TEST 2

PHYS 111, FALL 2010, SECTION 1

Name: ______________________________

By writing my name above, I affirm that this test represents my work only, without aid from outside sources. In all aspects of this course I perform with honor and integrity.

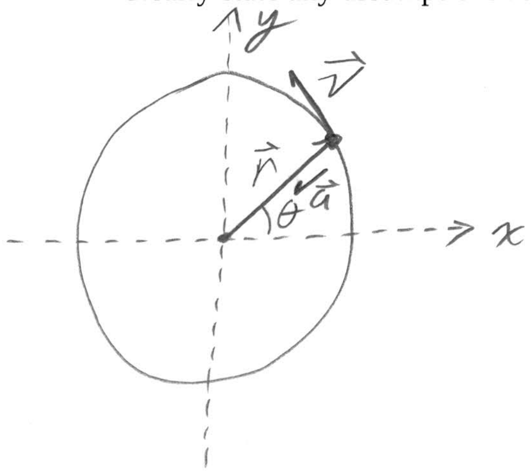
SHOW YOUR WORK ON ALL OF THE PROBLEMS. YOUR APPROACH TO THE PROBLEM IS AS IMPORTANT AS, IF NOT MORE IMPORTANT THAN, YOUR ANSWER. DRAW **CLEAR AND NEAT PICTURES** SHOWING COORDINATE SYSTEMS AND ALL OF THE RELEVANT PROBLEM VARIABLES. ALSO, **EXPLICITLY** SHOW THE **BASIC EQUATIONS** YOU ARE USING. BE NEAT AND THOROUGH. THE EASIER IT IS FOR ME TO UNDERSTAND WHAT YOU ARE DOING, THE BETTER YOUR GRADE WILL BE.

1) Starting with the unit vector expression for the position, r, of a particle constrained to move in a circle, derive an expression for the magnitude of the velocity vector and the magnitude of the acceleration vector assuming that the particle is moving in **uniform circular motion.**

Include a picture with the position vector, r, the velocity vector, v, the acceleration vector, a, and the position angle, θ, clearly marked.

assume: r and $\frac{d\theta}{dt}$ are const.

Clearly state any assumptions required by the proof.



$$\vec{r} = r_x\hat{i} + r_y\hat{j}$$

$$\vec{r} = r\cos\theta\,\hat{i} + r\sin\theta\,\hat{j}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = r\frac{d}{dt}(\cos\theta\,\hat{i} + \sin\theta\,\hat{j})$$

$$= r\left(-\frac{d\theta}{dt}\sin\theta\,\hat{i} + \frac{d\theta}{dt}\cos\theta\,\hat{j}\right)$$

$$\vec{v} = r\frac{d\theta}{dt}(-\sin\theta\,\hat{i} + \cos\theta\,\hat{j})$$

$$\boxed{v = r\omega} \quad \omega \equiv \frac{d\theta}{dt}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = r\omega\frac{d}{dt}(-\sin\theta\,\hat{i} + \cos\theta\,\hat{j})$$

$$= r\omega\frac{d\theta}{dt}(-\cos\theta\,\hat{i} - \sin\theta\,\hat{j})$$

$$\boxed{a = r\omega^2}$$

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2) Multiple Choice, 4 points each.

2.1) A piece of mud flies off of the rim of a spinning bicycle wheel. In what direction will it be going after leaving the wheel? (ignore gravity)

A) On a line directed away from the center of the wheel.

B) On a line tangent to the rim of the wheel.

C) On a curved path spiraling outward from the wheel.

D) Between the lines described in A and B above.

2.2) A sports car collides with a truck. Which experiences the greatest magnitude force during the collision?

A) The car
B) The truck
C) They magnitudes of the forces are the same.
D) It depends on their relative speeds

2.3) Which experiences the greatest magnitude acceleration?

A) The car.
B) The truck.
C) The magnitudes of their accelerations are the same.
D) It depends on their relative speeds.

