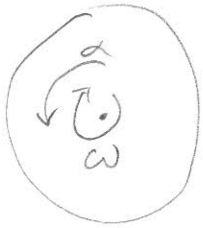
Rotation – Set 1

An aging hippie is jammin' to his old "In-A-Gadda-Da-Vida" album. The turntable on which the record sits spins it with a constant angular speed of 33.33 revolutions per minute. The hippie notices that when he turns off the turntable motor, the record makes exactly three revolutions before stopping.

a) What's the angular deceleration of the record ~~in radians per second~~, man? (Assume it's a constant.)

b) He also notices that when he starts the turntable, it takes the record 3.0 seconds to come up to speed. After the record gets up to speed, "In-A-Gadda-Da-Vida" plays for an agonizing 3.50 minutes. What is the total angular displacement of the record at the end of the song?

a)



$\theta_0 = 0$ $\qquad \alpha = ?$

$\theta = 3 \text{ rev}$

$\omega_0 = 33.3 \text{ rev/min} = 33.3 \text{ rev/min} \cdot \frac{1}{60} \frac{\text{min}}{\text{s}} = 0.55 \text{ rev/s}$

$\omega = 0$

$$\theta = \theta_0 + \omega_0 t + \tfrac{1}{2}\alpha t^2 \qquad \omega = \omega_0 + \alpha t$$

$$\theta = \omega_0 t + \tfrac{1}{2}\alpha t^2 \qquad 0 = \omega_0 + \alpha t$$

$$\Rightarrow t = -\frac{\omega_0}{\alpha}$$

$$\theta = -\frac{\omega_0^2}{\alpha} + \frac{1}{2}\alpha \frac{\omega_0^2}{\alpha^2}$$

$$\theta = -\frac{1}{2}\frac{\omega_0^2}{\alpha} \Rightarrow \boxed{\alpha = -\frac{1}{2}\frac{\omega_0^2}{\theta}}$$

$$\alpha = -\frac{1}{2}\frac{33.3^2}{3} = -184 \text{ rev/min}^2$$

or $$\alpha = -\frac{1}{2}\frac{(0.55)^2}{3} = 0.05 \text{ rev/s}^2 = 0.314 \text{ rad/s}^2$$

Rotation Problems, Set 1, P1 continued

b) Let: $\theta_0 = 0$, $\omega_0 = 0$, $\omega = 33.3\ \text{rev/min}$, $t_1 = 3.0\,\text{s}$, $t_2 = 3.5\ \text{min}$

<u>During acceleration</u>

$$\theta = \cancel{\theta_0}^{=0} + \cancel{\omega_0 t}^{=0} + \tfrac{1}{2}\alpha t^2$$

$$\theta_1 = \tfrac{1}{2}\alpha t_1^2$$

$$\Rightarrow \theta_1 = \tfrac{1}{2}\frac{\omega}{\cancel{t}} t_1^{\cancel{2}}$$

$$\boxed{\theta_1 = \tfrac{1}{2}\omega_1 t_1}$$

$$\omega_1 = \omega_0 + \alpha t$$

$$\omega_1 = \alpha t$$

$$\Rightarrow \alpha = \frac{\omega}{t}$$

<u>After acceleration:</u> $\omega_0 = \omega_1 = 33.3\ \text{rev/min}$, $\theta_0 = \theta_1$

$$\theta = \theta_0 + \omega_0 t + \cancel{\tfrac{1}{2}\alpha t^2}^{=0}$$

$$\theta_2 = \theta_1 + \omega_1 t_2$$

$$t_1 = 3.0\,\text{s} \cdot \frac{1}{60}\frac{\text{min}}{\text{s}} = \underline{0.05\,\text{m}}$$

