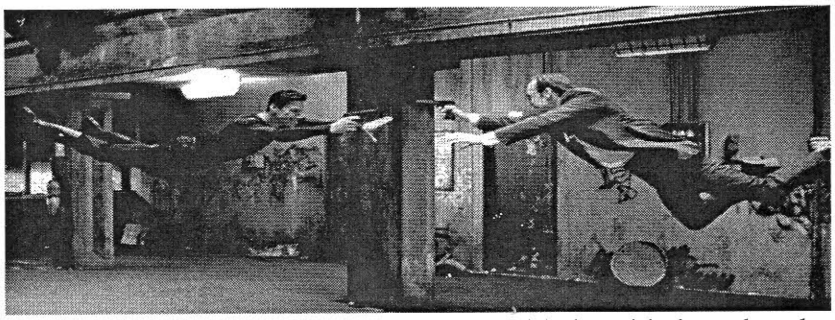
Systems of Particles – Set 4



Neo and Agent Smith are flying towards each other. They collide in mid air and grab onto each other (they stick together).

a) Assume that momentum is conserved in the Matrix and find an expression relating their initial velocities to their final velocity.

m_N v_N v_S m_S $(m_N + m_S)$ v_F

$$m_N v_N - m_S v_S = (m_N + m_S) v_F$$

$$\Rightarrow \boxed{v_F = \frac{m_N v_N - m_S v_S}{m_N + m_S}}$$

b) Let M_N = 70 kg, V_{NI} = 50 m/s, M_S = 100 kg, and V_{SI} = 35 m/s. Put these numbers into your expression and solve for their final velocity.

$$v_F = \frac{(70\,kg)(50\,m/s) - (100\,kg)(35\,m/s)}{(70\,kg + 100\,kg)} = \underline{0}$$

c) Calculate the pre-collision and post-collision kinetic energy of the system. Does this system conserve kinetic energy through the collision?

Pre

$K_I = K_N + K_S$

$K_I = \frac{1}{2} m_N v_N^2 + \frac{1}{2} m_S v_S^2$

$=$ a positive number

Post

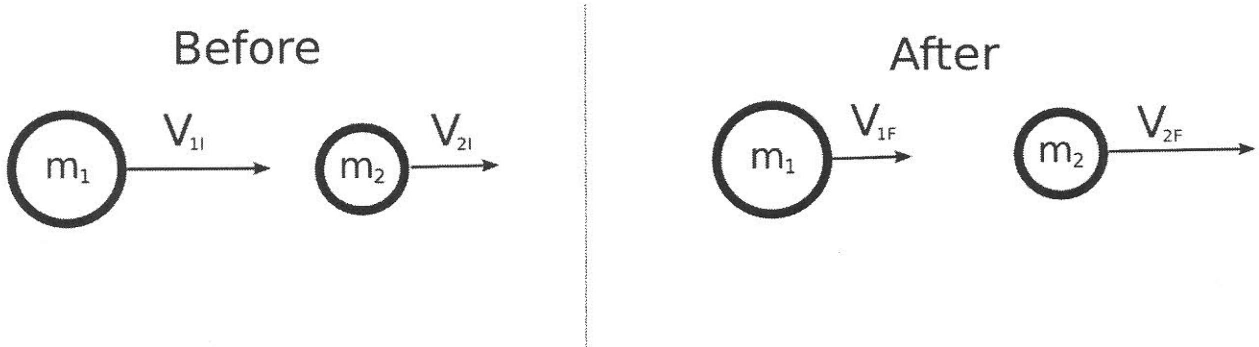
$K_F = \frac{1}{2}(m_N + m_S) v_F^2$

$\underline{K_F = 0}$

$\boxed{K \text{ is } \underline{\text{not}} \text{ conserved}}$

If Neo and Agent Smith conserved energy as well as momentum, the would bounce off of each other and the collision would be *elastic*. Let's derive a general expression relating the initial and final velocities in an elastic collision.

Step 1:
Starting with the picture below, write two equations, one for the *conservation of momentum* and one for the *conservation of kinetic energy*.