2.4) A mass, m, is pushed with a horizontal force, F, on a horizontal surface but does not move. The coefficient of static friction is μ_s. The magnitude of the static friction force on the mass is

a) $\mu_s mg$
b) $\mu_s F$
c) F
d) mg

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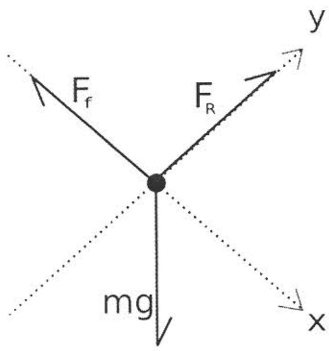
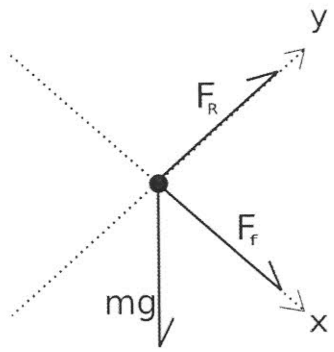
2.4) A block is accelerating down a rough incline plane. Which is the correct free body diagram?

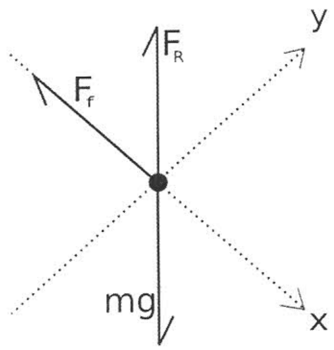
A)



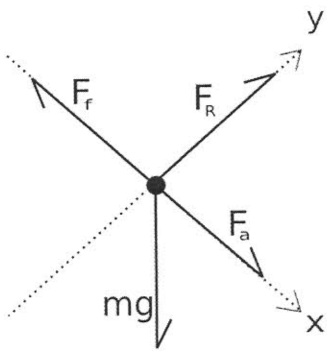
B)



C)



D)



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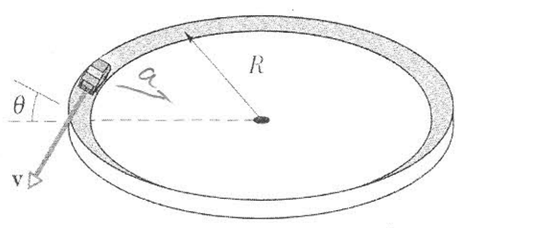
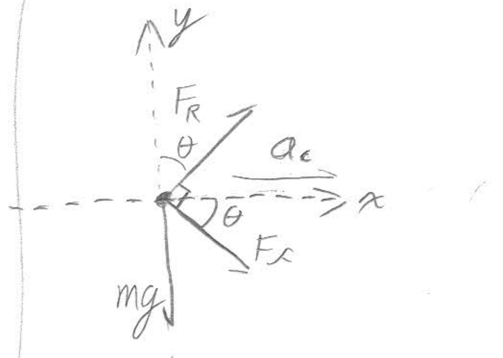
4) Curves on roadways are often banked so that the reaction force from the road provides part of the centripetal acceleration required to keep the car moving around the circle.



Assume the road is a circle of radius R, the car is going some velocity v, and the tires have some coefficient of static friction μ_s.

Find an expression for the minimum angle required to keep the car on the road in terms of R, v, and μ_s.

FBD - Side view



(perpendicular)

F_R is always $\perp$ to road surface

* F_f is always $\parallel$ to road surface.

(parallel)

NSL

x: $\sum F_x = ma_x$

centripetal

$$F_R \sin\theta + F_f \cos\theta = m a_c$$

$$\Rightarrow F_R \sin\theta + \mu_s F_R \cos\theta = m \frac{v^2}{R} \quad ①$$

no accel in y

y: $\sum F_y = ma_y$

$$F_R \cos\theta - F_f \sin\theta - mg = 0$$

$$\Rightarrow F_R \cos\theta - \mu_s F_R \sin\theta - mg = 0 \quad ②$$

From ①: $F_R(\sin\theta + \mu_s \cos\theta) = m\frac{v^2}{R}$

From ②: $F_R(\cos\theta - \mu_s \sin\theta) = mg$

Divide $\frac{1}{2}$: $\frac{\cancel{F_R}(\sin\theta + \mu_s\cos\theta)}{\cancel{F_R}(\cos\theta - \mu_s \sin\theta)} = \frac{\cancel{m}\, v^2/R}{\cancel{m} g}$