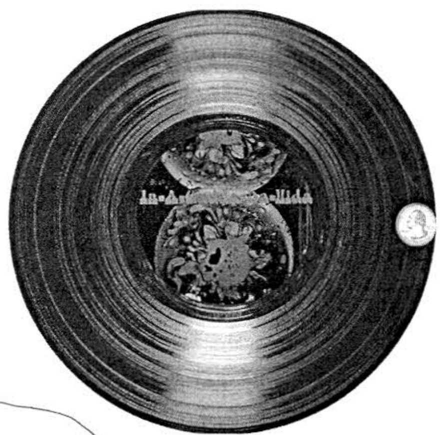
$$\Rightarrow \theta_2 = \tfrac{1}{2}\omega_1 t_1 + \omega_1 t_2$$

$$\Rightarrow \boxed{\theta_2 = \omega_1\left(\tfrac{t_1}{2} + t_2\right)}$$

$$\theta_2 = 33.3\ \text{rev/min}\left(\tfrac{1}{2}(0.05\,\text{m}) + 3.5\,\text{m}\right)$$

$$\boxed{\theta_2 = 117\ \text{revolutions}}$$

Your hippie friend is fascinated by watching a quarter on his record go around and around and around. He's somehow managed to formulate a few questions about the quarter that he needs your help answering.



a) So dude, like, if I totally put the quarter a distance r from the center, like, what distance, c, does it go in one revolution?

$C = 2\pi r$, Because $\boxed{\pi = \frac{C}{D}}$ ← It's the definition of π!

C = Circumference
D = diameter

b) Groovy, so then, like, how far does the quarter go, let's call the distance s, if the record rotates through an angle θ.

I like to use a ratio.

$\frac{\theta}{\theta_1} = \frac{s}{c} \Rightarrow \boxed{\frac{\theta}{\theta_1} = \frac{s}{2\pi r}}$

$\theta_1 = \theta$ for 1 rev

θ in degrees

$\frac{\theta}{360} = \frac{s}{2\pi r}$

$\Rightarrow \boxed{s = \frac{\pi}{180} r\theta}$

θ in radians

$\frac{\theta}{\cancel{2}\pi} = \frac{s}{\cancel{2\pi} r}$

$\boxed{s = r\theta}$ ← My favorite!

c) Woah! Trippy... so then how FAST, is the quarter moving, call it v, in terms of the angular velocity ω?

If: $s = r\theta$ and $\omega = \frac{d\theta}{dt}$, then let's take the derivative

$\frac{ds}{dt} = r\frac{d\theta}{dt} \Rightarrow \boxed{v = r\omega}$

d) Right on! So like, let's say the record totally has angular acceleration, α! What's the *tangential* acceleration, a_T, of the quarter?

Again, $v = r\omega$ and $\alpha = \frac{d\omega}{dt}$

So: $\frac{dv}{dt} = r\frac{d\omega}{dt} \Rightarrow \boxed{a_T = r\alpha}$

what's a_c? oh yea! $\boxed{a_c = r\omega^2}$

Rotation – Set 1

The hippie wants to explore rolling the record across the floor now. Thankfully, this means that the terrible music has stopped playing. Assume that the record rolls in a perfectly straight line.

a) If the radius of the record is r = 30 cm, how far has it rolled after one revolution?

b) What is it's total angular displacement after it has rolled 5 meters?

c) If the initial *translational* velocity of the center of the record is 2 m/s, what is it's angular velocity?

d) If the record comes to a stop after 5 m, what was is its *angular* acceleration (assuming constant acceleration).

a) It has to roll one circumference, so:

$C = 2\pi r \Rightarrow C = 2\pi(0.30m) = \boxed{1.9\ m}$

b) From problem #2, we know:

$s = r\theta$, with θ in radians.

$\Rightarrow \theta = \frac{s}{r} \Rightarrow \theta = \frac{5m}{0.3m} = \boxed{16.7\ radians}$

$\frac{16.7\ rad}{2\pi\ rad/rev} = \boxed{2.7\ revolutions}$

c) We know $v = r\omega \Rightarrow \omega = \frac{v}{r} \Rightarrow \omega = \frac{2.0\ m/s}{0.3m} = \boxed{6.7\ rad/s}$

$\omega = 6.7\ \cancel{rad}/s \cdot \frac{1\ rev}{2\pi\ \cancel{rad}}$

$\boxed{\omega = 1.1\ rev/s}$

continued

d) We know $a_T = r\alpha$. But what's a_T?

