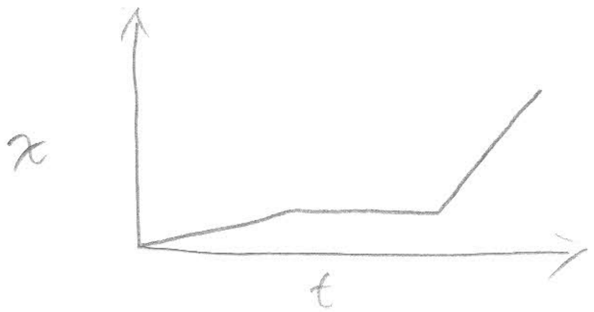
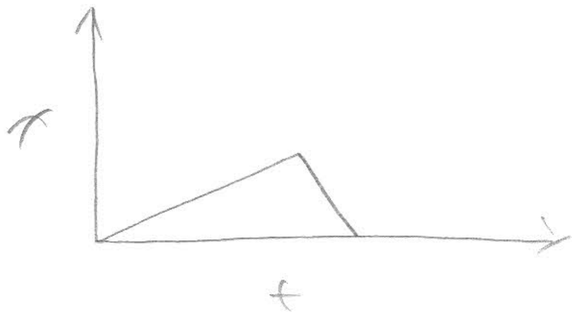
Kinematics Problems – Set 2

Sketch position vs. time graphs for the following situations. You should label your axes, but you don't need to include numbers.

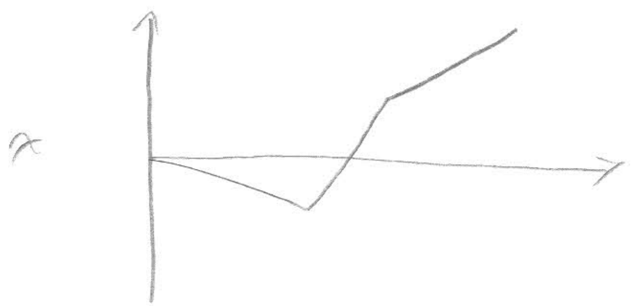
(a) A student walks to the bus stop, waits for the bus, then rides to campus. Assume that all the motion is along a straight street.



(b) A student walks slowly from home to the bus stop, realizes he forgot his paper that is due, and *quickly* walks home to get it.



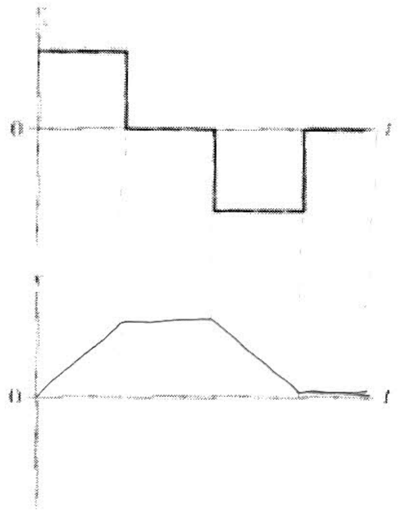
(c) The quarterback drops back 10 yards from the line of scrimmage, then throws a pass 20 yards to the receiver, who catches it and sprints 20 yards to the goal. Draw your graph for the *football*. Think carefully about what the slopes of the lines should be.



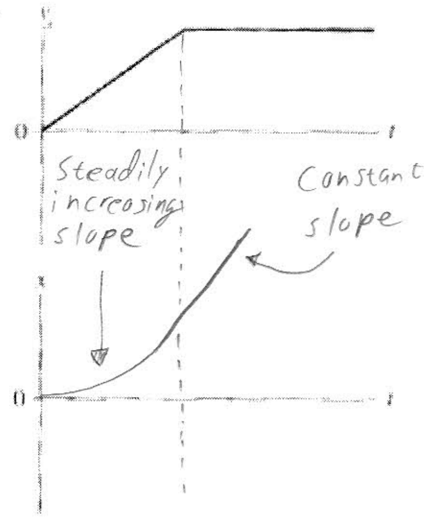
Kinematics Problems – Set 2

Below are four velocity vs. time graphs. For each, draw the corresponding position vs. time graph.