$$\Rightarrow gR\sin\theta + \mu_s gR\cos\theta = v^2\cos\theta - \mu_s v^2 \sin\theta$$

$$(gR + \mu_s v^2)\sin\theta = (v^2 - \mu_s gR)\cos\theta$$

$$\Rightarrow \boxed{\tan\theta = \frac{v^2 - \mu_s gR}{gR + \mu_s v^2}}$$

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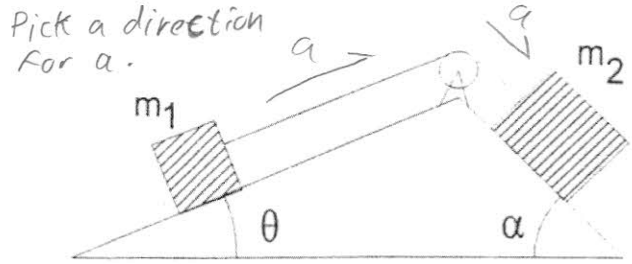
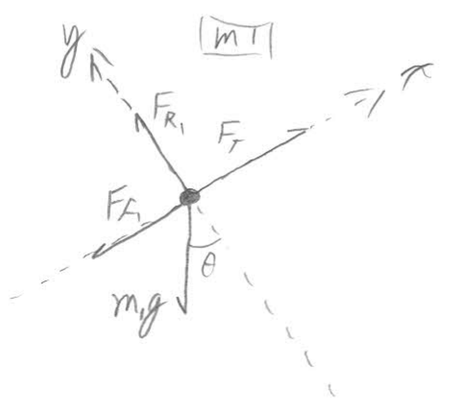
3) Consider the figure shown below. Assume the pulley is massless and frictionless.

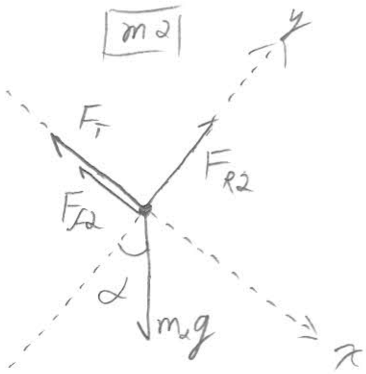
$\theta = 30^{\circ}$
$\alpha = 45^{\circ}$
$\mu_k = 0.10$
$m_1 = 5.0$ kg
$m_2 = 15.0$ kg



a) Find an expression (no numbers) for the acceleration of the system.
b) Plug in the numbers and find the numeric answer.

Free Body Diagrams





a) Write NSL

m1:

x: $\Sigma F_x = ma_x$

① $F_T - F_{f1} - m_1 g \sin\theta = m_1 a$

y: $\Sigma F_y = ma_y$

② $F_{R1} - m_1 g \cos\theta = 0$

m2:

x: $\Sigma F_x = ma_x$

③ $m_2 g \sin\alpha - F_T - F_{f2} = m_2 a$

y: $\Sigma F_y = ma_y$

④ $F_{R2} - m_2 g \cos\alpha = 0$

continued ↓

Sample Test 2 - P3 continued

* Rewrite friction terms using: $F_f = \mu_K F_R$

From 1: $F_T - \mu_K F_{R1} - m_1 g \sin\theta = m_1 a$ (5)

From 3: $m_2 g \sin\alpha - F_T - \mu_K F_{R2} = m_2 a$ (6)

