

SAMPLE TEST 3  
PHYS 111 SPRING 2010

Name: \_\_\_\_\_

By writing my name above, I affirm that this test represents my work only, without aid from outside sources. In all aspects of this course I perform with honor and integrity.

SHOW YOUR WORK ON ALL OF THE PROBLEMS – YOUR APPROACH TO THE PROBLEM IS AS IMPORTANT AS (IF NOT MORE) IMPORTANT THAN) YOUR FINAL ANSWER.

1) Starting with the definition of work, derive the **Work Energy Theorem**.

$$W = \int \vec{F} \cdot d\vec{s}, \quad \vec{F} = F_x \hat{x} + F_y \hat{y} + F_z \hat{z}$$
$$d\vec{s} = dx \hat{x} + dy \hat{y} + dz \hat{z}$$

$$W = \int (F_x \hat{x} + F_y \hat{y} + F_z \hat{z}) \cdot (dx \hat{x} + dy \hat{y} + dz \hat{z})$$

$$\Rightarrow W = \int (F_x dx + F_y dy + F_z dz) \Rightarrow \boxed{W = \int F_x dx + \int F_y dy + \int F_z dz}$$

$$\boxed{W_{\text{net}} = \sum W} = \sum (\int F_x dx + \int F_y dy + \int F_z dz)$$

$$\Rightarrow W_{\text{net}} = \sum \int F_x dx + \sum \int F_y dy + \sum \int F_z dz$$

$$\Rightarrow W_{\text{net}} = \int \sum F_x dx + \int \sum F_y dy + \int \sum F_z dz$$

Now:  $\sum F_x = ma_x$  (NSL) so we can subst.

consider just the  $x$ -axis.

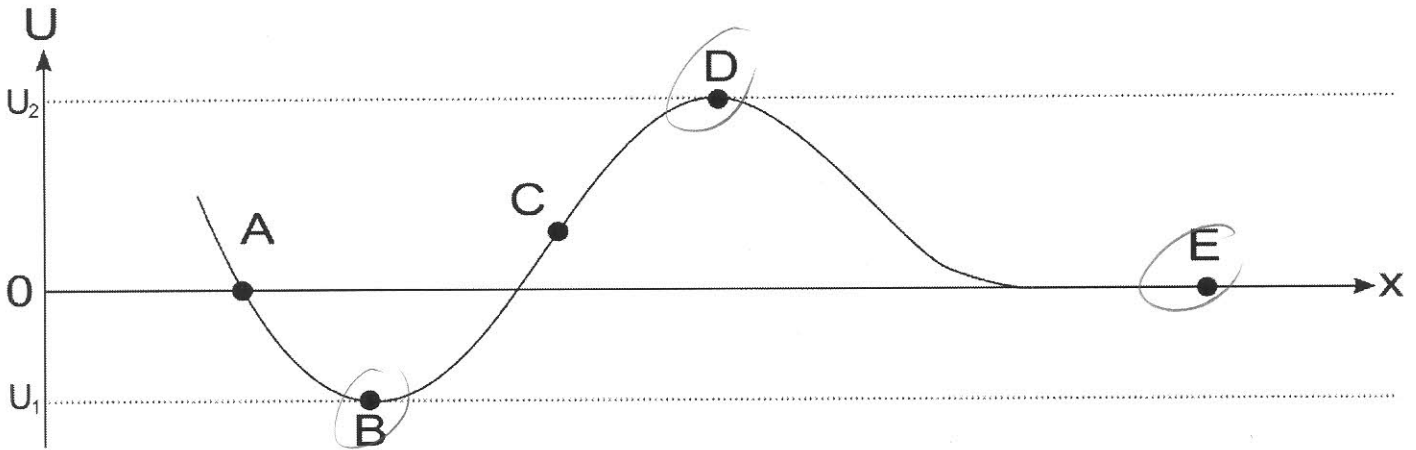
$$\int_{x_0}^x \sum F_x dx = \int_{x_0}^x m a_x dx = \int_{x_0}^x m \frac{dv_x}{dt} dx \stackrel{\text{Change Variable}}{=} \int_{v_{0x}}^{v_x} m \frac{dx}{dt} dv = \int_{v_{0x}}^{v_x} m v dv$$

$$= \frac{1}{2} m v_x^2 - \frac{1}{2} m v_{0x}^2 = \boxed{\Delta K_x}$$

$$\text{So: } \boxed{W_{\text{net}} = \Delta K} \quad \text{QED.}$$

SAMPLE TEST 3  
PHYS 111 FALL 2012

2) Conceptual Questions, 6 points each.