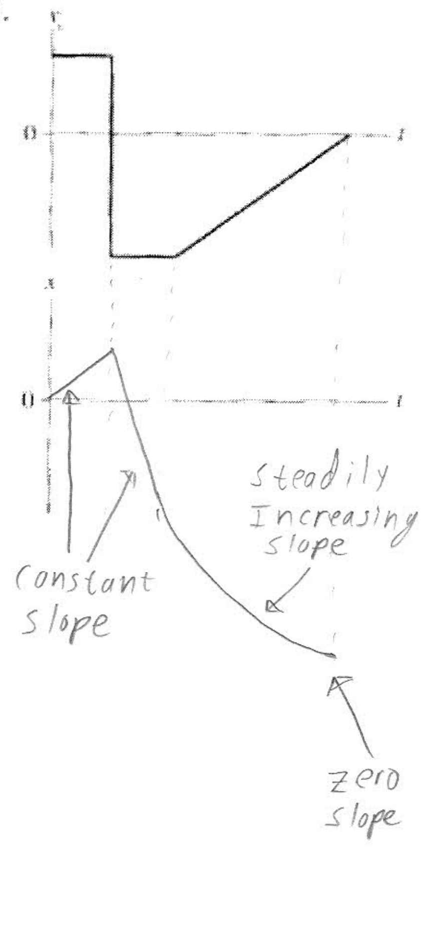
a.



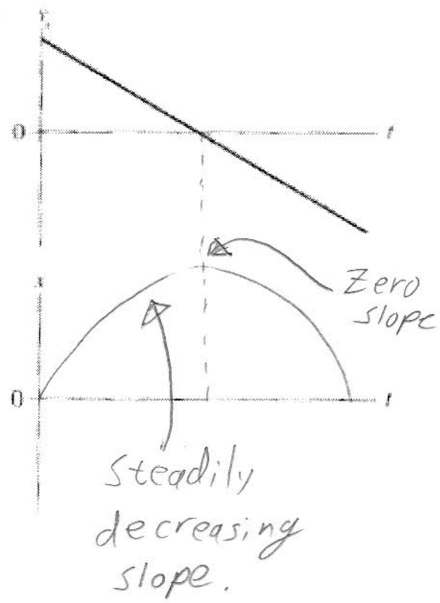
b.



c.



d.

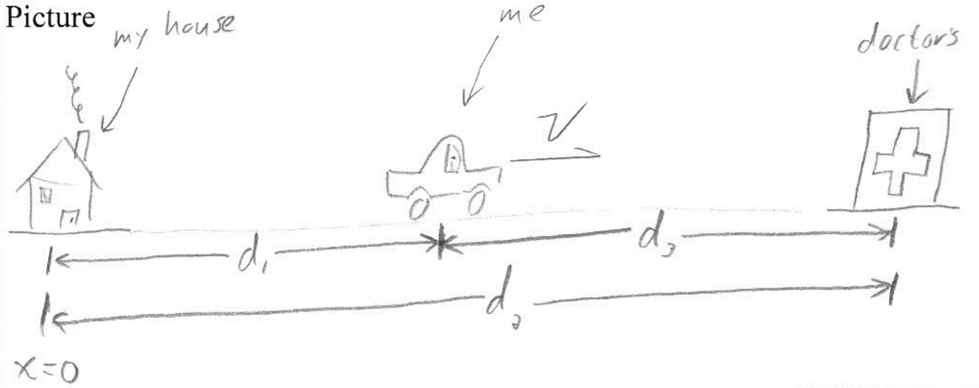


Kinematics Problems – Set 2

I am 10 miles away from my house traveling at 60 miles per hour. The doctor's office is ahead of me 17 miles from my house. How long will it take me to get to the doctor's office?

a) Think carefully about the problem. Play a movie in your mind of what's going on.
b) Draw a CLEAR picture of the situation. Pick an ORIGIN for your coordinate system.
c) List the GIVEN information and the WANTED information off to the side. Assign variables to each piece of information.
d) Label all of the relevant distances and velocities as given in the problem statement. Do NOT put numbers on your picture, use the variables that you assigned.

