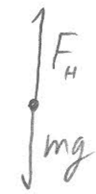
Force Problems – Set 1

You toss a rock straight up into the air by placing it on the palm of your hand (you're not gripping it) then pushing your hand up very rapidly.
Draw F*ree Body Diagrams* for the rock at the following points along its trajectory. Be sure to include a coordinate system on your diagram.
The lengths of the vector arrows should indicate the magnitudes of the applied forces.

a) As you hold the rock at rest on your palm, before moving your hand.



$a = \emptyset$, $F_H = mg$

b) As your hand is moving up but before the rock leaves your hand.

$F_H > mg$, a is positive ↑

c) One-tenth of a second after the rock leaves your hand.

No $F_{Aristotle}$! a is neg ↓

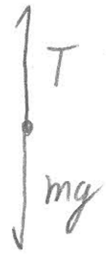
d) At the peak of the rock's trajectory

e) After the rock has reached its highest point and is now falling straight down.

Force Problems – Set 1

For the following situations, draw free-body diagrams to indicate all forces acting on the object(s) in question. *Indicate relative magnitudes of forces by drawing long, short, or equal-length vectors.*

a) An elevator suspended by a cable is descending at a constant velocity.

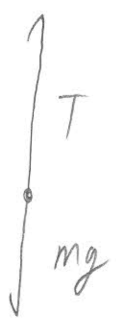


$T = mg$, $a = 0$, const v

b) An elevator suspended by a cable is ascending at a constant velocity.

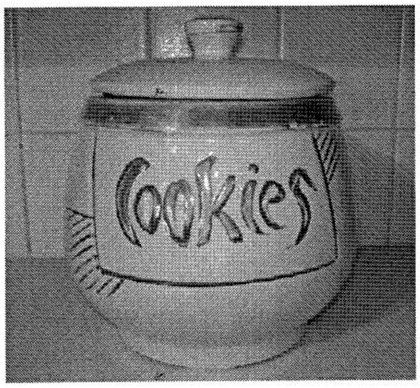
$T = mg$, $a = 0$, const v

c) An elevator suspended by a cable is accelerating upwards.



$T > mg$, a is positive $\uparrow$

Force Problems – Set 1

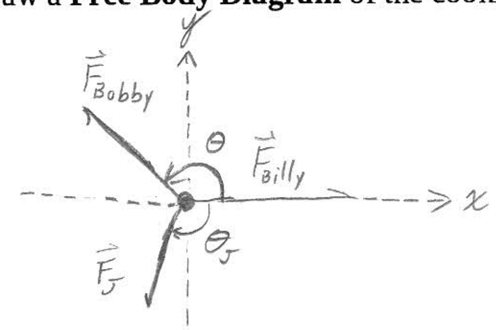


Bobby, Billy, and Jane are fighting over a cookie jar each pulling it in a different direction. As a result of their tussle, the jar's acceleration is zero. Bobby is pulling with a force of 10 N. Billy is pulling with a force of 12 N at an angle of 150° degrees with respect to the direction that Bobby is pulling.

a) What is the *Net Force* on the jar?

if $\vec{a}=0$, then $\vec{F}_{net}=0$

b) Draw a **Free Body Diagram** of the cookie jar. Include a *coordinate system.*



$|\vec{F}_{Bobby}| = 10N$

$|\vec{F}_{Billy}| = 12N$

$\theta = 150°$

c) Write **Newton's Second Law** for each axis.

$x: F_{Bobby\,x} + F_{Billy\,x} + F_{Jx} = ma_x$

$y: F_{Bobby\,y} + F_{Billy\,y} + F_{Jy} = ma_y$

$x: |\vec{F}_{Bobby}|\cos(\emptyset) + |\vec{F}_{Billy}|\cos\theta + F_{Jx} = 0$

$y: |\vec{F}_{Bobby}|\sin(\emptyset) + |\vec{F}_{Billy}|\sin\theta + F_{Jy} = 0$

b) Solve the above system of equations for the components of Jane's force and find the magnitude and angle of her force with respect to Billy's.

