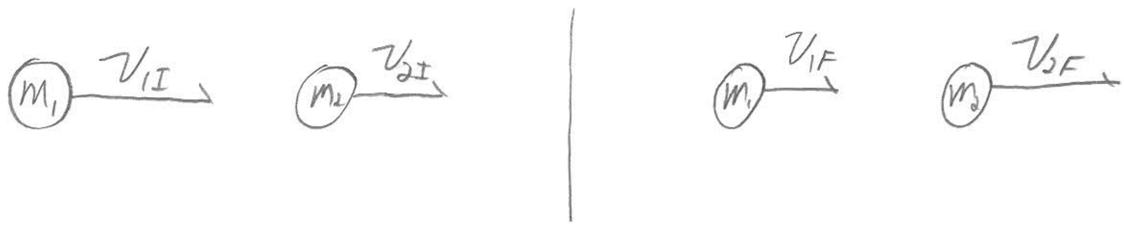
Sample Test 4
Phys 111 Spring 2010

Name: ______________________________

By writing my name above, I affirm that this test represents my work only, without aid from outside sources. In all aspects of this course I perform with honor and integrity.

SHOW YOUR WORK ON ALL OF THE PROBLEMS – YOUR APPROACH TO THE PROBLEM IS AS IMPORTANT AS (IF NOT MORE) IMPORTANT THAN) YOUR FINAL ANSWER.

1) Derive the equations for the final velocities of particles undergoing an elastic collision in 1 dimension.



Conserve momentum:

$$m_1 v_{1I} + m_2 v_{2I} = m_1 v_{1F} + m_2 v_{2F}$$

Conserve kinetic energy

$$\tfrac{1}{2} m_1 v_{1I}^2 + \tfrac{1}{2} m_2 v_{2I}^2 = \tfrac{1}{2} m_1 v_{1F}^2 + \tfrac{1}{2} m_2 v_{2F}^2$$

arrange by mass and divide

$$\frac{m_1 (v_{1I}^2 - v_{1F}^2)}{m_1 (v_{1I} - v_{1F})} = \frac{m_2 (v_{1F}^2 - v_{1I}^2)}{m_2 (v_{1F} - v_{1I})}$$

Factor numerator

$$\frac{(v_{1I} + v_{1F})(v_{1I} - v_{1F})}{(v_{1I} - v_{1F})} = \frac{(v_{2F} + v_{2I})(v_{2F} - v_{2I})}{(v_{2F} - v_{2I})}$$

continued ↓

Sample test 4, Q1 continued

$$v_{1I} + v_{1F} = v_{2I} + v_{2F}$$

Solve for v_{2F} and subst. into momentum eq.

$$v_{2F} = v_{1I} + v_{1F} - v_{2I}$$

Into momentum eq:

$$m_1 v_{1I} + m_2 v_{2I} = m_1 v_{1F} + m_2 (v_{1I} + v_{1F} - v_{2I})$$

$$\Rightarrow m_1 v_{1I} + m_2 v_{2I} = m_1 v_{1F} + m_2 v_{1I} + m_2 {}_{1F} - m_2 v_{2I}$$

$$\Rightarrow m_1 v_{1I} - m_2 v_{1I} + m_2 v_{2I} + m_2 v_{2I} = m_1 v_{1F} + m_2 v_{1F}$$

$$\Rightarrow (m_1 - m_2) v_{1I} + 2 m_2 v_{2I} = (m_1 + m_2) v_{1F}$$

$$\Rightarrow \boxed{v_{1F} = \frac{(m_1 - m_2)}{(m_1 + m_2)} v_{1I} + \frac{2 m_2}{(m_1 + m_2)} v_{2I}}$$

Swap 1 for 2 to get v_{2F}

$$\boxed{v_{2F} = \frac{(m_2 - m_1)}{(m_2 + m_1)} v_{2I} + \frac{2 m_1}{(m_2 + m_1)} v_{1I}}$$

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2) Multiple Choice, 4 points each.

2.1) Which of the following systems do **NOT** conserve momentum?

a. While slipping on a patch of ice a car collides with another car on the ice. (Ignore friction with the ice.) *System: Both cars.*
b. A single car sliding across the ice. (Ignore friction with the ice.) *System: The car.*
c. A ball drops to earth. (Ignore air friction.) *System: The ball.*
d. A billiard ball collides with another billiard ball on a pool table. (Ignore friction with the table during the collision.) *System: Both balls.*

A compact car and a large truck collide head-on and stick together.

2.2 which vehicle undergoes the larger magnitude momentum change?

a. Car.
b. Truck.
c. Same for both.
d. Can't tell without knowing the final velocity of the wreck.