Conserve momentum: $m_1 V_{1I} + m_2 V_{2I} = m_1 V_{1F} + m_2 V_{2F}$ (1)

Conserve energy: $\frac{1}{2} m_1 V_{1I}^2 + \frac{1}{2} m_2 V_{2I}^2 = \frac{1}{2} m_1 V_{1F}^2 + \frac{1}{2} m_2 V_{2F}^2$ (2)

Using the two equations above, work out the algebra required to get to the following equation:

$$\frac{V_{1I}^2 - V_{1F}^2}{V_{1I} - V_{1F}} = \frac{V_{2F}^2 - V_{2I}^2}{V_{2F} - V_{2I}}$$ This is waypoint 1

Put all m_1 on one side, all m_2 on the other side of both equations and divide.

From (2): $m_1 (V_{1I}^2 - V_{1F}^2) = m_2 (V_{2F}^2 - V_{2I}^2)$

From (1): $m_1 (V_{1I} - V_{1F}) = m_2 (V_{2F} - V_{2I})$

Divide: $\frac{\cancel{m_1} (V_{1I}^2 - V_{1F}^2)}{\cancel{m_1} (V_{1I} - V_{1F})} = \frac{\cancel{m_2} (V_{2F}^2 - V_{2I}^2)}{\cancel{m_2} (V_{2F} - V_{2I})}$

Step 2:

Starting with waypoint 1: $\frac{V_{1I}^2 - V_{1F}^2}{V_{1I} - V_{1F}} = \frac{V_{2F}^2 - V_{2I}^2}{V_{2F} - V_{2I}}$

Peform the required algebra to get to waypoint 2: $V_{1I} + V_{1F} = V_{2I} + V_{2F}$

The following relationship may prove useful: $(a+b)(a-b) = (a^2 - b^2)$

Applying our relationship to the numerator:

$$\frac{\cancel{(V_{1I} - V_{1F})}(V_{1I} + V_{1F})}{\cancel{(V_{1I} - V_{1F})}} = \frac{\cancel{(V_{2F} - V_{2I})}(V_{2F} + V_{2I})}{\cancel{(V_{2F} - V_{2I})}}$$

$$\Rightarrow \boxed{V_{1I} + V_{1F} = V_{2I} + V_{2F}} \quad 3$$

Step 3:

Combine the results of waypoint 2, $V_{1I}+V_{1F}=V_{2I}+V_{2F}$,with the equation for conservation of momentum from part 1 to arrive at waypoint 3:

$$(m_1-m_2)V_{1I}+2m_2 V_{2I}=(m_1+m_2)V_{1F}$$

Solve ③ for V_{2F}: $V_{2F} = V_{1I} + V_{1F} - V_{2I}$

Subst into ① : $m_1 V_{1I} + m_2 V_{2I} = m_1 V_{1F} + m_2 (V_{1I} + V_{1F} - V_{2I})$

$\Rightarrow m_1 V_{1I} + m_2 V_{2I} = m_1 V_{1F} + m_2 V_{1I} + m_2 V_{1F} - m_2 V_{2I}$

Move all initial velocities to the left and all final velocities to the right.

$\Rightarrow m_1 V_{1I} + m_2 V_{2I} - m_2 V_{1I} + m_2 V_{2I} = m_1 V_{1F} + m_2 V_{1F}$

$\Rightarrow \boxed{(m_1 - m_2) V_{1I} + 2 m_2 V_{2I} = (m_1 + m_2) V_{1F}}$

Step 4:

Solve waypoint 3 to get the general expression for V_{1F}:

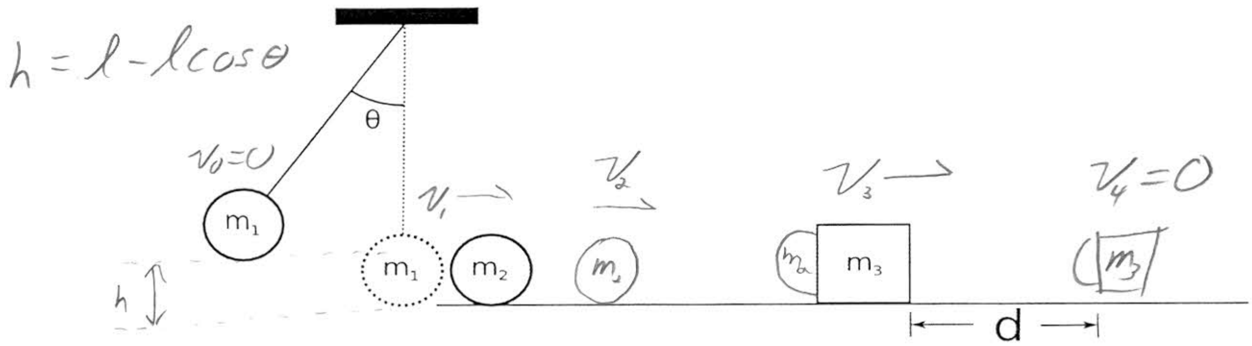
$$V_{1F} = \frac{(m_1 - m_2)}{(m_1 + m_2)} V_{1I} + \frac{2m_2}{m_1 + m_2} V_{2I}$$