2.1) Refer to the potential energy curve above. At which point(s) does the Force go to zero?

- a) A, C
- b) B, D
- c) B, D, E
- d) B

$F = -\frac{dU}{dx}$ , so  $F=0$  at min/max points.

2.2) Your car travels at a *constant velocity* up a hill that makes an angle  $\theta$  with the horizontal. After traveling a distance  $d$ , the **net work** done on the car is:

- a)  $W_{net} = mgd \sin(\theta)$
- b)  $W_{net} = -mgd \sin(\theta)$
- c)  $W_{net} = 0$
- d)  $W_{net} = mgd \sin(\theta) - N\mu_k d$

Constant Velocity  $\Rightarrow \Delta K = 0$   
and because  $W_{net} = \Delta K$ ,  $W_{net} = 0$

2.3) Two objects, one of mass  $m$  and the other of mass  $2m$ , are dropped from the top of a building. When they hit the ground:

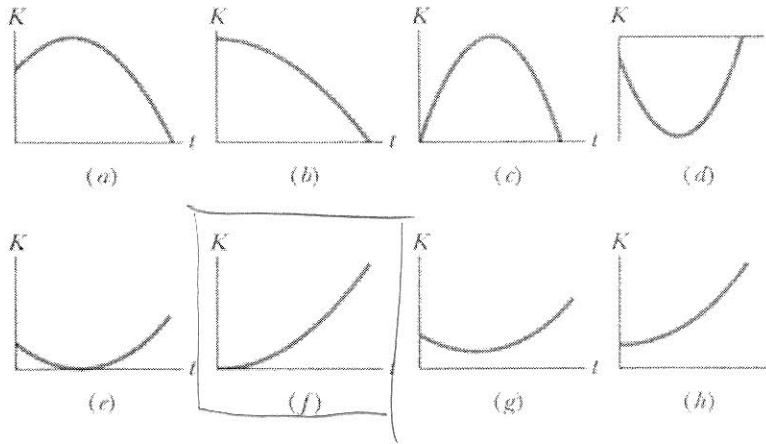
- a) both will have the same kinetic energy.
- b) the heavier one will have twice the kinetic energy of the lighter one.
- c) the heavier one will have half the kinetic energy of the lighter one.
- d) the lighter one will be moving at half the speed of the heavier one.

$mgh = K_F \Rightarrow K$  goes as  $m$   
so if  $m$  doubles,  $K$  doubles

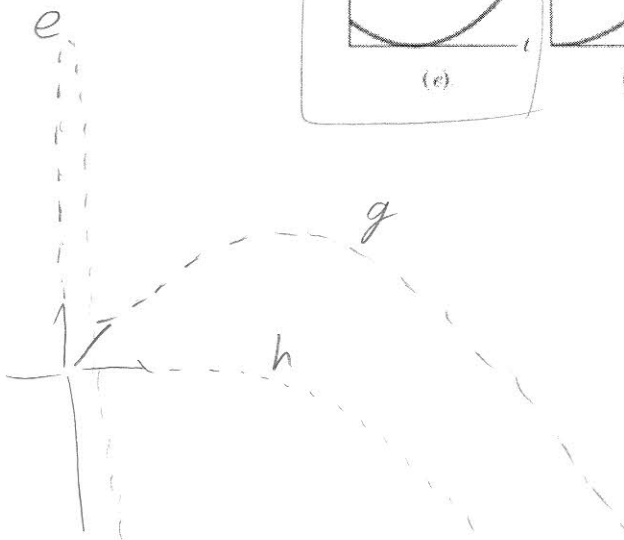
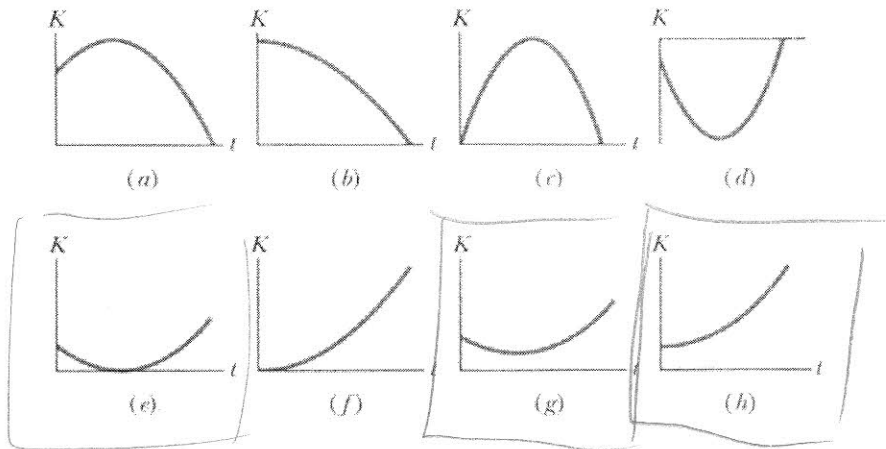
SAMPLE TEST 3  
 PHYS 111 FALL 2012

