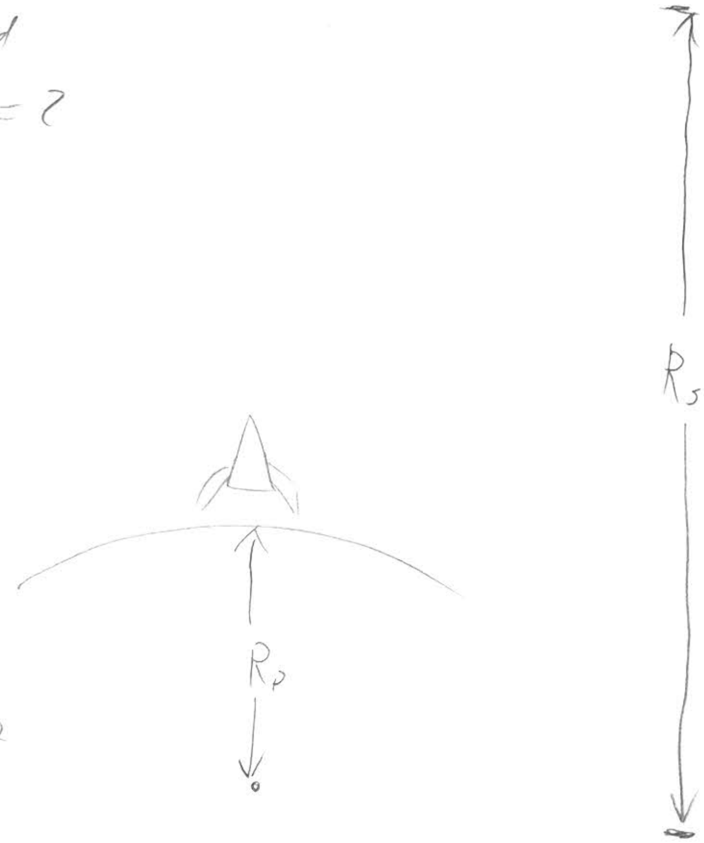
Energy Problems #5, P1



Given

a) $M_p = 5.0 \times 10^{23}\ kg$

$R_p = 3.0 \times 10^{6}\ m$

$M_s = 10\ kg$

$V_0 = 3{,}000\ m/s$

$R_s = 4 \times 10^{6}\ m$

$G = 6.67 \times 10^{-11}\ m^3/kg \cdot s^2$

Only G gravity

Wanted

$V_{\cancel{F}s} = ?$

$$U_I = -\frac{G M_p M_s}{R_p} \qquad K_I = \frac{1}{2} M_s V_0^2$$

$$U_F = -\frac{G M_p M_s}{R_s} \qquad K_F = \frac{1}{2} M_s V_s^2$$

$$-\frac{G M_p \cancel{M_s}}{R_p} + \frac{1}{2}\cancel{M_s} V_0^2 = -\frac{G M_p \cancel{M_s}}{R_s} + \frac{1}{2}\cancel{M_s} V_s^2$$

negative #

$$\Rightarrow 2 G M_p \left[\frac{1}{R_s} - \frac{1}{R_p}\right] + V_0^2 = V_s^2$$

But $R_s > R_p \Rightarrow \frac{1}{R_s} < \frac{1}{R_p}$

$$\Rightarrow \boxed{V_s^2 = V_0^2 - 2 G M_p \underbrace{\left[\frac{1}{R_p} - \frac{1}{R_s}\right]}_{\text{Positive \#}}}$$

continued

↓

a) continued

Plug in numbers

$$v_s = \left[\left(3\times10^3\,\tfrac{m}{s}\right)^2 - (2)\left(6.67\times10^{-11}\,\tfrac{m^3}{kg\cdot s^2}\right)\left(5.0\times10^{23}\,kg\right)\left[\frac{1}{3.0\times10^6\,m} - \frac{1}{4.0\times10^6\,m}\right]\right]^{1/2}$$

$$\boxed{v_s = 1.86\times10^3\,m/s}$$

b) Now, Given v_0, Find H_{max}

$$U_I = -\frac{GM_pM_s}{R_p} \qquad K_I = \tfrac{1}{2}M_s v_0^2$$

$$U_F = -\frac{GM_pM_s}{H_{max}} \qquad K_F = 0$$

← stops and turns around at H_{max}

$$-\frac{GM_p\cancel{M_s}}{R_p} + \tfrac{1}{2}\cancel{M_s}v_0^2 = -\frac{GM_p\cancel{M_s}}{H_{max}}$$

← Need this upstairs

I want to invert the entire equation, so I'll put the left side under a common denominator. After I divide by GM_p

$$-\frac{1}{R_p} + \frac{v_0^2}{2GM_p} = -\frac{1}{H_{max}} \Rightarrow \frac{2GM_p - v_0^2R_p}{2GM_pR_p} = \frac{1}{H_{max}}$$

$$\boxed{\Rightarrow H_{max} = \frac{2GM_pR_p}{2GM_p - v_0^2R_p}}$$

Yay!

