SAMPLE TEST 5

PHYS 111, FALL 2010, SECTION 1

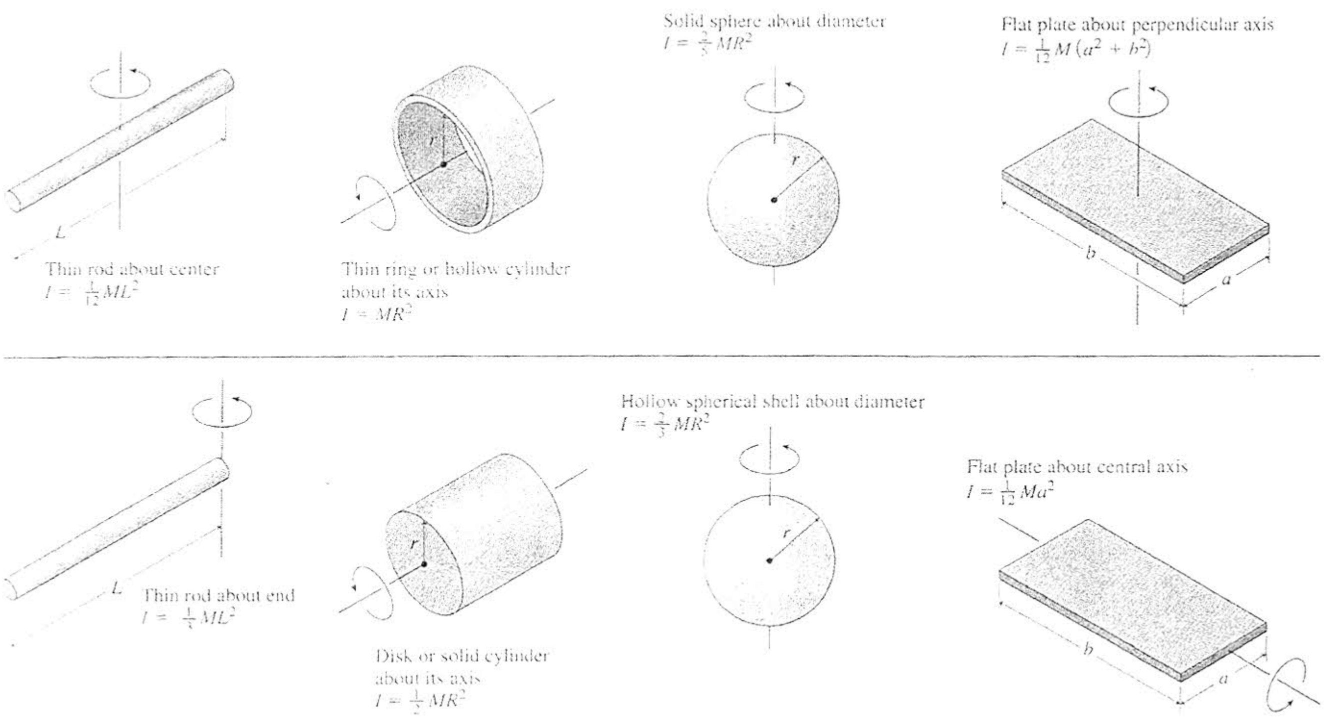
Name: ______________________________

By writing my name above, I affirm that this test represents my work only, without aid from outside sources. In all aspects of this course I perform with honor and integrity.

SHOW YOUR WORK ON ALL OF THE PROBLEMS. YOUR APPROACH TO THE PROBLEM IS AS IMPORTANT AS, IF NOT MORE IMPORTANT THAN, YOUR ANSWER. DRAW **CLEAR AND NEAT PICTURES** SHOWING COORDINATE SYSTEMS AND ALL OF THE RELEVANT PROBLEM VARIABLES. ALSO, <u>**EXPLICITLY**</u> SHOW THE **BASIC EQUATIONS** YOU ARE USING. BE NEAT AND THOROUGH. THE EASIER IT IS FOR ME TO UNDERSTAND WHAT YOU ARE DOING, THE BETTER YOUR GRADE WILL BE.