2.4) A glob of slime is dropped from rest from the edge of a cliff. Which graph(s) below could show how the kinetic energy changes as a function of time?

$v_0 = 0$   
 $\Rightarrow K_0 = 0$   
 and then increases...



2.5) A glob of slime is launched with an initial velocity from the top of a cliff. Which graph(s) below could show how the kinetic energy changes as a function of time?  
 (The direction of the initial velocity is intentionally left undefined.)



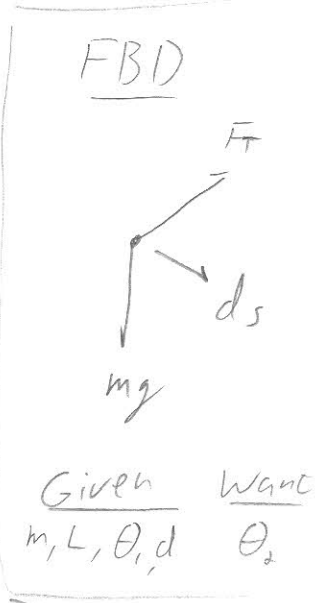
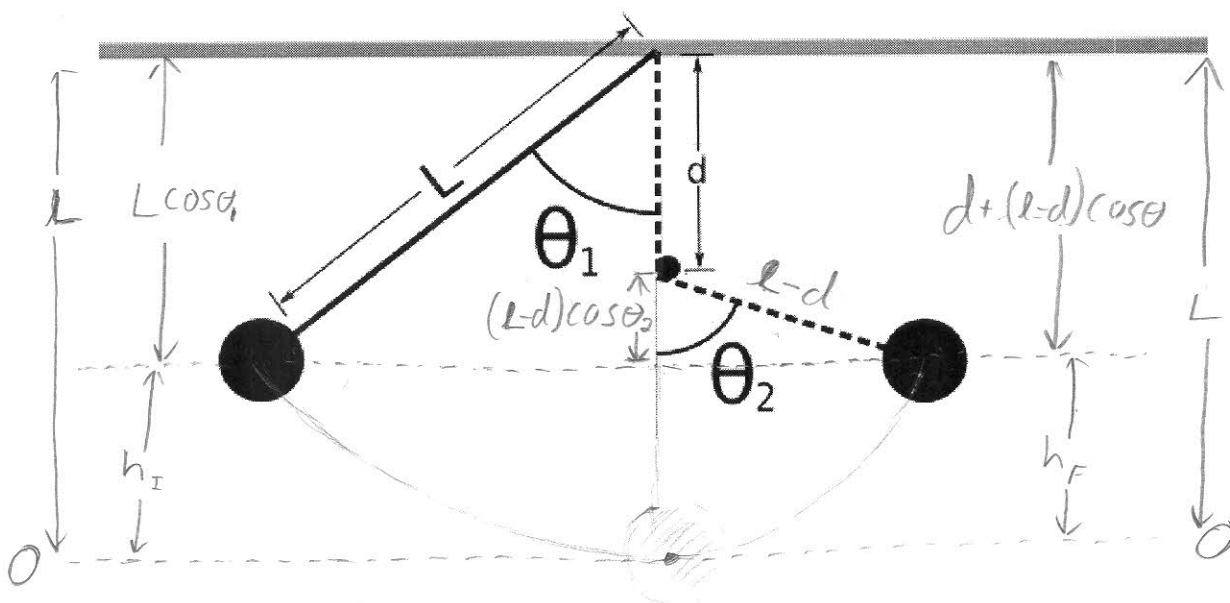
$K_F > 0$   
 Launched up gives e and g  
 Launched horizontal or down gives h