① $\underline{x} = \cancel{x_0}^{\,0} + \underline{v_0}t + \frac{1}{2}\overset{?}{\boxed{a_T}}\overset{?}{\boxed{t^2}}$ $\qquad$ $\cancel{v}^{\,0} = \underline{v_0} + \overset{?}{\boxed{a_T}}\overset{?}{\boxed{t}}$ ②

2 eq. 2 unknowns!

solve ② for t and plug into ①

From ②: $t = -\frac{v_0}{a_T}$

into ①: $x = -v_0 \frac{v_0}{a_T} + \frac{1}{2}\cancel{a_T}\frac{v_0^2}{a_T^{\cancel{2}}}$

$\Rightarrow x = -\frac{v_0^2}{a_T} + \frac{1}{2}\frac{v_0^2}{a_T}$

$\Rightarrow x = -\frac{1}{2}\frac{v_0^2}{a_T} \Rightarrow a_T = -\frac{1}{2}\frac{v_0^2}{x}$

Then!

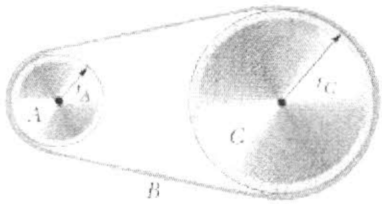
$a_T = r\alpha \Rightarrow -\frac{1}{2}\frac{v_0^2}{x} = r\alpha \Rightarrow \boxed{\alpha = -\frac{v_0^2}{2rx}}$

$$\alpha = -\frac{(2\,m/s)^2}{(2)(0.3\,m)(5\,m)} = 1.3\ rad/s^2$$

$$= (1.3\ rad/s^2)\left(\frac{1}{2\pi}\frac{rev}{rad}\right)$$

$$\boxed{= 0.2\ rev/s^2}$$

A pulley of radius $r_A = 10$ cm is coupled by a belt to a pulley of radius $r_C = 25$ cm. A motor is attached to the axle of pulley A giving it an angular acceleration of $\alpha_A = 1.6$ rad/s^2. How long does it take pulley C to achieve an angular velocity of 100 rev/min assuming that the belt does not slip?



$\alpha_A = 1.6\,\text{rad}/\text{s}^2$

$\omega_c = 100\,\frac{\text{rev}}{\text{min}} \cdot 2\pi\,\frac{\text{rad}}{\text{rev}} \cdot \frac{1\text{ min}}{60\text{ s}}$

$= 10.5\,\text{rad}/\text{s}$

We need to relate α_c to α_A.

Because they are coupled by the belt, a point on the rim of A must have the same linear acceleration as a point on the rim of C.

(1) $\boxed{a_A = r_A\alpha_A}$ and (2) $\boxed{a_c = r_c\alpha_c}$

But: $a_A = a_c = \underline{a}$

So, $r_A\alpha_A = r_c\alpha_c \Rightarrow \boxed{\alpha_c = \frac{r_A}{r_c}\alpha_A}$

Now, using kinematics:

$\omega = \cancel{\omega_0}^{0} + \alpha t \Rightarrow \omega_c = \alpha_c t$

$\Rightarrow t = \frac{\omega_c}{\alpha_c} \Rightarrow \boxed{t = \frac{\omega_c}{\alpha_A} \cdot \frac{r_c}{r_A}}$

$t = \frac{10.5\,\text{rad}/\text{s}}{1.6\,\text{rad}/\text{s}^2} \cdot \frac{25\,\text{cm}}{10\,\text{cm}} = 16.5\text{ s}$

Rotation – Set 1

4

A car's odometer works by counting the revolutions of the axle. A car's speedometer works by measuring the angular velocity of the axle. For the odometer and speedometer to work correctly, the correct tire radius has to be set at the factory.

