SAMPLE TEST 6
PHYS 111, FALL 2010, SECTION 1

Name: ______________________________

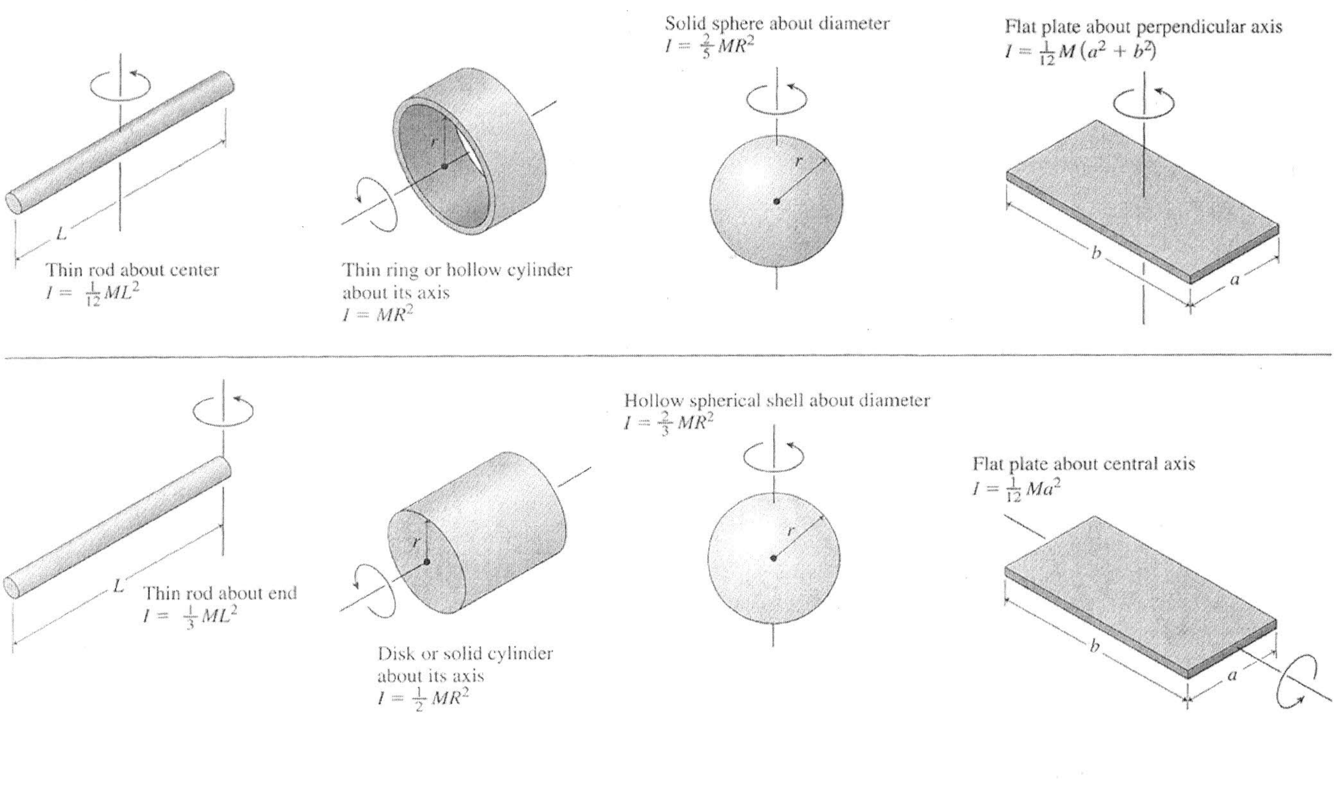
By writing my name above, I affirm that this test represents my work only, without aid from outside sources. In all aspects of this course I perform with honor and integrity.

SHOW YOUR WORK ON ALL OF THE PROBLEMS. YOUR APPROACH TO THE PROBLEM IS AS IMPORTANT AS, IF NOT MORE IMPORTANT THAN, YOUR ANSWER. DRAW **CLEAR AND NEAT PICTURES** SHOWING COORDINATE SYSTEMS AND ALL OF THE RELEVANT PROBLEM VARIABLES. ALSO, **EXPLICITLY** SHOW THE **BASIC EQUATIONS** YOU ARE USING. BE NEAT AND THOROUGH. THE EASIER IT IS FOR ME TO UNDERSTAND WHAT YOU ARE DOING, THE BETTER YOUR GRADE WILL BE.