SAMPLE TEST 3  
PHYS 111 FALL 2012

3. Use work-energy techniques to solve the following problem.

A pendulum consisting of a string of length  $L$  and a sphere of mass  $m$  starts from rest at an angle  $\theta_1$  from the vertical. The string hits a peg a distance  $d$  below the suspension point, as shown in the picture below.

- Use work/energy to find  $\theta_2$  in terms of  $\theta_1$ ,  $L$ , and  $d$ .
- If  $\theta_1 = 90^\circ$ , what is the minimum value of  $d$  (in terms of  $L$ ) so that the pendulum will make a full circle around the peg without the string going slack?



a)  $h_I = L - L \cos \theta_1$

$h_F = L - [d + (L-d) \cos \theta_2]$

$h_I = L - d - L \cos \theta_2 + d \cos \theta_2$

$U_I = mgh_I \quad K_I = 0$

$U_F = mgh_F \quad K_F = 0$

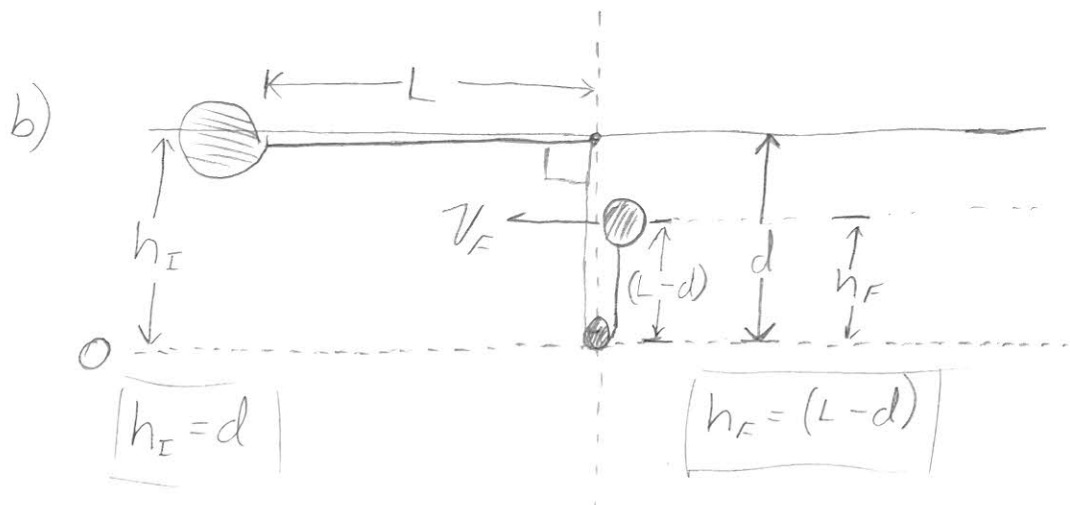
$mgh_I = mgh_F \Rightarrow h_I = h_F$

$\Rightarrow L - L \cos \theta_1 = L - d + (d - L) \cos \theta_2$

$\Rightarrow \cos \theta_2 = \frac{d - L \cos \theta_1}{d - L}$

Sample Test 3, P3 continued

(2)



Given  
 $\theta_i = 90^\circ$   
 $L$   
 $v_o = 0$

Want  
 $d$

$U_I = mgd$        $K_I = 0$

$U_F = mg(L-d)$        $K_F = \frac{1}{2} m v_F^2$

$\Rightarrow \boxed{mgd = mg(L-d) + \frac{1}{2} m v_F^2} \quad | \quad (1)$

✗ But what's  $v_F$ ?