$F_{Jx} = -|\vec{F}_{Bobby}|\cos(\emptyset) - |\vec{F}_{Billy}|\cos\theta$

$F_{Jx} = -10\cdot\cos(\emptyset) - 12\cos(150)$

$F_{Jx} = 0.39N$

$F_{Jy} = -\cancel{|\vec{F}_{Bobby}|\sin(\emptyset)}^{\,0} - |\vec{F}_{Billy}|\sin\theta$

$F_{Jy} = 0 - 12\cdot\sin(150)$

$F_{Jy} = -6.0$

$|\vec{F}_{Jx}| = (0.39^2 + 6^2)^{1/2} = \boxed{6.01N}$

$\theta_J = \tan^{-1}\left(\frac{-6.0}{0.39}\right)$

$\boxed{\theta_J = -86°}$

Force Problems – Set 1

A very talented stick skier is accelerating down a VERY slippery slope. (there's no friction, that's next period).

Given	Want
θ, d	$\vec{a}, \vec{V}_F$

a) List all of the forces on the skier.

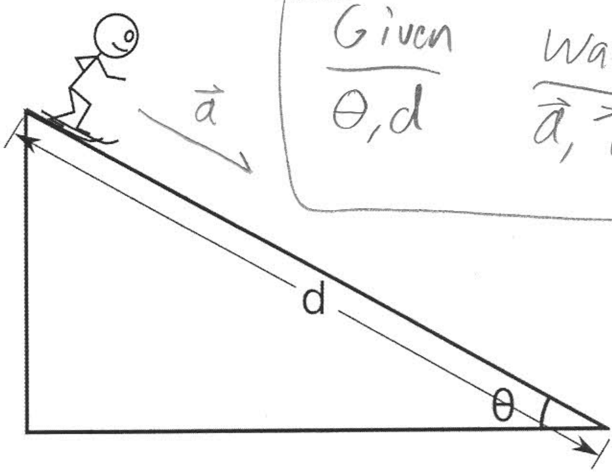
b) In what direction will the skier accelerate? Draw the acceleration vector in the picture at the right.

c) Draw a *Free Body Diagram* for the skier.

d) Choose a coordinate system and superimpose it on your free body diagram.

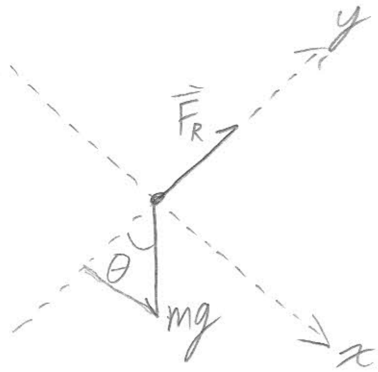
e) Write the x and y versions of Newton's Second Law for the skier based on your coordinate system. Solve these equations for acceleration.

g) What's the skier's velocity after accelerating a distance d?



a) Gravity, Reaction Force from Slope

c, d)



Rotate coordinate system to align x with the $\vec{a}$ vec.

e)

x

$\Sigma F_x = m a_x$

$\cancel{m} g \sin\theta = \cancel{m} a_x$

$\boxed{a_x = g \sin\theta}$

y — no accel in y

$\Sigma F_y = m a_y$ ↓ $a_y = 0$

$F_R - mg\cos\theta = 0$

$\boxed{F_R = mg\cos\theta}$

continued ↓

Force Problems - Set 1, P2 continued

g) Now use kinematics. But, only x since nothing is happening in y.

$$x = \cancel{x_0}^{0} + \cancel{v_{0x}t}^{0} + \tfrac{1}{2}a_x t^2 \qquad v_x = \cancel{v_0}^{0} + a_x t$$

$$d = \tfrac{1}{2}a_x t^2 \qquad v_x = at \Rightarrow t = \frac{v_x}{a}$$

$$d = \tfrac{1}{2}\cancel{a_x}\frac{v_x^2}{a_x^{\cancel{2}}} \Rightarrow d = \frac{v_x^2}{2a_x}$$

$$\Rightarrow v_x = (2da_x)^{1/2}$$

$$\Rightarrow \boxed{v_x = (2dg\sin\theta)^{1/2}}$$

Force Problems – Set 1

A block is given an initial velocity of 5 m/s up a frictionless 20° incline. How far up the incline does the block slide before coming to rest?