A few potentially useful equations

Moment of Inertia, discrete definition	$I=\sum m_i r_i^2$
Moment of Inertia, integral definition	$I=\int r^2\,dm$
Parallel Axis Theorem	$I=I_{cm}+Md^2$
Superposition	$I_{Total}=\sum I_i$

TABLE 10.2 Rotational Inertias



SAMPLE TEST 6

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1) Derivations

a) (10pts) Given a differential equation of the form $\frac{d^2 x(t)}{dt^2} = -\omega^2 x(t)$, write the general solution for $x(t)$, $v(t)$, and $a(t)$ in terms of the angular frequency ω, the amplitude A, and the phase angle φ.

assume: $x(t) = A\cos(\omega t + \phi)$ ①

then: $v(t) = \frac{dx}{dt} = -\omega A \sin(\omega t + \phi)$ ②

$a(t) = \frac{dv}{dt} = \frac{d^2x}{dt^2} = -\omega^2 A\cos(\omega t + \phi)$ ③

b) (10pts) Given the boundary conditions $x(t_0) = x_0$ and $v(t_0) = v_0$, derive an expression for the phase angle φ and the amplitude A.

From eq ①: $x_0 = A\cos(\omega t_0 + \phi)$

From eq ②: $v_0 = -\omega A \sin(\omega t_0 + \phi)$

Divide $\frac{②}{①}$: $\frac{v_0}{x_0} = \frac{-\omega \sin(\omega t + \phi)}{\cos(\omega t + \phi)} \Rightarrow \tan(\omega t + \phi) = -\frac{v_0}{x_0 \omega}$

$\Rightarrow \omega t + \phi = \tan^{-1}\left(-\frac{v_0}{x_0\omega}\right) \Rightarrow \boxed{\phi = \tan^{-1}\left(-\frac{v_0}{x_0\omega}\right) - \omega t}$

If we divide v_0 by ω, we have

$v_0/\omega = -A\sin(\omega t + \phi)$

then we can write: $\left(x_0^2 + \left(\frac{v_0}{\omega}\right)^2\right)^{1/2} = \left(A^2\cos^2(\omega t + \phi) + A^2\sin^2(\omega t + \phi)\right)^{1/2}$

$= A\left(\cos^2(\omega t + \phi) + \sin^2(\omega t + \phi)\right)^{1/2}$

$\Rightarrow \boxed{A = \left(x_0^2 + \left(\frac{v_0}{\omega}\right)^2\right)^{1/2}}$

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A mass attached to a spring is in simple harmonic motion. At the exact moment the mass moves through equilibrium:

2.1) its instantaneous acceleration

a) has maximum magnitude.

b) is zero.

c) has greater than zero magnitude (but not maximum).

2.2) its instantaneous speed

a) has maximum magnitude.

b) is zero.

c) has greater than zero magnitude (but not maximum).

2.3) A mass attached to the free end of an ideal spring is in simple harmonic motion with an amplitude A=0.5 m and an angular frequency 18 rad/s. What is the maximum velocity of the mass?

a) 36 m/s
b) 9 m/s
c) 3 m/s
d) None of the above

$$V_{max} = \omega A = 0.5 \cdot 18 = 9\ m/s$$

2.4) A person sits on a swing. When the person sits still, the swing oscillates back and forth at its natural frequency. If, instead, two people sit on the swing, the new frequency of the swing is

a) Greater
b) Less
c) The same

$$\omega = \sqrt{\frac{k}{m}} \quad \text{as } m\uparrow,\ \omega\downarrow$$

2.5) A person sits on a swing. When the person sits still, the swing oscillates back and forth at its natural frequency. If, instead, the person stands on the swing, the new natural frequency of the swing is

a) Greater
b) Less
c) The same

$$\omega = \frac{mg\,l_{cm}}{I} = \frac{mg\,l_{cm}}{Ml^2}$$

I increases faster than l_{cm}

So $I\uparrow$, $\omega\downarrow$

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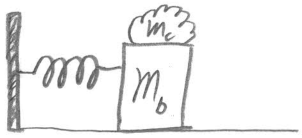
A block of mass m_b = 5.0 kg is attached to a spring of spring constant k = 4 N/m where it is allowed to oscillate horizontally on a frictionless surface. The spring is compressed a distance d = 0.5 m from equilibrium and released. As the block passes equilibrium, a wad of clay, m_c = 10.0 kg, falls from directly above and sticks to the block.

a) What is the angular frequency of the block/clay system?
b) What is the velocity of the block/clay system just after the collision?
c) What is the amplitude of the block/clay system?