→ if it's zero, the pendulum falls, but I want it to go in a circle... oh! a circle!

FBD - at  $h_F$       NSL



$F_T + mg = ma$ , but  $a = \frac{v^2}{r} \Rightarrow a = \frac{v^2}{L-d}$

string just goes slack when  $F_T = 0$

$\Rightarrow \boxed{mg = m \frac{v^2}{L-d} \Rightarrow v_F = \sqrt{g(L-d)}} \quad | \quad (2)$

continued ↓

Sample Test 3, P3 continued

3

So from (1) and (2)

$$d = L - d + \frac{1}{5}(L - d)$$

$$\Rightarrow d + d + \frac{1}{5}d = L + \frac{1}{5}L$$

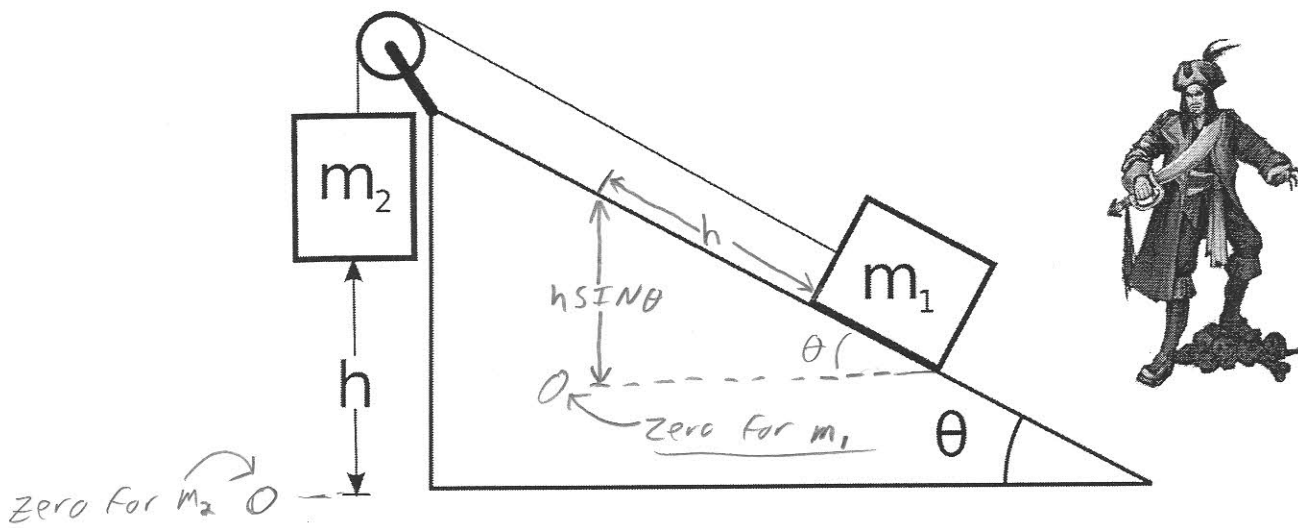
$$\frac{5}{5}d + \frac{1}{5}d = \frac{3}{5}L \Rightarrow \boxed{d = \frac{3}{5}L}$$

SAMPLE TEST 3

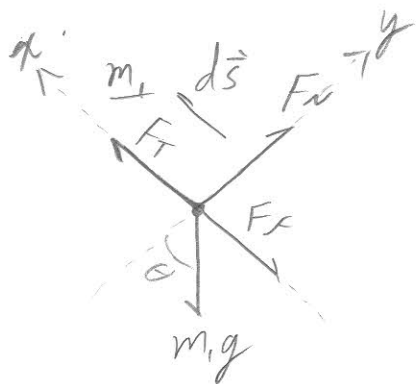
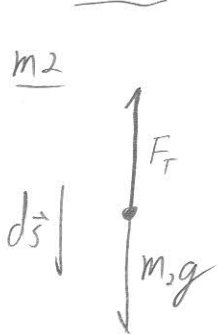
PHYS 111

3) Use Work-Energy techniques to solve the following problem

In the system below,  $m_2$  hangs from a rope that passes over a pulley (massless and frictionless) and is attached to  $m_1$ . Assuming that  $m_2 > m_1$ , the system starts from rest, and that an annoying evil pirate has added friction to the ramp, find an expression for the velocity of  $m_2$  after it falls through a distance  $h$ .



