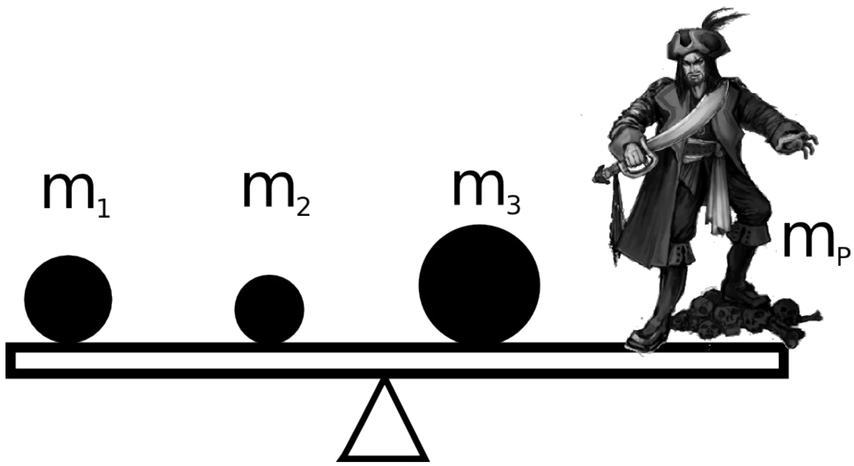
Systems of Particles – Set 1

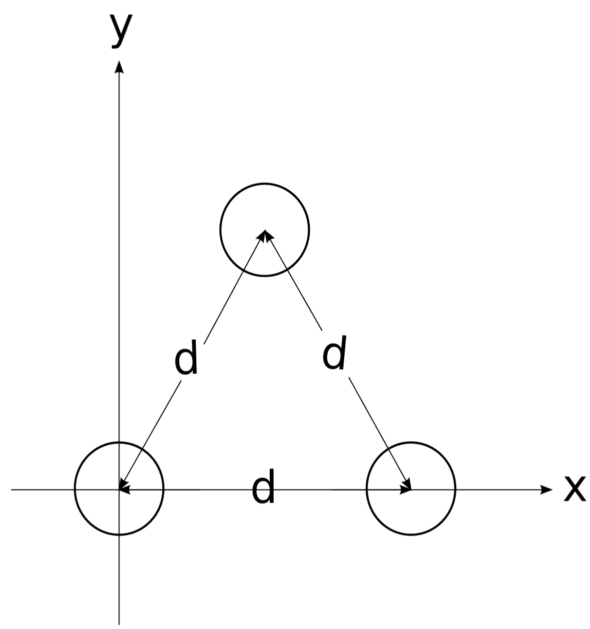
For reasons that nobody can explain, the Evil Pirate wants to stand on one end of a Plank of Negligible Mass (it's presumably used for walking) and be perfectly balanced by balls from the *Horrible Pendulum of Doom*. He's a little bit OCD, so he insists that all of the objects on the plank must be separated by the same distance *d*.



You can avoid walking the Plank of Negligible Mass if you can tell him where to put his fulcrum so that he can stand happily balanced and calm.

HINT: Put the fulcrum at the center of mass.

Three balls of equal mass form an equilateral triangle. Find the coordinates of the center of mass of the system.



We are going to calculate the location of the center of mass of a thin uniform rod of mass *M* and length *L*. The integral form of the center of mass equation is:

$$l_{cm} = \frac{\int_{l_0}^{L} l\,dm}{\int_{l_0}^{L} dm}$$

The variable of integration is *l*, but the differential is *dm*. We need to rewrite *dm* in terms of *l*. In other words, we need to perform a *Change of Variables*. The following steps will walk us through it.

a. You're in the hardware store and you notice that chain is on sale for \$1.00 per pound. The chain you want is 3 pounds per foot and you want 10 feet. How much do you have to spend?

b. You see that your neighbor is walking out the door with 50 feet of chain. He says gleefully that he paid only 25 dollars for it. What is the *linear density* (mass per unit length) of his chain ?

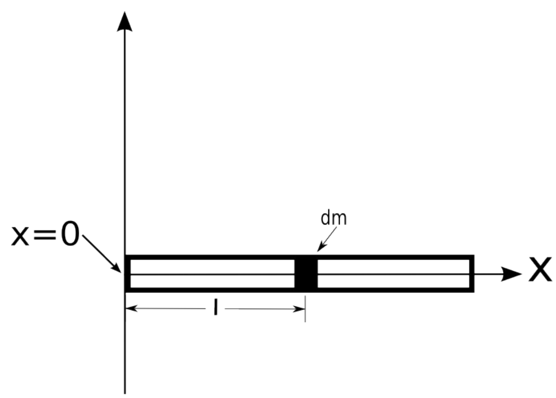
c. What would you pay for 30 feet of the chain that your neighbor bought?

d. We generally assign linear density the variable λ. What is the linear density (λ) of a uniform rod of mass *M* and length *L*? (refer to step b for assistance)

d. Now, write an equation for the mass m of a piece of this rod in terms of the total mass M, the total length L, and the length of the piece l. (refer to step c for assistance).

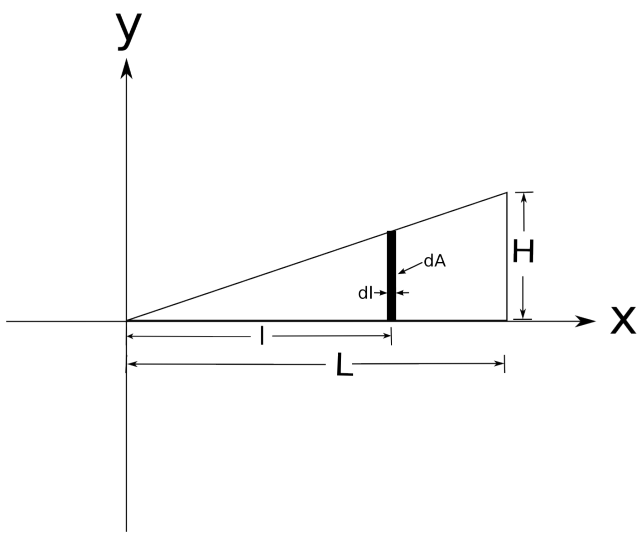
e. Calculate the derivative $\frac{dm}{dl}$ of the equation above. Now, what is dm in terms of M, L, and dl?

f. Taking the lower integral first, use the picture below and what we discovered in part e to calculate the total mass. Hopefully you get M.



g. Now calculate the the position of the center of mass. Hopefully, you get L/2.

Calculate the center of mass of a triangular chunk of aluminum of mass *M*, length *l*, and height *h*.



a) Write an expression for *dm* in terms of *dl* similarly to what we did for the uniform rod.
b) Using the result of part a, calculate X_{cm}.
c) Using the result of part a, calculate Y_{cm}.

A thin rod of length L has a linear density of $\lambda = \lambda_0 \left[2\frac{l^2}{L^2} + \frac{1}{3} \right]$.

where l is the distance from one end of the rod and λ_0 is a constant with units of mass per unit length.

a) Calculate the total mass of the rod.
b) Calculate the center of mass of the rod.

A flat piece of aluminum is cut into a semi-circle of radius R and mass M. Calculate the coordinates of the center of mass.