2.3 During the collision, The velocity of the center of mass of the system

a. is always zero
c. increases
c. decreases
d. stays the same

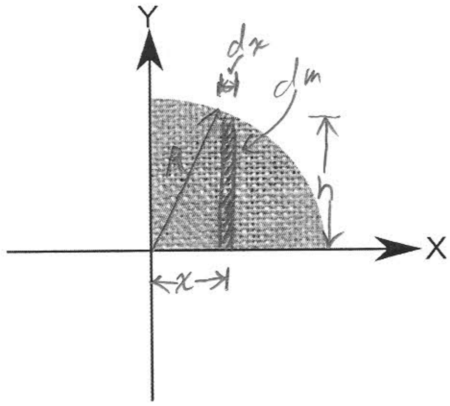
2.4 A lunar vehicle is tested on Earth at a speed of 10 km/hr. When it travles at 10 km/hr on the moon, it's momentum will be:

a. more
b. less
c. the same

2.5 When you fire a rifle, the bullet flies out and the rifle recoils. The magnitude of the momentum of the bullet is ______ the momentum of the rifle.

a. more than
b. the same as
c. less than

3. Find the location of the center of mass of a quarter of a circle of radius R and mass M.



$$dm = \sigma \, dA$$

$$\sigma = \frac{M}{\frac{1}{4}\pi R^2}, \quad dA = h\,dx$$

$$\boxed{dm = \frac{4M}{\pi R^2} h\,dx}$$

I'll just do x since it's the same in x and y.

$$x_{cm} = \frac{1}{M}\int_0^R x\,dm = \frac{1}{M}\int_0^R x \frac{4M}{\pi R^2} h\,dx$$

have to fix h:

$$h^2 + x^2 = R^2 \Rightarrow h = \sqrt{R^2 - x^2}$$

$$\Rightarrow x_{cm} = \frac{4}{\pi R^2}\int_0^R x\sqrt{R^2 - x^2}\,dx, \quad u = R^2 - x^2, \quad du = -2x\,dx$$

when $x = R$, $u = 0$

when $x = 0$, $u = R^2$

$$x_{cm} = \frac{-2}{\pi R^2}\int_{R^2}^{0} u^{1/2}\,du = \frac{2}{\pi R^2}\left(\frac{2}{3}u^{3/2}\right)\Big|_0^{R^2} = \frac{4}{3\pi R^2}(R^2)^{3/2} = \boxed{\frac{4}{3\pi}R}$$

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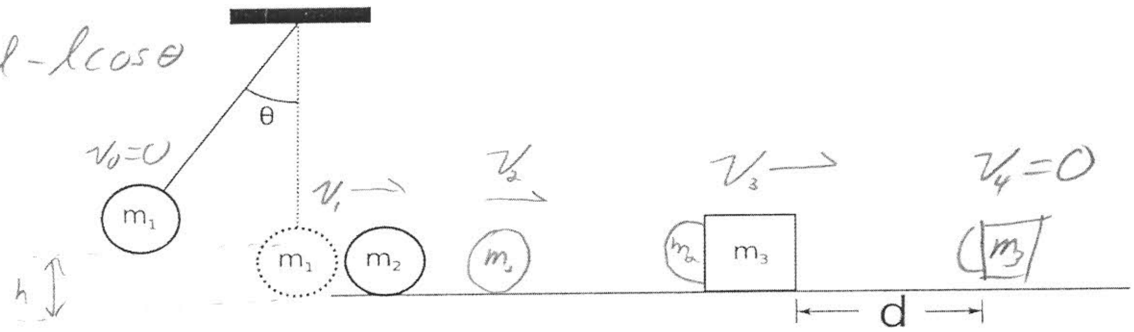
4. A mass $m_1 = 3$ kg is attached to a string of length $l = 4.0$ m to create a pendulum. The pendulum, initially making an angle θ with the vertical, is released from rest. At the bottom of its swing, it collides elastically with mass $m_2 = 5$ kg. Mass 2 rolls (no friction) and sticks to $m_3 = 5$ kg. The m_2, m_3 combination slides with $\mu_k = 0.3$ a distance $d = 0.2$ m before coming to rest.

What was the original value of θ?

$h = l - l\cos\theta$



Step ①: W/E for pendulum

$U_I = mg(l - l\cos\theta)$ $\quad$ $U_F = 0$

$K_I = 0$ $\quad$ $K_F = \frac{1}{2} m_1 v_1^2$

$\Rightarrow m_1 g l(1-\cos\theta) = \frac{1}{2} m_1 v_1^2$

$$\boxed{v_1 = \sqrt{2gl(1-\cos\theta)}} \quad ①$$

Step ②: Collide to get v_2

$$\boxed{v_2 = \frac{2m_1}{m_1 + m_2} v_1} \quad ②$$

Taken directly from our equation for elastic collisions.