continued ↓

Energy Problems #5, P1 - continued

c) If it never stops and comes back, H_{max} will be infinity.

$$H_{max} = \frac{2GM_pR_p}{\boxed{2GM_p - V_0^2 R_p}}$$

$H_{max} = \infty$ when this goes to zero.

$$2GM_p - V_0^2 R_p = 0 \Rightarrow \boxed{V_{esc} = \left(\frac{2GM_p}{R_p}\right)^{1/2}}$$

Escape Velocity

Alternate method to find V_{esc}

$U_I = -\dfrac{GMm}{R_p}$ $\qquad K_I = \frac{1}{2} m V_{esc}^2$

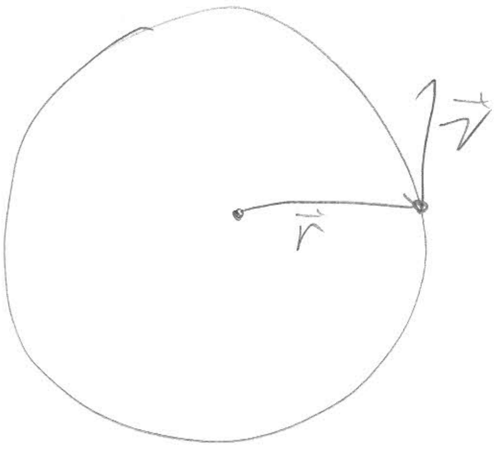
$U_F = \emptyset$ $\qquad K_F = \emptyset \leftarrow$ stops at ∞

$r \to \infty,\ U \to 0$

$$\Rightarrow -\frac{GM\cancel{m}}{R_p} + \frac{1}{2}\cancel{m} V_{esc}^2 = 0$$

$$\Rightarrow \boxed{V_{esc} = \left(\frac{2GM_p}{R_p}\right)^{1/2}}$$

Energy Problems Set 5, P2



uniform Circular motion!

$$\boxed{a = \frac{v^2}{r}}$$

a) FBD

NSL

$$\sum \vec{F} = m\vec{a}$$

$$\Rightarrow F_G = m\frac{v^2}{r}$$

$$\Rightarrow \frac{GM\cancel{m}}{r^{\cancel{2}}} = \cancel{m}\frac{v^2}{\cancel{r}}$$

$$\Rightarrow \boxed{v_{orb} = \left(\frac{GM}{r}\right)^{1/2}} \quad *$$

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b) So... The period of the Geosynchronous orbit is 24 hrs.

So we know P...

The velocity is distance over time

$$v_{orb} = \frac{dist.}{time} = \frac{2\pi r}{P} \quad \text{and} \quad v_{orb} = \left(\frac{GM}{r}\right)^{1/2}$$

$$\Rightarrow \frac{2\pi r}{P} = \left(\frac{GM}{r}\right)^{1/2} \Rightarrow \frac{4\pi^2 r^2}{P^2} = \frac{GM}{r}$$

$$\Rightarrow \boxed{r^3 = \frac{GM}{4\pi^2} P^2}$$

b) continued

$$r = \left[\frac{GM}{4\pi^2} P^2\right]^{1/3} = \left[\frac{(6.67\times10^{-11})(5.97\times10^{24})}{4\pi^2}\left((24)(3600)\right)^2\right]^{1/3}$$

$$r = 4.2\times10^{7}\,m = \underline{4.2\times10^{4}\,km}$$

c) Now that I have r:

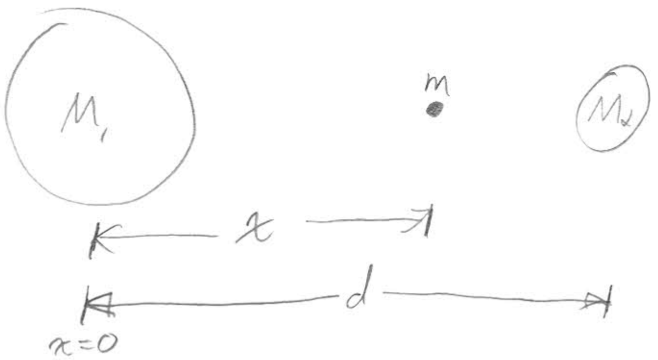
$$V_{orb} = \frac{2\pi r}{P} = \frac{2\pi}{P}\left[\frac{GM}{4\pi^2}P^2\right]^{1/3} = \left[\frac{\cancel{8}^{2}\cancel{\pi^3}^{\pi}}{\cancel{P^3}^{P}}\cdot\frac{GM}{\cancel{4}\cancel{\pi^2}}\cancel{P^2}\right]^{1/3}$$

$$\boxed{V_{orb} = \left[\frac{2GM}{P}\right]^{1/3}}$$

$$V_{orb} = \left[\frac{2(6.67\times10^{-11})(5.97\times10^{24})}{(24)(3600)}\right]^{1/3} = \underline{2.1\times10^{3}\,m/s}$$