Simply divide both sides by $(m_1 + m_2)$ and you get the final equation.

Sample Test 4
Phys 111 Spring 2010

4. A mass $m_1 = 3$ kg is attached to a string of length $l = 4.0$ m to create a pendulum. The pendulum, initially making an angle θ with the vertical, is released from rest. At the bottom of its swing, it collides elastically with mass $m_2 = 5$ kg. Mass 2 rolls (no friction) and sticks to $m_3 = 5$ kg. The m_2, m_3 combination slides with $\mu_k = 0.3$ a distance $d = 0.2$ m before coming to rest.

What was the original value of θ?



Step ①: W/E for pendulum

$U_I = mg(l - l\cos\theta)$ $\quad U_F = 0$

$K_I = 0$ $\quad K_F = \frac{1}{2} m_1 V_1^2$

$\Rightarrow \cancel{m_1} g l (1-\cos\theta) = \frac{1}{2} \cancel{m_1} V_1^2$

$$\boxed{V_1 = \sqrt{2gl(1-\cos\theta)}} \quad (1)$$

Step ②: Collide to get V_2

$$\boxed{V_2 = \frac{2m_1}{m_1 + m_2} V_1} \quad (2)$$

Taken directly from our equation for elastic collisions.

Step ③: Collide to get V_3

$m_2 V_2 = (m_2 + m_3) V_3$ conserve momentum.

$$\boxed{V_3 = \frac{m_2}{m_2 + m_3} V_2} \quad (3)$$

continued ↓

Sample test 4, P4 continued

Step 4: W/E to slide to a stop

$$U_I = U_F = 0$$

$$K_I = \tfrac{1}{2}(m_2 + m_3) v_3^2 \qquad K_F = 0$$

$$W_{NCF} = -\mu_k (m_2 + m_3) g d$$

$$\tfrac{1}{2}\cancel{(m_2 + m_3)} v_3^2 - \mu_k \cancel{(m_2 + m_3)} g d = 0$$

$$\boxed{v_3^2 = 2\mu_k g d} \quad 4$$

Put everything together:

$$3 \to 4 \qquad \frac{m_2^2}{(m_2+m_3)^2} v_2^2 = 2\mu_k g d$$

$$2 \to \qquad \frac{m_2^2}{(m_2+m_3)^2} \cdot \frac{4m_1^2}{(m_1+m_2)^2} v_1^2 = 2\mu_k g d$$

$$1 \to \qquad \left(\frac{2m_1 m_2}{(m_2+m_3)(m_1+m_2)}\right)^2 \cancel{2g}\, l(1-\cos\theta) = \cancel{2}\mu_k \cancel{g} d$$

Let's let: $R = \dfrac{2m_1 m_2}{(m_2+m_3)(m_1+m_2)}$

then: $R^2 l(1-\cos\theta) = \mu_k d$

$$\Rightarrow R^2 l - R^2 l\cos\theta = \mu_k d$$

$$\Rightarrow R^2 l\cos\theta = R^2 l - \mu_k d \Rightarrow \boxed{\cos\theta = 1 - \frac{\mu_k d}{R^2 l}}$$

continued ↓

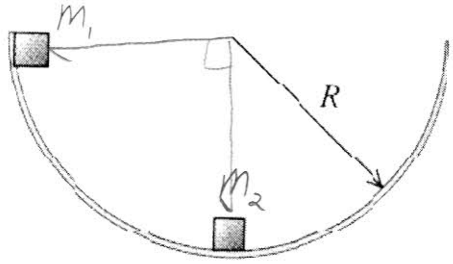
Sample test 4, P4 continued

$$R = \frac{2 m_1 m_2}{(m_1 + m_2)(m_1 + m_3)} = \frac{(2)(3)(5)}{(8)(10)} = \boxed{0.375}$$

