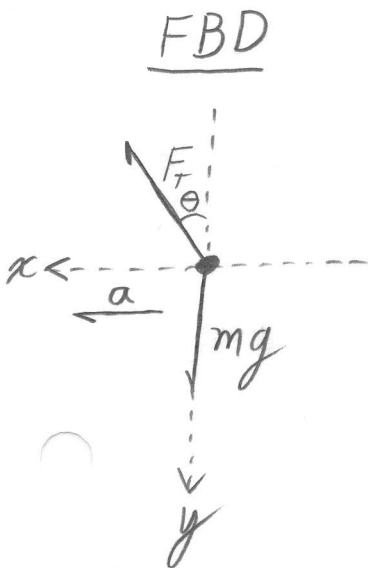
$$\boxed{F_R = mg} \quad (2)$$

Force Problems – Set 3

Napoleon lands a flippin' sweet blow to the tether ball, which is attached to a rope of whose length is $L = 0.60\text{m}$ sending it whipping around the pole with a period of of 0.75s .



- a) Napoleon is jealous of your awesome physics skills and needs your help finding an expression for the angle, θ , that the rope makes with the pole.
- b) Calculate a numeric value for θ using your expression from part a and use it to calculate the tension in the rope.



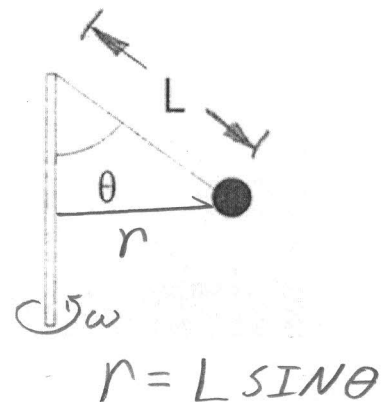
NSL

$x: \sum F_x = ma_x$

① $F_T \sin \theta = m \frac{v^2}{r}$

$y: \sum F_y = ma_y$

② $F_T \cos \theta - mg = 0$



Let's eliminate F_T Between ① and ② First and then deal with the period.

From ②: $F_T \cos \theta = mg$ ③

Divide ①: $\frac{F_T \sin \theta}{F_T \cos \theta} = \frac{m \frac{v^2}{r}}{mg} \Rightarrow \boxed{\tan \theta = \frac{v^2}{rg}}$ ④

Now we can relate v and r to the period.

$v = \frac{d}{t} \Rightarrow \boxed{v = \frac{2\pi r}{P}}$ | Plug into ④

Napoleon - continued

From (4): $\tan\theta = \frac{4\pi^2 r^2}{p^2} \frac{1}{kg}$

$$\tan\theta = \frac{4\pi^2 r}{g p^2} \quad (5)$$

But $r = L \sin\theta$

so: $\tan\theta = \frac{4\pi^2}{g p^2} L \sin\theta$

$$\Rightarrow \frac{\cancel{\sin\theta}}{\cos\theta} = \frac{4\pi^2 L}{g p^2} \cancel{\sin\theta}$$

$$\Rightarrow \boxed{\cos\theta = \frac{4\pi^2 L}{g p^2}}$$

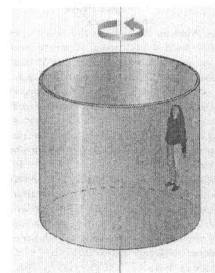
b) $\theta = \cos^{-1}\left(\frac{g p^2}{4\pi^2 L}\right) = \cos^{-1}\left[\frac{(9.8)(0.75)^2}{(4)\pi^2(0.6)}\right]$

$$\boxed{\theta = 76^\circ}$$

Force Problems – Set 3

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The Cyclone is an exciting state fair ride consisting of a large vertical cylinder that spins about its axis. Riders stand inside the cylinder with their backs against the wall. When the cylinder is spinning fast enough, the floor drops away and the rider “sticks” to the wall.



If the coefficient of static friction between a wall and person is μ_s , and the radius of the cylinder is R , show that the maximum period of revolution is given by:

$$P = \left(\frac{4\pi^2 R \mu_s}{g} \right)^{\frac{1}{2}}$$

NSL

$$x: \sum F_x = ma_x$$

$$\textcircled{1} F_R = mr\omega^2$$

$$y: \sum F_y = ma_y$$

$$F_c - mg = 0$$

$$\textcircled{2} \mu_s F_R - mg = 0$$

Subst $\textcircled{1} \rightarrow \textcircled{2}$: $\mu_s mr\omega^2 = mg$

$$\omega = \frac{2\pi}{P}$$

$$\mu_s r \frac{4\pi^2}{P^2} = g$$

$$P = \left(\frac{4\pi^2 \mu_s r}{g} \right)^{\frac{1}{2}}$$