Picture



Given

$d_1 = 10$ miles
$d_2 = 17$ miles
$v = 60$ mi/hr

Wanted

$t = ?$

e) Clearly identify the physical relationships that you'll use to solve the problem. What variables are you trying to relate? How are they related? Write down the appropriate relationship(s) using your variables.

relate distance to velocity and time:

$$x = x_0 + vt \Rightarrow \boxed{d_2 = d_1 + vt} \quad (1)$$

f) Solve the equation above for the variable of interest. Put in number only after you have an analytical answer.

$$d_2 = d_1 + vt$$

$$\Rightarrow d_2 - d_1 = vt$$

$$\Rightarrow \boxed{t = \frac{d_2 - d_1}{v}}$$

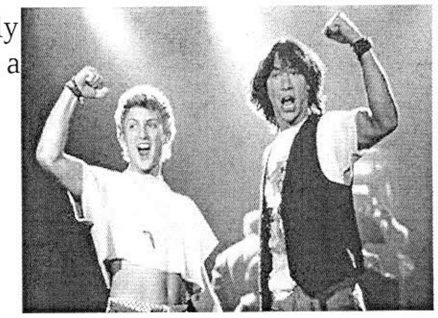
The "answer"

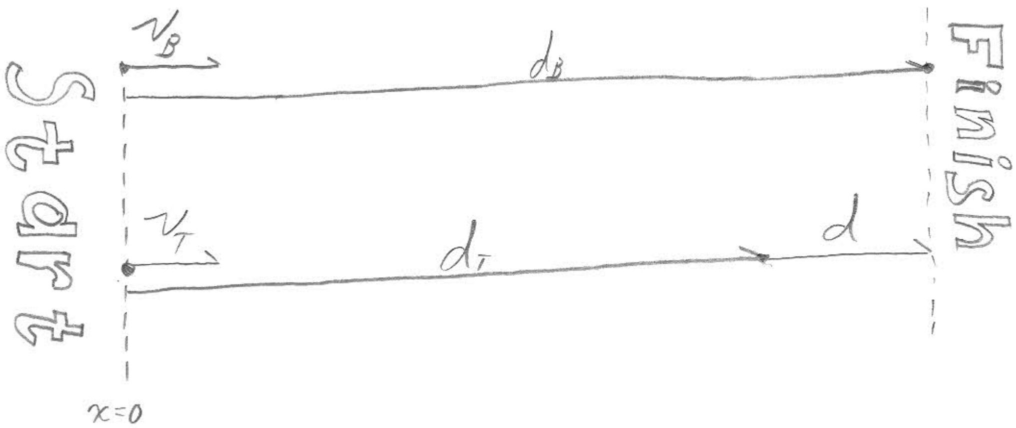
$$t = \frac{17_m - 10_m}{60\ m/hr} = \frac{7\text{ miles}}{60\text{ miles/hr}} = \boxed{0.12\text{ hr}}$$

$$t = 0.12\ \cancel{hr} \cdot 60\ \frac{min}{\cancel{hr}} = 7.2 \text{ minutes}$$

Kinematics Problems – Set 2

Bill and Ted are going to race. Bill can run 15 mi/hr and Ted can only run 11 mi/hr. How far from the finish line is Ted when Bill finishes a quarter mile race?





Given

$v_B = 15$ mi/hr

$v_T = 11$ mi/hr

$d_B = 0.25$ mi

$d = ?$

Little vector sum: $d_B = d_T + d$ (1)

Relate distance, velocity, and time.
Need one displacement relationship per actor.

Bill

$x(t) = x_0 + v_x t$

(2) $d_B = 0 + v_B t$

Ted

$x(t) = x_0 + v_x t$

(3) $d_T = 0 + v_T t$

From (3): $t = \frac{d_T}{v_T}$ (4)

Plug (4) into (2): $d_B = v_B \frac{d_T}{v_T}$

Solve for d_T: $d_T = \frac{v_T}{v_B} d_B$ (5)

Plug (5) back into (1): $d_B = \frac{v_T}{v_B} d_B + d$ (6)

continued ↓

Kinematics problems: Set2, P1 continued

Solve (6) for d

$$d = d_B - d_B \frac{v_T}{v_B}$$

$$\Rightarrow d = d_B\left(1 - \frac{v_T}{v_B}\right)$$

$$\Rightarrow \boxed{d = d_B\left[\frac{v_B - v_T}{v_B}\right]}$$ The answer

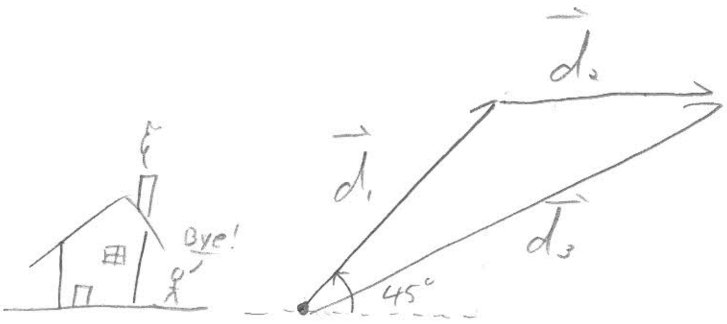
Put numbers in

$$d = 0.25\,mi\left[\frac{15\,mi/hr - 11\,mi/hr}{15\,mi/hr}\right] = \underline{0.067\text{ miles}}$$