and: $$\theta = \cos^{-1}\left[1 - \frac{\mu_k d}{R^2 \ell}\right]$$

$$\theta = \cos^{-1}\left[1 - \frac{(0.3)(0.2)}{(0.375)^2(4)}\right] = \cos^{-1}(0.89)$$

$$\boxed{\theta = 26.7°}$$



Two masses are released from rest in a frictionless hemispherical bowl of radius R from the positions shown in the figure. Derive an expression for their final height in the case of :

a) An elastic collision

b) An inelastic collision

c) How much bigger than the second mass does the first mass have to be so that the second mass gets out of the bowl.

Stage 1: Release m_1 from rest at the top of the bowl.

$U_I = m_1 g R \qquad K_I = 0$

$U_F = 0 \qquad K_F = \frac{1}{2} m_1 V_1^2$

$\Rightarrow \boxed{V_1 = \sqrt{2gR}}$ (1) $\Rightarrow$ Works for parts a and b

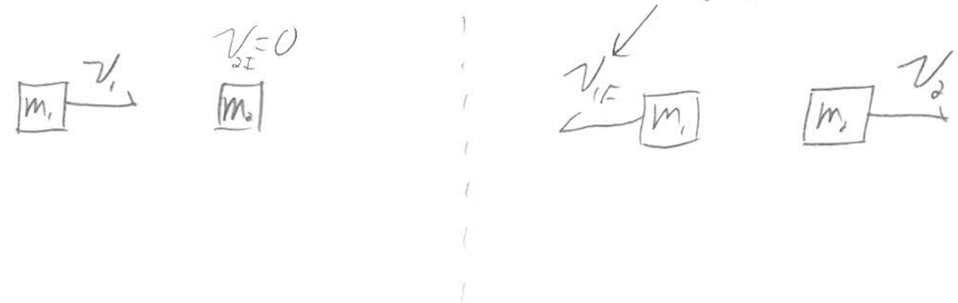
continued

Systems of Particles - Set 4, P8 continued

Stage 2: Collision!

a) elastic collision

Don't care about this one



General Form of the elastic collision eq.

$$V_{2F} = \frac{m_2 - m_1}{m_1 + m_2} V_{2I} + \frac{2m_1}{m_1 + m_2} V_{1I}$$

And putting in variables from the picture

$$V_2 = \cancel{\frac{m_2 - m_1}{m_1 + m_2} V_{2I}}^{\,0} + \frac{2m_1}{m_1 + m_2} V_1$$

$$\boxed{V_2 = \frac{2m_1}{m_1 + m_2} V_1} \quad 2a$$

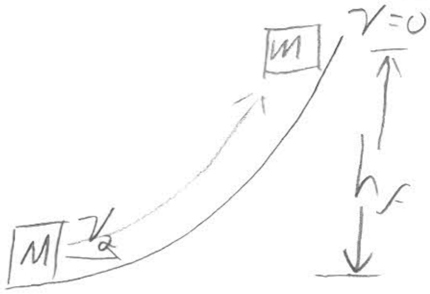
b) Inelastic

conserve momentum: $m_1 V_1 + \cancel{m_2 V_{2I}}^{\,0} = (m_1 + m_2) V_2$

$$\Rightarrow \boxed{V_2 = \frac{m_1}{m_1 + m_2} V_1} \quad 2b$$

continued ↓

Stage 3: Final mass reaches some new height h



$U_I = 0$ $\qquad K_I = \frac{1}{2} M v_2^2$

$U_F = Mgh_F$ $\qquad K_F = 0$

$$\cancel{M}gh_F = \frac{1}{2}\cancel{M}v_2^2 \Rightarrow \boxed{h_F = \frac{v_2^2}{2g}} \ ③$$