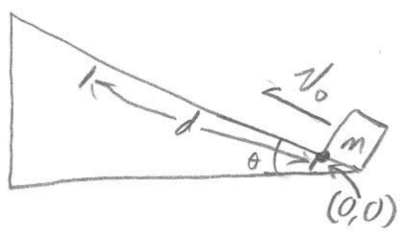
a) Draw a free body diagram of the block

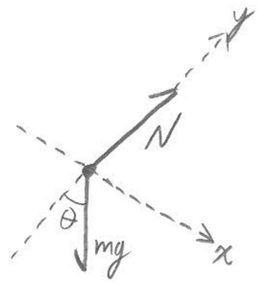
Given

$V_0 = 5\,m/s$

$\theta = 20^\circ$

$d = ?$ ← want





b) Put a coordinate system on your free body diagram and, from the resulting picture, write Newton's second law for the x axis and for the y axis. Solve these equations for the acceleration of the block.

$$x:\ \cancel{m}g\sin\theta = \cancel{m}a_x \Rightarrow a_x = g\sin\theta$$

$$y:\ N - mg\cos\theta = 0$$

c) Use the kinematics equations and the acceleration from part b to find the distance.

$$x = x_0 + V_0 t + \tfrac{1}{2}a_x t^2 \qquad V = V_0 + a_x t$$

$$-d = 0 - V_0 t + \tfrac{1}{2} g\sin\theta\, t^2 \qquad 0 = -V_0 + g\sin\theta\, t$$

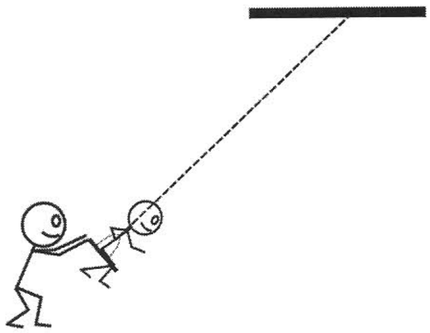
$$t = \frac{V_0}{g\sin\theta}$$

$$-d = \frac{-V_0^2}{g\sin\theta} + \frac{1}{2}\frac{V_0^2}{g\sin\theta}$$

$$\boxed{d = -\frac{V_0^2}{2g\sin\theta}} = \frac{-5^2}{(2)(9.8)\sin(20)} = \boxed{-3.7\,m}$$

Force Problems – Set 1

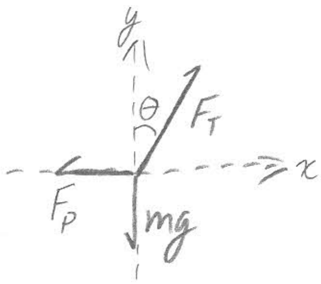
A 30-kg child is seated in a swing of negligible mass. Her father is pulling her back and is just about to let her go. How much **horizontal** force must her father apply so that the child and swing is held stationary the chain makes an angle of 32° with the vertical?



a) What is the child's acceleration?

$a = 0$, child is not moving

b) Draw a free body diagram of the system. Choose and label a coordinate system.



Given	Want
$m = 30\ kg$	$F_p = ?$
$\theta = 32^\circ$	
$a = 0$	

c) Using the picture from part b, write Newton's Second Law for the x-axis and the y-axis. Solve these equations for the required force.

$$\sum \vec{F} = m\vec{a} \Rightarrow \vec{F}_p + \vec{F}_T + m\vec{g} = \cancel{m\vec{a}}^{\,0}$$

$$x: F_T \sin\theta - F_p = 0 \Rightarrow F_T \sin\theta = F_p$$

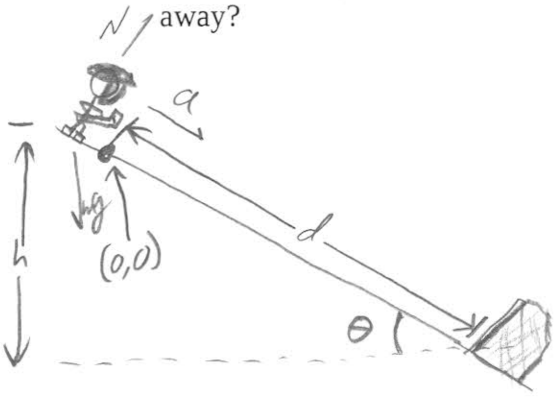
$$y: F_T \cos\theta - mg = 0 \Rightarrow F_T \cos = mg$$