Kinematics Problems – Set 2

I'm running errands in my car. I start at my house and drive due northeast at 40mi/hr for 10 minutes. Then, I turn East and drive 60 mi/hr for 4 minutes.

a) Find the components of my final displacement from my house.
b) If I drive directly home from this spot, how fast do I have to be going to get home in 5 minutes?



Given

$|\vec{v}_1| = 40 mi/hr$ $\quad t_1 = 10_m$

$\theta_1 = 45°$

$|\vec{v}_2| = 60 mi/hr$ $\quad t_2 = 4_m$

$\theta_2 = 0°$

$\vec{d}_3 = ?, \ |\vec{v}_3| = ?$

$t_3 = 5_{min}$

Vector problem:

$\vec{d}_3 = \vec{d}_1 + \vec{d}_2 \Rightarrow (1)\ d_{3x} = d_{1x} + d_{2x}$

$(2)\ d_{3y} = d_{1y} + d_{2y}$

I'm not given d_1 or d_2, so I have to find them.

Velocity is constant, so I can use:

$x(t) = x_0 + v_x t$ and $y(t) = y_0 + v_y t$

d_1

$(3)\ d_{1x} = 0 + |\vec{v}_1| \cos\theta_1 t_1$ $\qquad (4)\ d_{1y} = 0 + |\vec{v}_1| \sin\theta_1 t_1$

d_2 – Referenced from the head of d_1

$(5)\ d_{2x} = 0 + |\vec{v}_2| \cos\theta_2 t_2$ $\qquad (6)\ d_{2y} = 0 + |\vec{v}_2| \sin\theta_2 t_2$

continued ↓

Kinematics Problems- Set2, P2

Combine equations 1, 3, and 5

$$\boxed{d_{3x} = |\vec{v}_1|\cos\theta_1 t_1 + |\vec{v}_2|\cos\theta_2 t_2}$$

Combine equations 2, 4, and 6

$$\boxed{d_{3y} = |\vec{v}_1|\sin\theta_1 t_1 + \cancel{|\vec{v}_2|\sin\theta_2 t_2}^{\,0}} \leftarrow \sin(0) = 0$$

$$d_{3x} \cdot 40\,mi/\cancel{hr}\cos(45)\cdot 10\,\cancel{min}\cdot\frac{1}{60}\frac{\cancel{hr}}{\cancel{min}} + 60\,mi/\cancel{hr}\cos(0)\cdot 4\,\cancel{min}\frac{1}{60}\frac{\cancel{hr}}{\cancel{min}}$$
$$= \boxed{8.7\,miles}$$

$$d_{3y} = 40\,mi/\cancel{hr}\cdot\sin(45)\cdot\frac{10\,\cancel{min}}{60\,\cancel{min}/\cancel{hr}} = \boxed{4.7\,miles}$$

b) $$|\vec{d}_3| = (d_{3x}^2 + d_{3y}^2)^{1/2}$$

$$x = x_0 + vt$$

$$|\vec{d}_3| = 0 + |\vec{v}_3|t_3 \Rightarrow |\vec{v}_3| = \frac{|\vec{d}_3|}{t_3}$$