Moment of Inertia, discrete definition	$I=\sum m_i r_i^2$
Moment of Inertia, integral definition	$I=\int r^2\,dm$
Parallel Axis Theorem	$I=I_{cm}+Md^2$
Superposition	$I_{Total}=\sum I_i$

TABLE 10.2 Rotational Inertias



SAMPLE TEST 5

PHYS 111, FALL 2010, SECTION 1

1) Derivations

a) (10pts) Starting with the definition of linear Kinetic energy ($K=\frac{1}{2}mV^2$), show that rotational kinetic energy of a rigid body is $K=\frac{1}{2}I\omega^2$ where $I=\int r^2 dm$.

b) (10pts) Starting with the definition of angular momentum ($L=m|\vec{r}\times\vec{V}|$), show that the angular momentum of a rigid body is $L=I\omega$ where $I=\int r^2 dm$.

Proofs given in another post.

SAMPLE TEST 5

PHYS 111, FALL 2010, SECTION 1

2) Multiple Choice, 4 points each.

2.1 A boy and a girl are riding on a merry-go-round that is turning. The boy is twice as far from the merry-go-round's center as the girl. The boy and the girl have the same mass. Which statement is true about the boy's moment of inertia with respect to the axis of rotation.

a) It is four times the girl's.
b) It is twice the girl's.
c) It is the same for both.
d) The boy has greater moment of inertia but it is impossible to say how much more than the girl's it is.

$I_B = m_B R_B^2$ $\quad R_B = 2R_G$

$I_G = m_G R_G^2$ $\quad m_B = m_G$

$\Rightarrow \frac{I_B}{I_G} = \frac{m_B (2R_G)^2}{m_G R_G^2} = \boxed{4}$

2.2 A disk, a hoop, a solid sphere, and a hollow sphere, all with the same mass and radius, are having a race down an incline plane. Rank them in the order that they will arrive at the bottom of the ramp, 1 = winner, 4 = loser.

2 Disk $\frac{1}{2}$ .5

4 Hoop 1 1.0

1 Solid Sphere $\frac{2}{5}$ .4

3 Hollow Sphere $\frac{2}{3}$ .6

acceleration will depend on I

$\tau = I\alpha \Rightarrow \alpha = \frac{\tau}{I}$ so, as $I\uparrow$, $\alpha\downarrow$

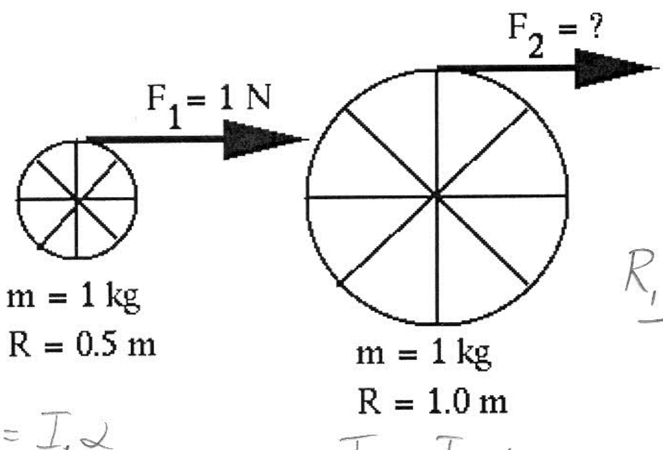
so, smallest I wins!

(Torque is the same on all because R and M are the same)

2.3 Two wheels with fixed axles each have a mass of 1.0 kg. All of the mass is concentrated at the rim so that $I = mR^2$ for each. What does F_2 have to be for both wheels to have the same angular acceleration?