Step ③: Collide to get v_3

$m_2 v_2 = (m_2 + m_3) v_3$ $\quad$ conserve momentum.

$$\boxed{v_3 = \frac{m_2}{m_2 + m_3} v_2} \quad ③$$

continued ↓

$$\boxed{v_3^2 = 2\mu_K g d} \quad ④$$

Put everything together:

③ → ④ $$\frac{m_2^2}{(m_2+m_3)^2} v_2^2 = 2\mu_K g d$$

② → $$\frac{m_2^2}{(m_2+m_3)^2} \cdot \frac{4m_1^2}{(m_1+m_2)^2} v_1^2 = 2\mu_K g d$$

① → $$\left(\frac{2m_1m_2}{(m_2+m_3)(m_1+m_2)}\right)^2 \cancel{2g} l(1-\cos\theta) = \cancel{2}\mu_K \cancel{g} d$$

Let's let: $$R = \frac{2m_1m_2}{(m_2+m_3)(m_1+m_2)}$$

then: $$R^2 l(1-\cos\theta) = \mu_K d$$

$$\Rightarrow R^2 l - R^2 l\cos\theta = \mu_K d$$

$$\Rightarrow R^2 l\cos\theta = R^2 l - \mu_K d \Rightarrow \boxed{\cos\theta = 1 - \frac{\mu_K d}{R^2 l}}$$

continued
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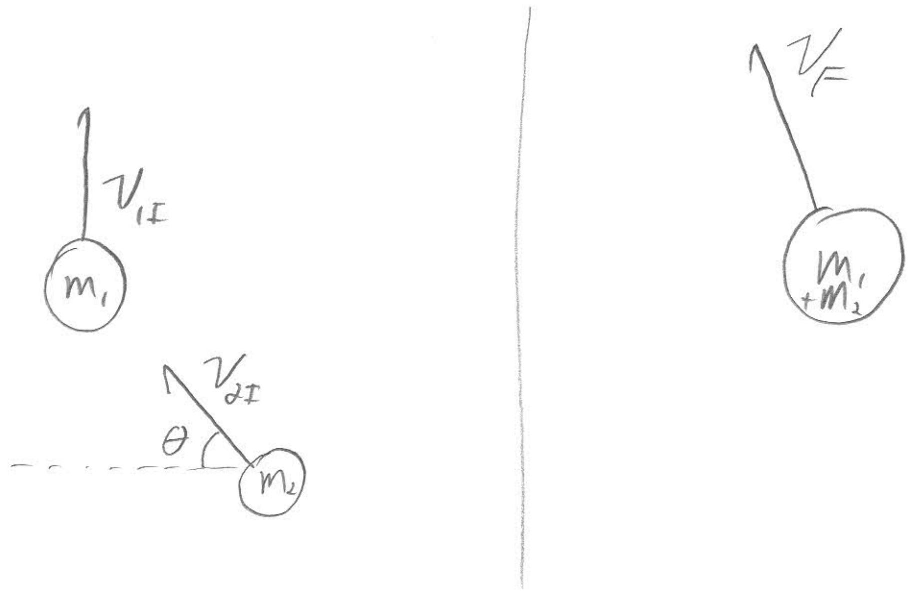
5. A 90 kg fullback is running towards the endzone (North) at 5.0 m/s. He is tackled by a 95.0 kg opponent who is running North-East (45 degrees East of North) at 3 m/s. They collide inelastically. What is their velocity just after the collision?



$$x: \quad m_2 v_{2I} \cos\theta = (m_1 + m_2) v_{Fx} \Rightarrow \boxed{v_{Fx} = \frac{m_2 v_{2I} \cos\theta}{m_1 + m_2}}$$

$$y: \quad m_1 v_{1I} + m_2 v_{2I} \sin\theta = (m_1 + m_2) v_{Fy}$$

$$\Rightarrow \boxed{v_{Fy} = \frac{m_1 v_{1I} + m_2 v_{2I} \sin\theta}{m_1 + m_2}}$$

$$v_{Fx} = \frac{(95\,kg)(3.0\,m/s)\cos(45)}{185} \qquad v_{Fy} = \frac{(90\,kg)(5\,m/s) + (95\,kg)(3\,m/s)\sin(45)}{185}$$

$$\boxed{v_{Fx} = 1.1\,m/s \qquad v_{Fy} = 3.5\,m/s}$$