Energy Problems, Set 5, P3

a)



Looking for $\sum \vec{F} = 0 \Rightarrow \vec{F}_1 + \vec{F}_2 = 0$

$$\Rightarrow -\frac{GM_1 m}{x^2} + \frac{GM_2 m}{(d-x)^2} = 0$$

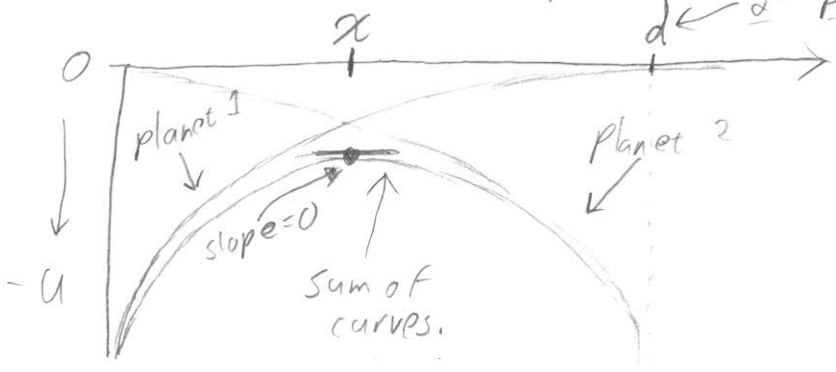
$$\Rightarrow \frac{M_1}{x^2} = \frac{M_2}{(d-x)^2} \Rightarrow \frac{\sqrt{M_1}}{x} = \frac{\sqrt{M_2}}{(d-x)}$$

$$\Rightarrow \sqrt{M_1}\, d - \sqrt{M_1}\, x = \sqrt{M_2}\, x$$

$$\Rightarrow x\left(\sqrt{M_1} + \sqrt{M_2}\right) = \sqrt{M_1}\, d$$

$$\Rightarrow \boxed{x = \frac{\sqrt{M_1}}{\sqrt{M_1} + \sqrt{M_2}}\, d}$$

b) The total potential is due to both masses. We can simply add them. $U_G = -\frac{GMm}{r}$ goes as $-\frac{1}{r}$, always negative.



equilibrium when slope = 0

c) Now mathematically:

$$U_T = U_{M1} + U_{M2} \Rightarrow U_T = -\frac{GM_1 m}{x} - \frac{GM_2 m}{(d-x)}$$

Let's find the extrema, when $\frac{dU_T}{dx} = 0$,
That's the equilibrium point.

$$\frac{dU_T}{dx} = \frac{d}{dx}\left[-\frac{GM_1 m}{x} - \frac{GM_2 m}{(d-x)}\right] = 0$$

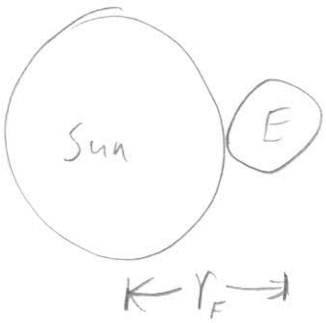
$$= -GM_1 m \frac{d}{dx}\left(\frac{1}{x}\right) - GM_2 m \frac{d}{dx}\frac{1}{(d-x)} = 0$$

$$= \boxed{+\frac{GM_1 m}{x^2} - \frac{GM_2 m}{(d-x)^2} = 0}$$

same as part a!!

$$\boxed{x = \frac{\sqrt{M_1}}{\sqrt{M_2} + \sqrt{M_1}} d}$$

Energy Problems Set 5 P4



Given

$M_\odot = 1.99 \times 10^{30}$ kg

$R_\odot = 6.955 \times 10^5$ km

$R_\oplus = 6.3781 \times 10^3$ km

$M_\oplus = 5.9742 \times 10^{24}$ kg

$v_i = 0$

Want

v_F

$$U_I = -\frac{G M_\odot m_\oplus}{r_I} \qquad K_I = 0 \qquad r_I = 1.49 \times 10^8 \text{ km}$$