a) 0.5 N
b) 1.0 N
c) 2.0 N
d) 4.0 N

$\tau = I\alpha$



$R_1 = \frac{1}{2}R_2$

$\tau_1 = I_1\alpha$ $\quad \tau_2 = I_2\alpha$

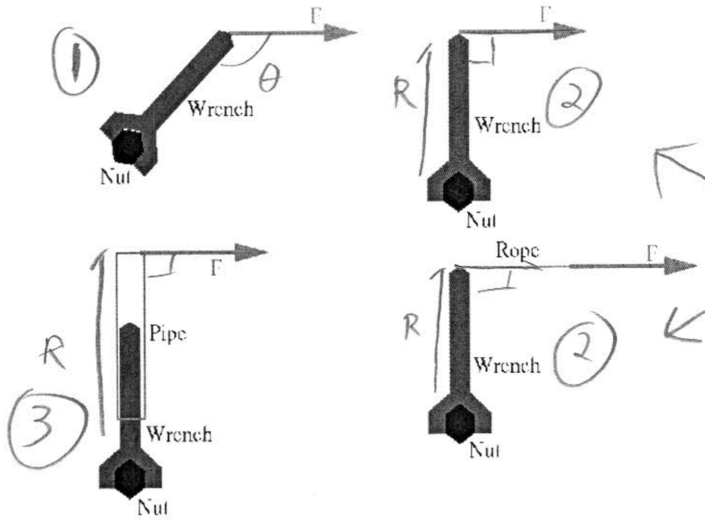
$F_1 \cancel{R_1} = mR_1^{\cancel{2}}\alpha$ $\quad F_2 \cancel{R_2} = mR_2^{\cancel{2}}\alpha$

$\frac{F_1}{F_2} = \frac{\cancel{m}R_1}{\cancel{m}R_2} = \frac{\frac{1}{2}R_2}{R_2} \Rightarrow \boxed{F_2 = 2F_1}$

Sample Test 5
Phys 111, Fall 2010, Section 1

2.4 You are trying to turn a nut with a wrench. The same force is applied in each picture. Rank the pictures by torque, 1 = smallest. If any of the torques are the same, give them the same ranking.

$T = RF \sin\theta$

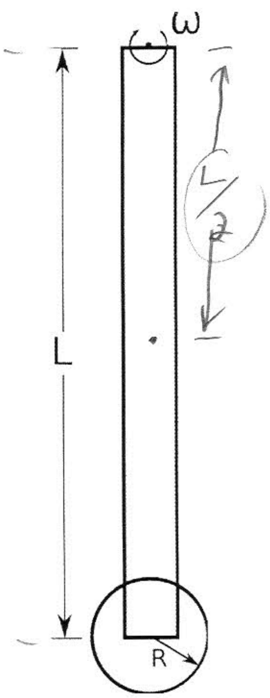


same

2.5 A pendulum is a combination of a solid bar with a flat disk attached, as shown in the picture. The mass of the rod is equal to the mass of the flat disk.

The moment of inertia of the rod about its center of mass is $I_{cm,rod} = \frac{1}{12} ML^2$

The moment of inertia of the disk about its center of mass is $I_{cm,disk} = \frac{1}{2} MR^2$



The total moment of inertia of this object about the axis of rotation shown is:

a) $I_T = \frac{25}{12} ML^2 + \frac{1}{2} MR^2$

b) $I_T = \frac{4}{3} ML^2 + \frac{1}{2} MR^2$

c) $I_T = \frac{1}{12} ML^2 + \frac{1}{2} MR^2$

d) $I_T = \frac{1}{3} ML^2 + \frac{1}{2} MR^2 + M(R+L)^2$

$I_T = I_{Rod} + I_{Disk}$

$I_{Rod} = I_{cm,Rod} + Md^2$

$= \frac{1}{12} ML^2 + M\left(\frac{L}{2}\right)^2$

$= \left(\frac{1}{12} + \frac{1}{4}\right) ML^2$

$I_{Rod} = \frac{1}{3} ML^2$

$I_{Disk} = \frac{1}{2} MR^2 + Md^2$

$= \frac{1}{2} MR^2 + ML^2$

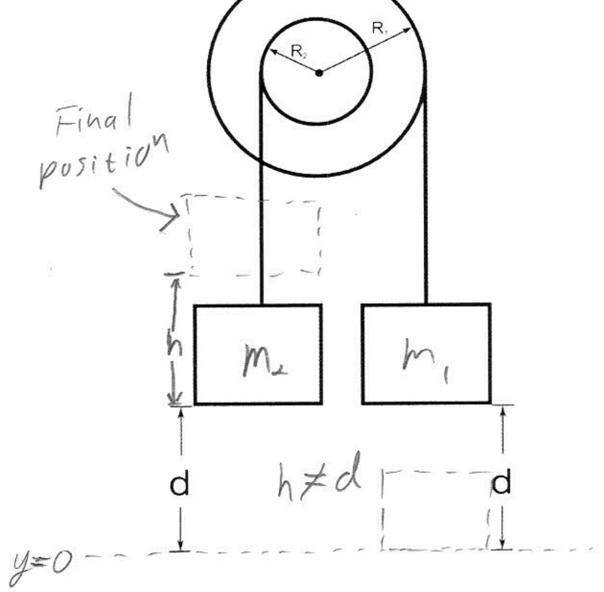
$I_T = \frac{1}{3} ML^2 + ML^2 + \frac{1}{2} MR^2 = \boxed{\frac{4}{3} ML^2 + \frac{1}{2} MR^2}$