Divide x by y: $\dfrac{\cancel{F_T}\sin\theta}{\cancel{F_T}\cos\theta} = \dfrac{F_p}{mg} \Rightarrow \boxed{F_p = mg\tan\theta}$

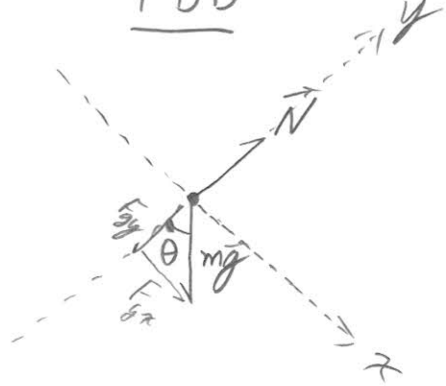
$F_p = (30)(9.8)(\tan 32) = \boxed{184\ N}$

Force Problems – Set 1

A terrible earthquake has happened in San Francisco right in the middle of a critical hockey tournament. As a result of the quake, the ice rink is tilted 15° from horizontal. The 80 kg goalie begins to slide down the slope uncontrollably from his net directly into the opposing goalies net. How fast is he when he crosses the opposite goal line 53 m away?



FBD



Given

$\theta = 15°$

$m = 80\ kg$

$d = 53\ m$

$V_F = ?$ ← want

Find the acceleration

$\vec{F}_{net} = m\vec{a} \Rightarrow NSL$

$x: N_x + F_{gx} = ma_x$

$0 + mg\,SIN\theta = ma_x \Rightarrow \boxed{a_x = g\,SIN\theta}$

$y: N_y + F_{gy} = 0$

$N_y - mg\cos\theta = \underline{0} \Rightarrow \underline{N_y = mg\cos\theta}$ okay... not useful here...

no acceleration in the y

continued ↓

Ice rink continued

We have acceleration, now find V_F using kinematics.
But there's only action in x:

$x = x_0 + V_{0x}t + \frac{1}{2}a_x t^2$ $\qquad$ $V = V_0 + at$

(1) $d = 0 + 0 + \frac{1}{2}g\sin\theta t^2$ $\qquad$ (2) $V_F = 0 + g\sin\theta t$

acceleration is in positive x

Solve (1) for t and plug into (2)

From (1): $t = \left[\frac{2d}{g\sin\theta}\right]^{1/2}$

into (2): $V_F = g\sin\theta\left[\frac{2d}{g\sin\theta}\right]^{1/2}$

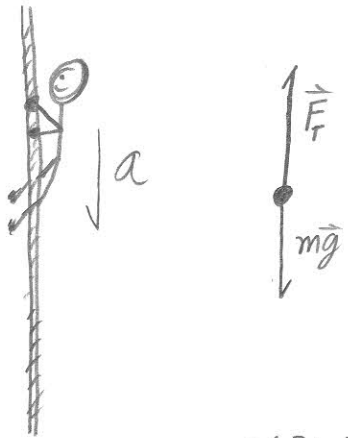
$\boxed{V_F = (2gd\sin\theta)^{1/2}}$ $\qquad$ hmm... $d\sin\theta = h$

$V_F = \left[(2)(9.8)(53)\sin(15)\right]^{1/2} = \boxed{16.4\ m/s}$

Force Problems – Set 1

A 52 kg circus performer slides down a rope that will break if the tension exceeds 425 N.

a) What happens if the performer hangs stationary from the rope?
b) At what acceleration will the performer just avoid breaking the rope?



Given

$F_{T max} = 425 N$

$m = 52 kg$

Want

Part a: F_T

Part b: a_{min}

NSL: $\Sigma F = ma$

$$F_T - mg = ma$$

a) Stationary performer, (or performer at constant v):

$a = 0$

$\Rightarrow F_T - mg = 0 \Rightarrow \boxed{F_T = mg}$

$F_T = (52 kg)(9.8 m/s^2) = \underline{510 N}$

Rope breaks.

b) Accelerating performer: $F_T = ma + mg < F_{T max}$

or: $\boxed{a < \frac{F_{T max}}{m} - g} \Rightarrow a < \frac{425N}{52kg} - 9.8 m/s^2$

$a < -1.6 m/s^2$ or less.