a)



$$\sum F = ma$$

$$-kx = (m_b + m_c)a \Rightarrow a = -\boxed{\frac{k}{m_b + m_c}}\,x$$

$$\boxed{\omega = \left(\frac{k}{m_b + m_c}\right)^{1/2}} = \left[\frac{4}{15}\right]^{1/2} = \underline{0.52\ \text{rad/s}}$$

b) To know the velocity after the collision, I need to know the velocity before the collision.

at $t=0$, $x(0) = d$, $v(0) = 0$

$x \Rightarrow$ so: $d = A\cos(\phi)$ ①
$v \Rightarrow$ and: $0 = -\omega \sin\phi$ ② $\Rightarrow \sin(\phi) = 0 \Rightarrow \boxed{\phi = 0}$

then: $d = A\cos(0) \Rightarrow \boxed{d = A}$

Before collision

At equilibrium, $v = \boxed{v_{max} = \omega A}$, where $\boxed{\omega = \sqrt{\frac{k}{m_b}}}$

collide: $P_I = P_F \Rightarrow m_b v_{max} = (m_b + m_c) v_F$

continued ↓

Sample Test 6 P_3 continued

$$m_b\sqrt{\frac{k}{m_b}}\,d = (m_b + m_c)\,v_F$$

$$\Rightarrow \boxed{v_F = \frac{d\sqrt{m_b k}}{m_b + m_c}} = \frac{(0.5)(5.0\cdot 4)^{1/2}}{15} = 0.15\ m/s$$

c) Let $t=0$, $v(0) = 0.15\ m/s$

$x(0) = 0$

$$x \Rightarrow 0 = A\cos(\phi) \Rightarrow \phi = \pi/2,\ 3\pi/2$$

$$v \Rightarrow 0.15 = -\omega A\, \cancel{\sin(3\pi/2)}^{-1}$$

$$0.15 = \omega A \Rightarrow A = \frac{0.15}{0.52} = \underline{0.29\ m}$$

Ⓘ

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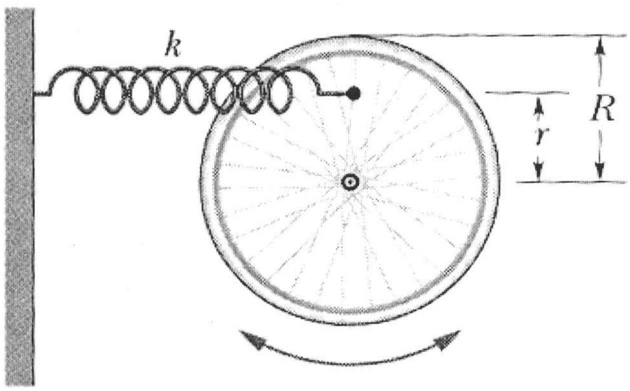
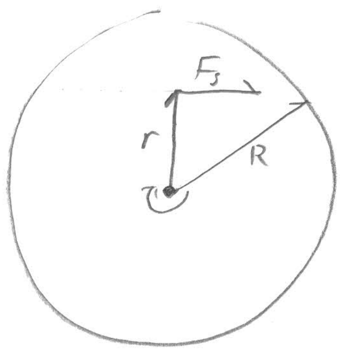
A wheel is free to rotate about a fixed axle. A spring with a spring constant k is attached to one of its spokes at a distacne r from the axle, as shown in the picture. Assume that the wheel is a hoop of mass m and radius R (the spokes have negligible mass).

a) Using **Newton's Second Law**, find the angular frequency of small oscillations in terms of m, R, r and the spring constant k.
b) Using **Energy techniques**, find the angular frequency of small oscillations in terms of m, R, r and the spring constant k.
b) What is the angular frequency if r = R.
c) What is the angular frequency if r = 0.





a) $\sum T = I\alpha$

$F_s r = I\alpha$

$-kxr = I\alpha \quad , \quad x = r\theta$

$-kr\theta r = I\alpha$

$$\alpha = -\boxed{\frac{kr^2}{I}}\,\theta \Rightarrow \omega = \left[\frac{kr^2}{I}\right]^{1/2}, \quad I = mR^2$$

$$\Rightarrow \boxed{\omega = \left[\frac{kr^2}{mR^2}\right]^{1/2}}$$

continued

Sample Test 6, P4 continued

b) $E_T = \frac{1}{2}kx^2 + \frac{1}{2}I\omega_v^2$ ← angular velocity!