* Solve 2 and 4 for F_R and subst. into 5 and 6

Subst 2 → 5:

$$F_T - \mu_K m_1 g \cos\theta - m_1 g \sin\theta = m_1 a \quad (7)$$

Subst 4 → 6:

$$m_2 g \sin\alpha - F_T - \mu_K m_2 g \cos\alpha = m_2 a \quad (8)$$

* Solve 7 for F_T and subst into 8

$$m_2 g \sin\alpha - \mu_K m_1 g \cos\theta - m_1 g \sin\theta - m_1 a - \mu_K m_2 g \cos\alpha = m_2 a$$

$$\Rightarrow g\left[m_2(\sin\alpha - \mu_K \cos\alpha) - m_1(\sin\theta + \mu_K \cos\theta)\right] = a(m_1 + m_2)$$

$$\boxed{a = g\,\frac{m_2(\sin\alpha - \mu_K\cos\alpha) - m_1(\sin\theta + \mu_K\cos\theta)}{(m_1 + m_2)}}$$

b)

$$a = 9.8\,\frac{(15)(\sin(45) - (0.10)\cos(45)) - (5.0)(\sin(30) + (0.10)\cos(30))}{5.0 + 15.0}$$

$$\boxed{a = 3.24\ m/s^2}$$

SAMPLE TEST 2

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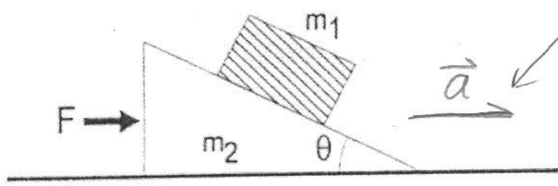
4) In the figure shown below all surfaces are frictionless. Find an expression for the the force applied to block m_2 so that block m1 does not slide down the ramp.

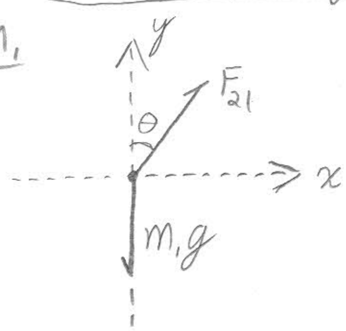
$\theta = 30^\circ$
$m_1 = 2.0$ kg
$m_2 = 5.0$ kg



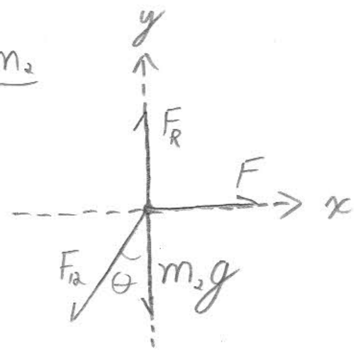
Both blocks are accelerating parallel to the floor. with the same accel.

Free Body diagrams

m_1



m_2



NSL

m_1

x: $\Sigma F_x = m a_x$

① $F_{21}\sin\theta = m_1 a$

y: $\Sigma F_y = m a_y$

② $F_{21}\cos\theta - m_1 g = 0$

m_2

x: $\Sigma F_x = m a_x$

③ $F - F_{21}\sin\theta = m_2 a$

④ y: $F_R - F_{12}\cos\theta - m_2 g = 0$

* To solve ③ for F, I'll eliminate a with eq ① and eliminate F_{21} with eq. ②.

continued

↓

Sample Test 2 Problem 4 continued

Divide $\frac{③}{①}$: $\frac{F - F_{21}\sin\theta}{F_{21}\sin\theta} = \frac{m_2 \cancel{a}}{m_1 \cancel{a}} \Rightarrow F - F_{21}\sin\theta = \frac{m_2}{m_1} F_{21}\sin\theta$

$$\Rightarrow F = F_{21}\sin\theta\left(1 + \frac{m_2}{m_1}\right) \quad ⑤$$

From ②: $F_{21} = \frac{m_1 g}{\cos\theta}$

$$\Rightarrow F = \frac{m_1 g}{\cos\theta}\sin\theta\left(1 + \frac{m_2}{m_1}\right)$$

$$\Rightarrow F = m_1\left(1 + \frac{m_2}{m_1}\right) g \tan\theta$$

$$\Rightarrow F = \left(m_1 + \cancel{m_1}\frac{m_2}{\cancel{m_1}}\right) g \tan\theta$$

$$\Rightarrow \boxed{F = (m_1 + m_2)\, g \tan\theta}$$

$$F = (2.0\text{ kg} + 5.0\text{ kg})(9.8\text{ m/s}^2)\tan(30^\circ)$$

$$\boxed{F = 40\text{ N}}$$