Sample Test 5
Phys 111, Fall 2010, Section 1

3. The picture below shows a modified atwood machine composed of two pulleys of different radii that have been glued together so that their angular velocities will be the same. Two blocks of equal mass are attached to the system by ropes. One rope is wound around the small pulley and the other rope is wound around the large pulley. The mass of the pulley is the same as the mass of the two blocks and $R_2 = \frac{1}{2} R_1$.

Assume that the moment of inertia of the pulley is $I = \frac{1}{2} M R_1^2$

a) If the masses are initially at rest, which way will the pulley rotate, clockwise or counter clockwise?
b) Using Work/Energy techniques, find an expression for the angular velocity of the pulleys after the mass attached to the large pulley has moved a distance d.



a) Imagine holding the pully so it can't rotate. Which will have the greater torque?

$m_1 = m_2$ and $F_T = mg$ (since nothing is moving...)

$T_1 = mg\,R_1$ and $T_2 = mg\,R_2$

$T_1 > T_2$ so it rotates clockwise.

b) $\underline{U_I} = mgd + mgd$ $\qquad$ $\underline{K_I} = 0$ $\qquad$ These are different

$\underline{U_F} = mg(h+d)$ $\qquad$ $\underline{K_F} = \frac{1}{2} m v_1^2 + \frac{1}{2} m v_2^2 + \frac{1}{2} I \omega^2$

Relate h to d

$d = R_1\theta$, $h = R_2\theta$; θ is the same for both.

$$\frac{h}{d} = \frac{R_2 \cancel{\theta}}{R_1 \cancel{\theta}} \Rightarrow h = \frac{R_2}{R_1} d = \frac{\frac{1}{2}\cancel{R_1}}{\cancel{R_1}} d \Rightarrow \boxed{h = \frac{1}{2} d}$$

Continued ↓

Sample Test 5, P3, continued

Conserve Energy

$$U_I + \cancel{K_I}^{\,0} + \cancel{W_{NCF}}^{\,0} = U_F + K_F$$

$$2mgd = mg(h+d) + \tfrac{1}{2}mv_1^2 + \tfrac{1}{2}mv_2^2 + \tfrac{1}{2}I\omega^2$$

Plug in $h = \tfrac{1}{2}d$, $v_1 = R_1\omega$, and $v_2 = R_2\omega$, $I = \tfrac{1}{2}mR_1^2$

$$2\cancel{m}gd = \cancel{m}g\left(\tfrac{1}{2}d + d\right) + \tfrac{1}{2}\cancel{m}R_1^2\omega^2 + \tfrac{1}{2}\cancel{m}R_2^2\omega^2 + \tfrac{1}{2}\left[\tfrac{1}{2}\cancel{m}R_1^2\right]\omega^2$$

$\leftarrow \tfrac{1}{2}R_1$

$$\Rightarrow 2gd - \tfrac{3}{2}gd = \left[\tfrac{1}{2}R_1^2 + \tfrac{1}{2}\left(\tfrac{1}{2}R_1\right)^2 + \tfrac{1}{4}R_1^2\right]\omega^2$$