$$r_F = R_\odot + R_\oplus$$

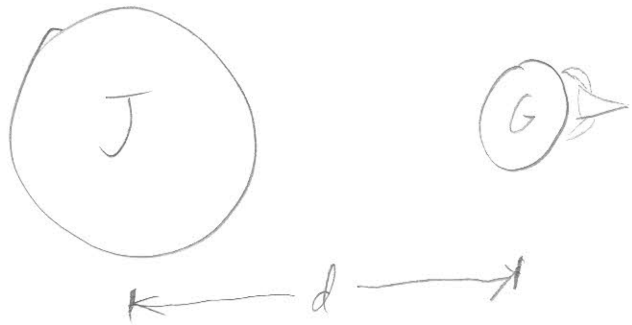
$$U_F = -\frac{G M_\odot m_\oplus}{r_F} \qquad K_F = \tfrac{1}{2} m_\oplus v_F^2$$

$$-\frac{G M_\odot \cancel{M_\oplus}}{r_I} = -\frac{G M_\odot \cancel{m_\oplus}}{r_F} + \tfrac{1}{2} \cancel{m_\oplus} v_F^2$$

$$v_F^2 = 2 G M_\odot \left[\frac{1}{r_F} - \frac{1}{r_I} \right] = \left[2(6.67 \times 10^{-11})(1.99 \times 10^{30}) \left[\frac{1}{7.02 \times 10^5} - \frac{1}{1.49 \times 10^8} \right] \right]^{1/2}$$

$$\boxed{v_F = 1.94 \times 10^7 \text{ m/s}}$$

Energy Problems Set 5, P5 -



Given

$R_G = 2.64 \times 10^6$ m

$M_G = 1.495 \times 10^{23}$ kg

$M_J = 1.900 \times 10^{27}$ kg

$d = 1.071 \times 10^9$ m

Want

V_{esc}

Escape when $U_F = 0$ ($r_F = \infty$) and $K_F = 0$

$$U_I = -\underbrace{\frac{G M_J m_R}{d + R_G}}_{\text{Jupiter}} - \underbrace{\frac{G M_G m_R}{R_G}}_{\text{Ganymede}}$$

$$K_I = \tfrac{1}{2} m_R V_{esc}^2$$

$$K_F = 0$$

$$U_F = 0$$

$$-\frac{G M_J \cancel{m_R}}{d + R_G} - \frac{G M_G \cancel{m_R}}{R_G} + \tfrac{1}{2} \cancel{m_R} V_{esc}^2 = 0$$

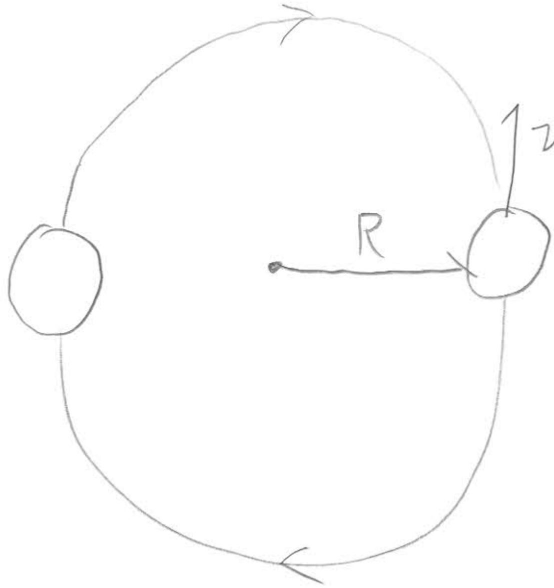
$$\boxed{V_{esc} = \left[G\left(\frac{M_J}{d + R_G} + \frac{M_G}{R_G} \right) \right]^{1/2}}$$

Energy Problems Set 5, P6

a) Given: M, R Want: F

$$\vec{F}_G = -\frac{G M_1 M_2}{r^2}\hat{r} \Rightarrow |\vec{F}_G| = \frac{GM^2}{4R^2}$$

b)



$$F_G = M\frac{v^2}{R}$$

$$\Rightarrow \frac{GM^{\cancel{2}}}{4R^{\cancel{2}}} = \cancel{M}\frac{v^2}{\cancel{R}}$$

$$\Rightarrow \boxed{v = \left[\frac{GM}{4R}\right]^{1/2}}$$

2 stars

c) $U_I = -\frac{GM^2}{2R}$ $K_I = \frac{1}{2}M v_{orb}^2 + \frac{1}{2}M v_{orb}^2$

$U_F = 0$ $K_F = 0$; $r \to \infty$

at $r = \infty$

continued ↓

Energy Problems Set 5, P6 — continued

$$U_I + K_I + W_{NCF} = U_F + K_F$$

$$-\frac{GM^2}{2R} + M V_{orb}^2 + W = 0$$

Energy to separate

$$W = \frac{GM^2}{2R} - M\boxed{\frac{GM}{4R}} \leftarrow V_{orb} \text{ from part b}$$

$$W = \frac{GM^2}{R}\left[\frac{1}{2} - \frac{1}{4}\right]$$

$$W = \frac{1}{4}\frac{GM^2}{R}$$