$$\frac{dE_T}{dt} = \cancel{\frac{1}{2}}k\cancel{2}x\frac{dx}{dt} + \cancel{\frac{1}{2}}I\cancel{2}\omega_v\frac{d\omega_v}{dt} = 0$$

$$\Rightarrow kxv + I\omega_v\alpha = 0 \quad , \quad v = r\omega_v \Rightarrow \omega_v = \frac{v}{r}$$

$$\Rightarrow kx\cancel{v} + mR^2\frac{\cancel{v}}{r}\cdot\frac{a}{r} = 0 \quad a = r\alpha \Rightarrow \alpha = \frac{a}{r}$$

$$\Rightarrow kx + m\frac{R^2}{r^2}a = 0$$

$$\Rightarrow a = -\boxed{\frac{k}{m}\frac{r^2}{R^2}}x \Rightarrow \boxed{\omega = \left[\frac{k}{m}\frac{r^2}{R^2}\right]^{1/2}}$$

c) if $R = r$, $\omega = \sqrt{\frac{k}{m}}$

d) if $r = 0$, $\underline{\omega = 0}$

Sample Test 6

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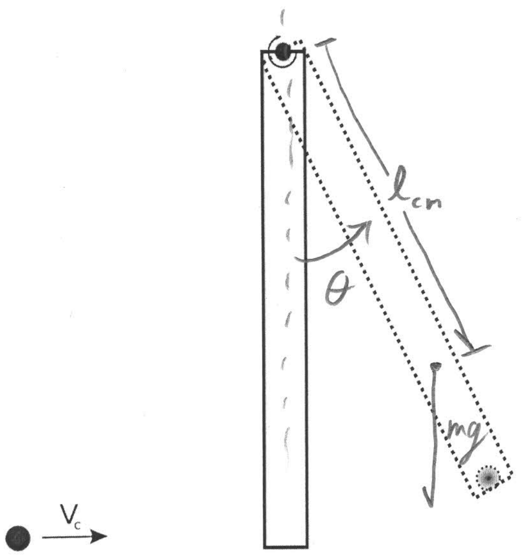
A 1 kg meter stick is hung from its and allowed to pivot. A small wad of clay with a mass of 0.25 kg with a velocity $V_c = 2$ m/s impacts the bottom of the meter stick. Assuming that the resulting oscillations are small:

a) (10 pts) Find the angular frequency of the resulting pendulum.

~~b) (10 pts) Find the amplitude of the resulting pendulum in terms of θ.~~



$$T = -mg\, l_{cm} \sin\theta$$

a) $\Sigma T = I\alpha$

$$-mg\, l_{cm} \sin\theta = I\alpha \Rightarrow \alpha = \frac{-mg\, l_{cm}}{I} \sin\theta$$

And for small θ, $\sin\theta \cong \theta$

$$\Rightarrow \alpha = \frac{-mg\, l_{cm}}{I}\theta \Rightarrow \omega = \left[\frac{m_T g\, l_{cm}}{I}\right]^{1/2}$$ where $\underline{m_T = m + M}$

Need to find I and l_{cm}

$I = I_s + I_c$, $I_s = I_{cm} + Md^2$

$$I_s = \frac{1}{12}ML^2 + M\left(\frac{L}{2}\right)^2 = \boxed{\frac{1}{3}ML^2}$$

$$I_c = mL^2$$

$$\boxed{I = \left(\tfrac{1}{3}M + m\right)L^2}$$

continued ↓

Sample Test 6, P5 continued

$$l_{cm} = \frac{\sum m_i l_i}{\sum m_i} = \frac{M\frac{L}{2} + mL}{M+m} = \boxed{\frac{\frac{1}{2}M + m}{M+m} L}$$

$$\omega = \left[\underbrace{\cancel{(m+M)}}_{m_T} g \underbrace{\frac{\frac{1}{2}M + m}{\cancel{M+m}} \cancel{L}}_{l_{cm}} \underbrace{\frac{1}{(\frac{1}{3}M + m)L^{\cancel{2}}}}_{\frac{1}{I}} \right]^{1/2}$$

$$\boxed{\omega = \left[\frac{\frac{1}{2}M + m}{\frac{1}{3}M + m} \frac{g}{L} \right]^{1/2}}$$

$m = \frac{1}{4}M, \quad L = 1$

$$\omega = \left[\frac{\frac{1}{2}\cancel{M} + \frac{1}{4}\cancel{M}}{\frac{1}{3}\cancel{M} + \frac{1}{4}\cancel{M}} \frac{g}{1} \right]^{1/2} \Rightarrow \boxed{\omega = \left[\frac{9}{7} g \right]^{1/2}}$$

b) ~~conserve angular momentum to~~ find