$$\Rightarrow \tfrac{1}{\cancel{2}}gd = \tfrac{7}{\cancel{8}_{4}}R_1^2\omega^2$$

$$\Rightarrow \boxed{\omega^2 = \tfrac{4}{7}\frac{gd}{R_1^2}}$$

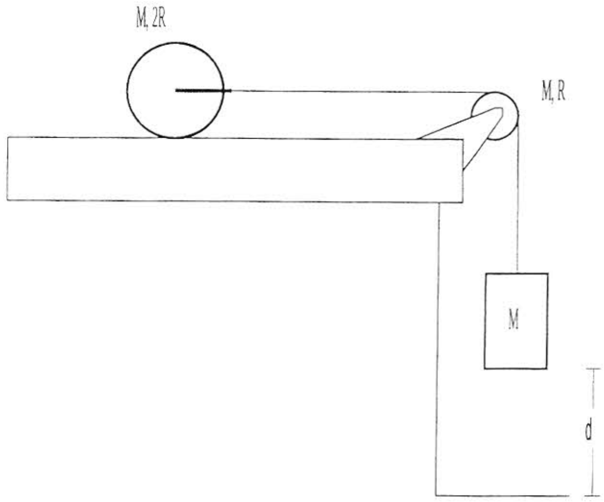
Units? $\omega^2 = \dfrac{\cancel{m}/s^2\ \cancel{m}}{\cancel{m^2}} = 1/s^2$ yay!

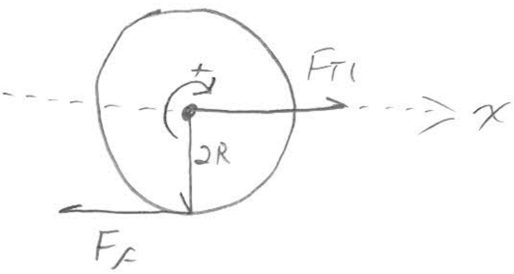
4. Use **Torque and Newton's Second Law** solve this problem.

A solid cylinder (radius = 2R, mass = M) rolls without slipping as it is pulled by a massless yoke attached to a string. The string goes over a frictionless pulley shaped as a solid disk (radius = R, mass = M) and is attached to a hanging weight (mass = M).

$$I_{cylinder} = \frac{1}{2}MR^2$$

What is the acceleration of the system?





$\tau = I\alpha$

$0 \cdot F_{T1} + 2RF_f = I\alpha$

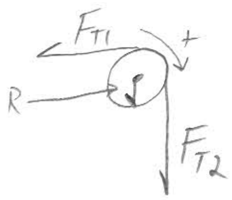
$\Rightarrow 2RF_f = \frac{1}{2}M(2R)^2\alpha$

$\Rightarrow \cancel{2R}F_f = \cancel{2}MR^{\cancel{2}}\alpha$

$\Rightarrow \boxed{F_f = MR\alpha}$ (1)

$F = ma$

$\boxed{F_{T1} - F_f = Ma}$ (2)

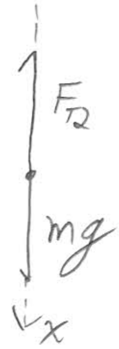


$\tau = I\alpha$

$RF_{T2} - RF_{T1} = I\alpha$

$\cancel{R}[F_{T2} - F_{T1}] = \frac{1}{2}MR^{\cancel{2}}\alpha$

$\boxed{F_{T2} - F_{T1} = \frac{1}{2}MR\alpha}$ (3)



$F = ma$

$Mg - F_{T2} = Ma$

$\Rightarrow \boxed{F_{T2} = M(g-a)}$ (4)

continued ↓

*combine ① and ② to eliminate F_f

From ②: $F_{T1} - MR\alpha = Ma$ $\boxed{\alpha = \frac{a}{2R} \text{ for this cylinder}}$

$\Rightarrow F_{T1} = MR\left(\frac{a}{2R}\right) + Ma$

$\Rightarrow F_{T1} = \frac{1}{2}Ma + Ma$

$\Rightarrow \boxed{F_{T1} = \frac{3}{2}Ma}$ ⑤

* Now comine ③, ④, and ⑤ and solve for a

From ③: $\cancel{M}(g-a) - \frac{3}{2}\cancel{M}a = \frac{1}{2}\cancel{M}R\alpha \leftarrow \boxed{\alpha = \frac{a}{R} \text{ for this pulley}}$

