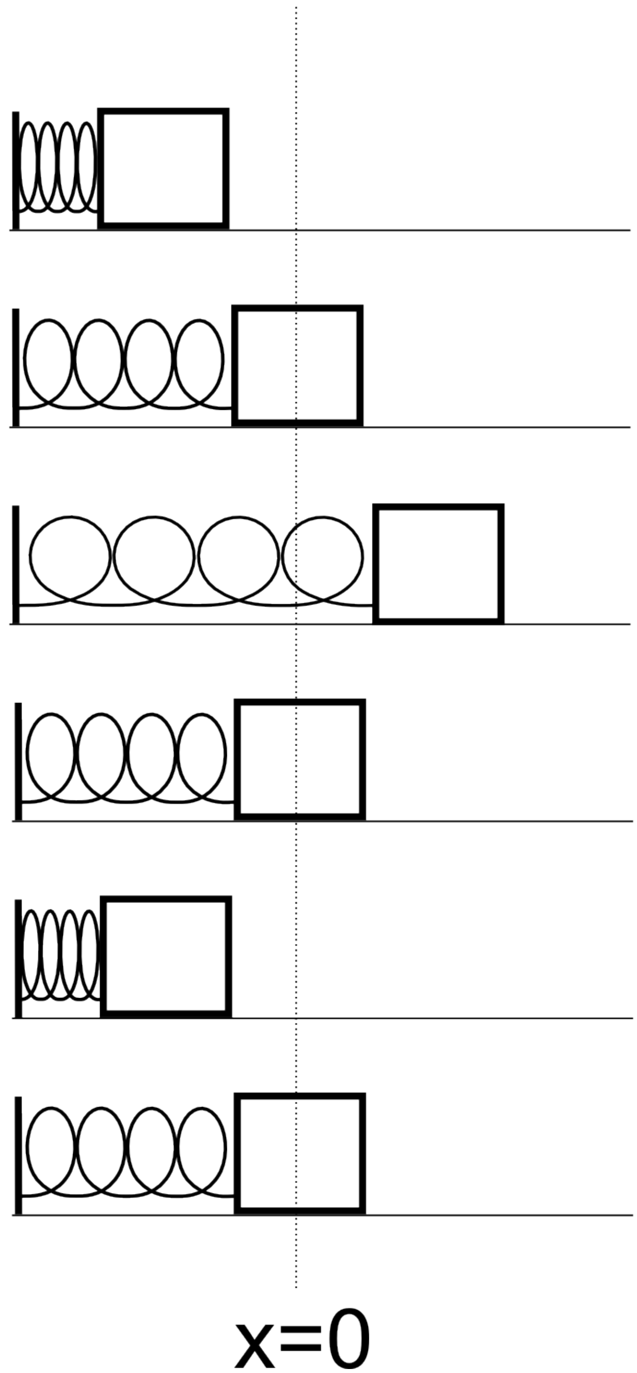
Oscillation – Set 1

Each row in the table below represents a snapshot of a mass attached to a spring. Assume that the mass starts from rest in the first row. In the second row, it is passing through x=0. In the third row, it has reached its maximum extension. In the fifth row, it has reached its maximum compression. In the cells below, mark an arrow indicating the direction of the associated force, acceleration, velocity, and position vectors for each row. If the magnitude is zero, put a zero in the cell.

F	a	v	x



The general form of the differential equation describing a simple harmonic oscillator is:

$$\frac{d^2x}{dt^2} = -\omega^2 x$$

The function *x(t)* describing the position of the oscillator that satisfies the above equation is:

$$x(t) = A\cos(\omega t + \phi)$$

where A, ω, and ϕ are constants.

a) Derive expressions for the *velocity* and *acceleration* of a simple harmonic oscillator.
HINT: How are position and velocity related? How are velocity and acceleration related?