FBD



$$W_{FN} = 0, \quad \vec{F}_N \perp d\vec{s}$$

$W_T = 0$ , Tension on  $m_2$  does work  $(-F_T h)$  and on  $m_1$  does work  $F_T h$

That leaves Friction and gravity:

$$U_I = m_2 g h + 0$$

$$K_I = 0 + 0$$

$$U_F = 0 + m_1 g h \sin \theta$$

$$K_F = \frac{1}{2} m_1 v^2 + \frac{1}{2} m_2 v^2$$

$$W_{NCF} = \int_0^h \vec{F}_x \cdot d\vec{s} = - \int_0^h \mu_s m g \cos \theta dx = -\mu_s m g \cos \theta h$$

continued ↓

Extra Space for #3

$$m_2gh - \mu_k m_1 g \cos \theta h = m_1 gh \sin \theta + \frac{1}{2} (m_1 + m_2) v^2$$

$$\frac{1}{2} (m_1 + m_2) v^2 = m_2gh - m_1gh \sin \theta - \mu_k m_1gh \cos \theta$$

$$v = \left[ \frac{2gh(m_2 - m_1(\sin \theta + \mu_k \cos \theta))}{(m_1 + m_2)} \right]^{\frac{1}{2}}$$



SAMPLE TEST 3

PHYS 111

5) Use Work-Energy techniques to solve the following problem

After getting your physics degree, you end up on the planning committee for the 2040 Olympic games. You are considering a new sport: Asteroid Jumping. You need to pick an asteroid that the high jumpers will not accidentally escape from after their jump (the initial velocity of their jump must be below the escape velocity of the asteroid).

- a) On Earth, the absolute highest a high jumper could go is 3.0m. On the Earth's surface, what initial velocity is required to reach a height of 3.0m?
- b) Asteroids are made out of rock that has a density of  $2500 \text{ kg/m}^3$ . Assuming spherical asteroids, find the radius of an asteroid with an escape velocity equal to the velocity from part a.

(HINT: The volume of a sphere is  $\frac{4}{3}\pi r^3$ , where  $r$  is the radius of the sphere.)



Given  
 $h = 3.0 \text{ m}$

Want  
 $v$

$$U_I = 0 \quad K_I = \frac{1}{2} m v^2$$

$$U_F = mgh \quad K_F = 0$$

$$v = \sqrt{2gh} \text{ / again!}$$

$$v = ((2)(9.8)(3.0))^{1/2} = \underline{7.7 \text{ m/s}}$$

b)

Given  
 $\rho = 2500 \text{ kg/m}^3$   
 $v_0 = 7.7 \text{ m/s}$

Want  
 $r_A$



$$U_I = -\frac{GM_A m}{r_A} \quad U_F = 0$$

$$K_I = \frac{1}{2} m v_0^2 \quad K_F = 0$$

$$\text{so: } -\frac{GM_A m}{r_A} + \frac{1}{2} m v_0^2 = 0$$

Continued



Extra Space for #5

$$\Rightarrow \frac{GM_A}{r_A} = \frac{1}{2} v_0^2$$

But:  $M = \rho V$  where  $V = \text{Volume}$

$$V = \frac{4}{3} \pi r^3$$

$$\text{So: } M_A = \rho \frac{4}{3} \pi r_A^3$$

$$\text{And: } \frac{G \rho \frac{4}{3} \pi r_A^3}{r_A} = \frac{1}{2} v_0^2$$

$$\Rightarrow r_A = \left[ \frac{3 v_0^2}{8 G \rho \pi} \right]^{1/2}$$

$$\Rightarrow r_A = \left[ \frac{3 (7.7)^2}{8 (6.67 \times 10^{-11}) (2500) (3.14)} \right]^{1/2}$$

$$r_A = 6,500 \text{ m}$$