Put the stages together:

a) elastic

$$h_F = \frac{1}{2g}\left[\frac{2m_1}{m_1+m_2}\right]^2 v_1^2$$

$$\Rightarrow h_F = \frac{1}{\cancel{2g}}\left[\frac{2m_1}{m_1+m_2}\right]^2 \cancel{2g}R$$

$$\Rightarrow \boxed{h_F = \left[\frac{2m_1}{m_1+m_2}\right]^2 R}$$

b) inelastic

$$h_F = \frac{1}{2g}\left[\frac{m_1}{m_1+m_2}\right]^2 v_1^2$$

$$h_F = \frac{1}{\cancel{2g}}\left[\frac{m_1}{m_1+m_2}\right]^2 \cancel{2g}h \Rightarrow \boxed{h_F = \left[\frac{m_1}{m_1+m_2}\right]^2 R}$$

continued ↓

Systems of Particles, Set 4 P8 - continued

i) consider the elastic case.

m_2 escapes the bowl when $h_f > R$

$h_f > R$

$\Rightarrow \left[\frac{2m_1}{m_1+m_2}\right]^2 \cancel{R} > \cancel{R} \Rightarrow \left[\frac{2m_1}{m_1+m_2}\right]^2 > 1$

$\Rightarrow \frac{2m_1}{m_1+m_2} > 1 \Rightarrow 2m_1 > m_1 + m_2$

$\Rightarrow \boxed{m_1 > m_2}$

$\boxed{\text{So } m_2 \text{ escapes when } m_1 > m_2}$

* Let's consider the inelastic case

$h_f > R$

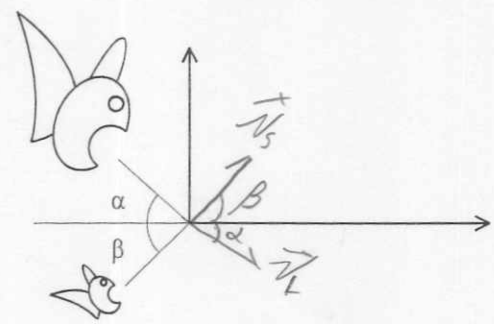
$\Rightarrow \left[\frac{m_1}{m_1+m_2}\right]^2 \cancel{R} > \cancel{R} \Rightarrow \left[\frac{m_1}{m_1+m_2}\right]^2 > 1$

$\Rightarrow m_1 > m_1 + m_2 \Rightarrow 0 > m_2$

Which isn't possible ...

1. A large fish will soon make a dish of a smaller fish. What is the velocity of the large fish and his dinner immediately after he eats? Give both the magnitude and direction of the final velocity with respect to the x-axis.

$m_{\text{large fish}} = 4.0$ kg	$m_{\text{small fish}} = 0.20$ kg
$v_{o\text{ large fish}} = 1.0$ m/s	$v_{o\text{ small fish}} = 5.0$ m/s
$\alpha_{\text{large fish}} = 25.0°$	$\beta_{\text{small fish}} = 50.0°$



Conserve momentum in both axis

$$x: m_L V_L \cos\alpha + m_s V_s \cos\beta = (m_L + m_s) V_F \cos\theta$$

$$y: -m_L V_L \sin\alpha + m_s V_s \sin\beta = (m_L + m_s) V_F \sin\theta$$

Divide y by x to eliminate V_F

$$\frac{-m_L V_L \sin\alpha + m_s V_s \sin\beta}{m_L V_L \cos\alpha + m_s V_s \cos\beta} = \frac{\cancel{(m_L+m_s)}\cancel{V_F}\sin\theta}{\cancel{(m_L+m_s)}\cancel{V_F}\cos\theta}$$

$$\tan\theta = \frac{-m_L V_L \sin\alpha + m_s V_s \sin\beta}{m_L V_L \cos\alpha + m_s V_s \cos\beta} \Rightarrow$$

$$\theta = \tan^{-1}\left[\frac{-(4.0)(1.0)\sin(25) + (0.2)(5)\sin(50)}{(4.0)(1.0)\cos(25) + (0.2)(5)\cos(50)}\right] = \boxed{-12°}$$

Plug back into x (or y) to get V_F

$$V_F = \frac{m_L V_L \cos\alpha + m_s V_s \cos\beta}{(m_L + m_s)\cos\theta} = \frac{(4)(1)\cos 25 + (0.2)(5)\cos(50)}{(4 + 0.2)\cos(-12)} = \boxed{1.0 \text{ m/s}}$$