$\Rightarrow g - a - \frac{3}{2}a = \frac{1}{2}R\frac{a}{R}$

$\Rightarrow g = \left(1 + \frac{3}{2} + \frac{1}{2}\right)a$

$\Rightarrow \boxed{a = \frac{1}{3}g}$ *

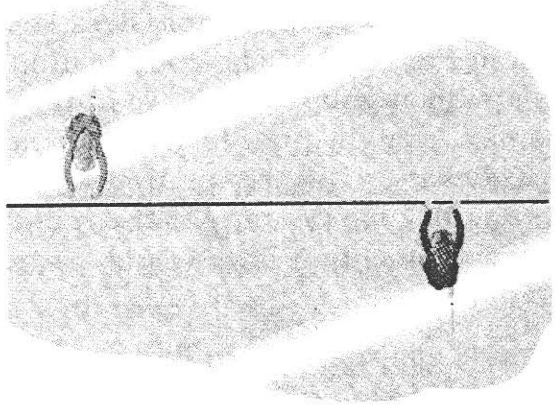
SAMPLE TEST 5

PHYS 111, FALL 2010, SECTION 1

4) Two skaters, each with a mass of 50 kg, approach each other along parallel paths separated by 3.0 m. They have equal and opposite velocities of 1.4 m/s. The first skater is holding one end of a long pole with negligible mass. As the skaters pass, the second skater grabs the other end of the pole. Assume that the ice is completely frictionless.

a) What is the moment of inertia about the center of mass of the resulting skater-pole system?

b) What is the resulting angular velocity of the skater-pole system?

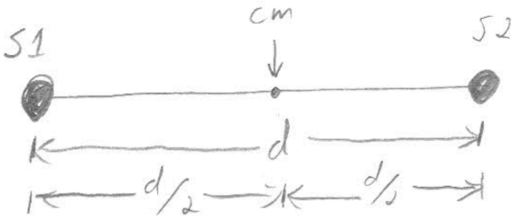


$m = 50\ kg$

$v = 1.4\ m/s$

$d = 3.0\ m$

a)



$$I = \sum m_i r_i^2$$

$$I = m\left(\frac{d}{2}\right)^2 + m\left(\frac{d}{2}\right)^2 = \boxed{\tfrac{1}{2} m d^2}$$

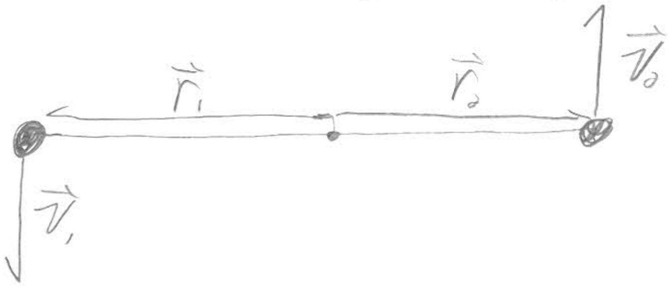
b) $L_I = L_F$

The skaters will rotate about the center of mass. But, prior to grabbing the pole, They are not a rigid body...

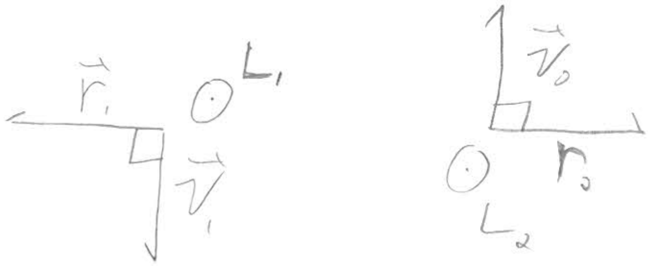
$$\vec{r}_1 \times m\vec{v}_1 + \vec{r}_2 \times m\vec{v}_2 = I\omega$$

continued ↓

At the instant of pole grabbing, it looks like this:



$\vec{r}_1 \times \vec{v}_1$ and $\vec{r}_2 \times \vec{v}_2$ give the same sign for $\vec{L}$



Both are out of the page.

So:

$$m r_1 v_1 + m r_2 v_2 = I\omega, \quad r_1 = r_2 = \frac{d}{2}$$

$$v_1 = v_2 = v$$

$$\frac{1}{2} m d v + \frac{1}{2} m d v = I\omega$$

$$m d v = I\omega \Rightarrow \omega = \frac{m d v}{I}$$

And plug in I from part a:

$$\omega = \frac{\cancel{m} d v}{\frac{1}{2}\cancel{m} d^2} \Rightarrow \boxed{\omega = 2\frac{v}{d}}$$

$$\boxed{\omega = 2\,\frac{1.4}{3} = 0.27\ \text{rad/s}}$$