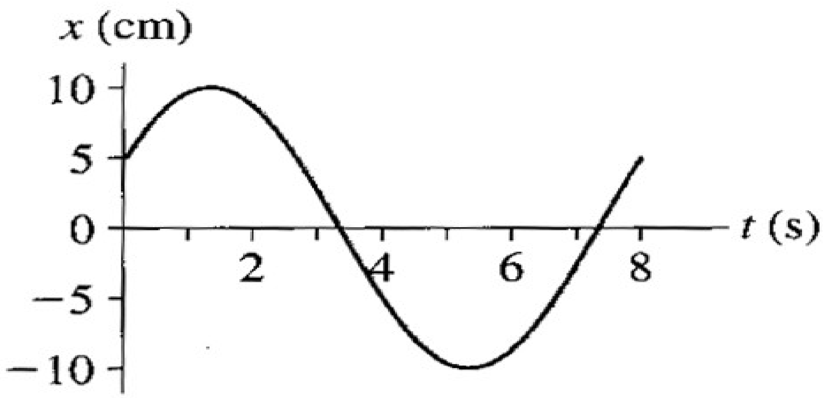
b) Show that the equation for *x(t)* actually satisfies the differential equation.

c) Why can't we apply the kinematics equations to a simple harmonic oscillator?

The figure below is a position versus time graph of a particle in simple harmonic motion. Assume that its position as a function of time is given by

$$x(t) = A\cos(\omega t + \phi)$$

where A, ω and ϕ are constants.



a) What is the maximum displacement (amplitude) of the particle?

b) Which constant in the above equation gives the maximum displacement, or **amplitude**, of the oscillations? *(HINT: What's the maximum possible value of cosine?)*

c) What is the value $x(0)$ (x when $t = 0$)?

d) Given your answer to part c, solve $x(0) = A\cos(\omega t + \phi)$ for ϕ (the phase constant) when $t = 0$.

e) What is the period, *T*, of the oscillations?

f) What are the units of ω? (*HINT: What are the units of the input to the cosine function?)*

g) What is the mathematical relationship between ω and T?

h) Good! Now calculate the numerical value of ω.

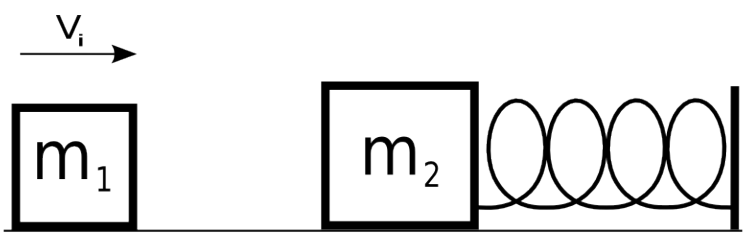
d) What is the maximum velocity of the particle?
(HINT: What's the maximum possible value of sine?)

e) What is the maximum acceleration of the particle?
(HINT: What's the maximum possible value of cosine?)

A block with a mass of m = 2.00 kg is attached to a spring with a spring constant k = 100 N/m. When t=1.00 s, the position and velocity of the block are $x(1)$=0.129 m and $v(1)$=3.415 m/s.

a) Find the angular frequency, ω, of the oscillator.
b) Find the phase constant, ϕ.
c) Find the amplitude, A.
d) What was the position of the block at t = 0.00 s?

A block with a mass of $m_1 = 10$ kg is moving to the right with a velocity V_i. It collides and sticks to a block with a mass of $m_2 = 15$ kg. The second mass is attached to a spring with spring constant k=3 N/m. Before the collision, the spring is at rest in it's equilibrium position.



a) What is the angular frequency of the resulting oscillator after the collision?
b) Assuming that the moment of collision is t=0, find the phase constant of the oscillator.
c) If the resulting amplitude of the oscillator is A = 3 m, what was the initial velocity of m_1?

You are given the position and velocity of a simple harmonic oscillator (SHO) at some time *t*:

$x(t)=x_0$ and $v(t)=v_0$.

Starting with the equations for position and velocity:

$$x(t)=A\cos(\omega t+\phi) \ , \quad x(t)=Acos(\omega t+\phi)$$

a) find an expression for the amplitude, *A*, of a **S**imple **H**armonic **O**scillator in terms of x_0 and v_0.

b) find an expression for the phase angle, ϕ, of a **S**imple **H**armonic **O**scillator in terms of x_0 and v_0.