My car came from the factory with tires with a diameter of $d_1 = 60$ cm. They wore out and I replaced them with tires with a diameter of $d_2 = 80$ cm.

a) With the proper tires on the car, What is the angular displacement of the tire after 1.0 km in radians and in revolutions? What is the angular velocity of the axle if I'm driving 100 km/hr in rad/s and in rev/s?

b) What are the answers to the questions in part a with the improper tires on the car?

c) If I've added 10,000 miles to the odometer since putting the new tires on, how many miles have I actually driven the car? (HINT: The odometer *assumes* that the proper tires are on the car.)

d) If the speed limit is 65 mph, what should my speedometer read so that I don't get a speeding ticket? (HINT: The speedometer *assumes* that the proper tires are on the car.)

a) $s = r\theta \Rightarrow \theta = \frac{s}{r}$, $\quad s = 1.0\,km$

$r = 60\,cm \cdot 1\times10^{-5}\,km/cm = 6.0\times10^{-4}\,km$

$$\theta = \frac{1.0\,km}{6.0\times10^{-4}\,km} = \boxed{1.7\times10^{3}\ radians}$$

$$= 1.7\times10^{3}\,rad \cdot \frac{1}{2\pi}\frac{rev}{rad}$$

$$= \boxed{271\ revolutions}$$

$$v = r\omega \Rightarrow \omega = \frac{v}{r} \Rightarrow v = 100\,km/hr \cdot \frac{1}{3600}\frac{hr}{s} = 2.8\times10^{-2}\,km/s$$

$$\omega = \frac{2.8\times10^{-2}\,km/s}{6.0\times10^{-4}\,km} = \boxed{47\,rad/s}$$

$$\omega = 47\,rad/s \cdot \frac{1}{2\pi}\frac{rev}{rad} = \boxed{7.5\,rev/s}$$

continued ↓

Rotation Set 1, P4 continued.

b) Improper tires, $r = 80\,cm \cdot 1\times10^{-5}\,km/cm = 8.0\times10^{-4}\,km$

$$\theta = \frac{1.0\,km}{8.0\times10^{-4}\,km} = \underline{1250 \text{ radians}}$$

$$\theta = \frac{1250\,rad}{2\pi\,rad/rev} = \underline{199\,rev.}$$

$$\omega = \frac{2.8\times10^{-2}\,km/s}{8.0\times10^{-4}\,km} = \underline{35\,rad/s}$$

$$\omega = \frac{35\,rad/s}{2\pi\,rad/rev} = \underline{5.6\,rev/s}$$

c) The odometer assumes $r = d_1$ and displays:

① $S_o = d_1\theta$, θ = axle rotation in radians

S_o = Odometer reading

In reality, the distance traveled by the car is given by:

② $S_R = d_2\theta$ θ = axle rotation in radians

S_R = actual distance traveled

θ is common between ① and ②, so dividing $\frac{②}{①}$

$$\frac{S_R}{S_o} = \frac{d_2\cancel{\theta}}{d_1\cancel{\theta}} \Rightarrow \boxed{S_R = \frac{d_2}{d_1}S_o} \Rightarrow S_R = \frac{80}{60}10{,}000 = \boxed{13{,}333 \text{ miles}}$$

continued ↓

Rotation Set 1 P ~~3~~ 4, continued

d) The argument is similar to part c:

The speedometer assumes $r = d_1$ and displays:

① $V_0 = d_1 \omega$ ω = angular velocity of axle in rad/s

V_0 = Speedometer reading

In reality, the velocity of the car is given by:

② $V_R = d_2 \omega$ ω = angular velocity of the axle in rad/s

V_R = actual speed of car.

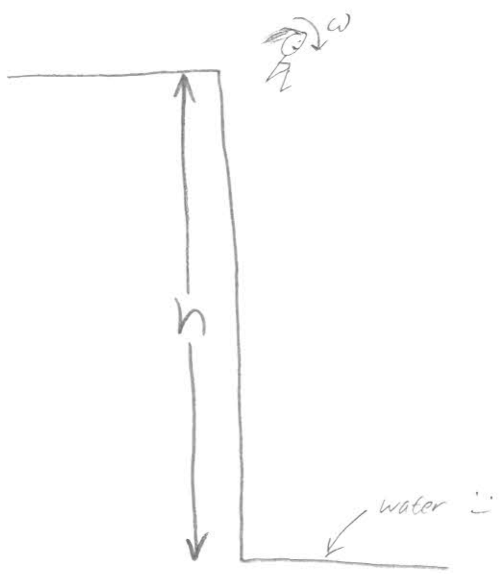
ω is common so dividing $\frac{①}{②}$ yields:

$$\frac{V_0}{V_R} = \frac{d_1 \cancel{\omega}}{d_2 \cancel{\omega}} \Rightarrow \boxed{V_0 = \frac{d_1}{d_2} V_R}$$

so when $V_R = 65$ mi/hr:

$$V_0 = \frac{60}{80} \cdot 65 \text{ mi/hr} = \boxed{49 \text{ mi/hr}}$$

A diver makes 2.5 complete revolutions on the way from a 10 m high platform to the water. Assume that her initial vertical velocity was zero. What was her angular velocity, assuming that it was constant throughout the trip.



$\theta = 2.5\ rev.$ $\quad \omega_0 = ?$

$h = 10m$

$v_0 = 0$

$\alpha = 0$

Rotation

$\theta = \theta_0 + \omega_0 t + \cancel{\tfrac{1}{2}\alpha t^2}$

$\theta = \omega_0 t$

$\Rightarrow \boxed{\omega_0 = \frac{\theta}{t}}$ ①

translation

$y = y_0 + \cancel{v_{0y} t} + \frac{1}{2} a_y t^2$

$0 = h - \frac{1}{2} g t^2$

$\Rightarrow \boxed{t = \left(\frac{2h}{g}\right)^{1/2}}$ ②

combine ① and ②

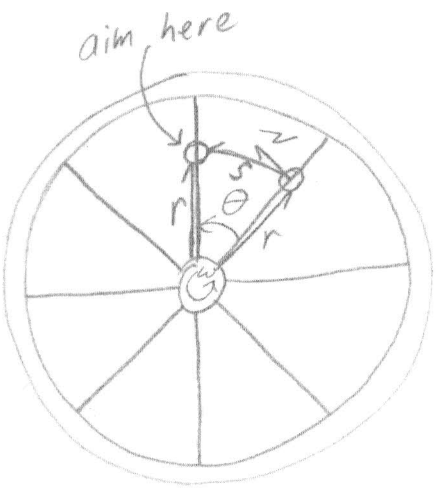
$\boxed{\omega_0 = \theta \left(\frac{g}{2h}\right)^{1/2}} \Rightarrow 2.5\,rev \left(\frac{9.8\,m/s^2}{(2)(10m)}\right)^{1/2} = \omega_0$

$\boxed{\omega_0 = 1.75\ rev/s}$

Rotation, moment of inertia

A wagon wheel with a radius of R=30cm with 8 equally spaced spokes is spinning with an angular velocity of ω=2.5 revolutions/sec. You want to shoot a 20 cm arrow parallel to the wheel's axle between the spokes without hitting one. Assuming that the spokes and the arrow are very thin:

a) What minimum speed must the arrow have?
b) Does it matter where between the axle and the rim you aim? If so, where is the best location?



Circle is 2π radians.
There are 8 equal wedges:
at: $\boxed{\theta = \frac{2\pi}{8} = \frac{\pi}{4}}$ each

Let's consider that we aim the arrow a distance r above the axle. If it just misses a spoke, It has to be all the way through before the next spoke gets there.

The spoke is moving at $v = r\omega$
and has to travel a distance $s = r\theta$

$$s = vt \Rightarrow \cancel{r}\theta = \cancel{r}\omega t$$

$\theta = \omega t$ $\Rightarrow$ time is independant of r.

continued ↓

So: $t = \frac{\theta}{\omega}$ So, the arrow travels it's length in $l = Vt = V\frac{\theta}{\omega}$

So it has to go

$$\boxed{V = l\frac{\omega}{\theta}}$$

$$V = 20\,cm \cdot \frac{2.5\,\cancel{rev}/s \cdot 2\cancel{\pi}\,\cancel{rad}/\cancel{rev}}{\cancel{\pi}/4\,\cancel{rad}}$$

$$V = (2)(2.5)(20)\,cm/s = 100\,cm/s$$