$$\Rightarrow \boxed{|\vec{v}_3| = \frac{(d_{3x}^2 + d_{3y}^2)^{1/2}}{t_3}}$$

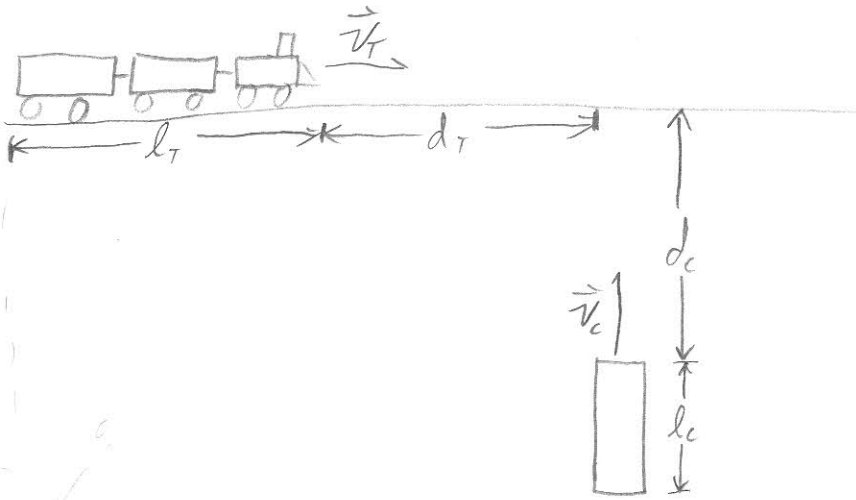
$$|\vec{v}_3| = \frac{(8.7^2 + 4.7^2)^{1/2}}{5\,min\cdot\frac{1}{60}\,hr/min} = \boxed{118\,mi/hr}$$

Kinematics

You are traveling north in your car approaching a rail-road crossing. A 200 ft passenger train traveling east at 50 mi/hr is 40 ft away from the intersection. The front of your 10 ft long car is 30 ft away from the intersection.

a) What minimum speed does your car need so that the back end of your car clears the tracks before the train gets to the intersection?

b) What is the maximum speed that your car can be traveling so that it clears the back end of the train?



$l_T = 200\,ft$

$V_T = 50\,mi/hr \cdot \frac{5280\,ft/s}{3600\,s/hr} = 73\,ft/s$

$d_T = 40\,ft$

$l_c = 10\,ft$

$d_c = 30\,ft$

a) car

$y = y_0 + V_0 t$

$d_c + l_c = 0 + V_c t$

train

$x = x_0 + V_0 t$

$d_T = V_t t \Rightarrow t = \frac{d_T}{V_t}$

$$d_c + l_c = V_c \frac{d_T}{V_T}$$

$$\boxed{V_c = V_T \frac{d_c + l_c}{d_T}}$$

$$\boxed{V_c = 73\,ft/s \,\frac{30\,ft + 10\,ft}{40\,ft} = 73\,ft/s}$$

same speed as train

continued ↓

Train vs car continued

b) car

$y = y_0 + v_0 t$

$d_c = v_c t$

$t = \frac{d_c}{v_c}$

train

$x = x_0 + v_0 t$

$l_T + d_T = v_T t$

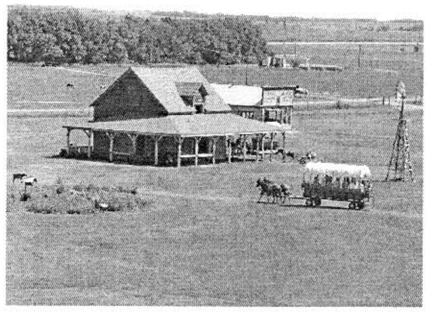
$l_T + d_T = v_T \frac{d_c}{v_c}$

$$\boxed{v_c = v_T \frac{d_c}{l_T + d_T}}$$

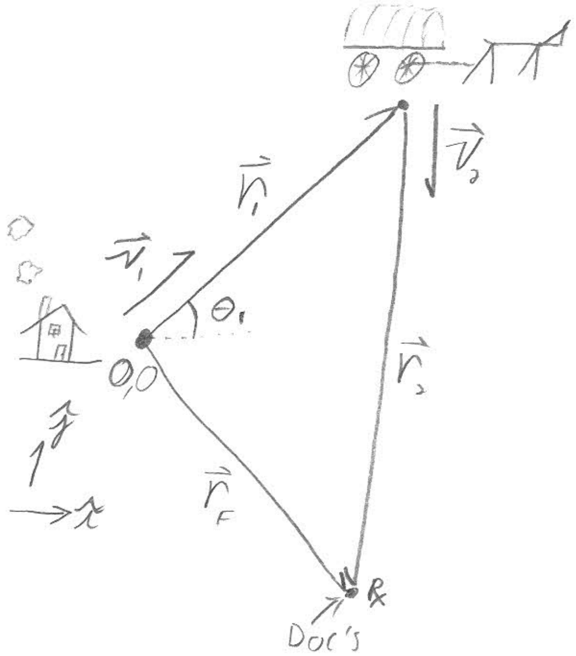
$$\boxed{v_c = 73 \cdot \frac{30}{200 + 40} = 9.1\ ft/s}$$

Kinematics

Ma and Pa Ingalls are going to have another baby! Ma says "It's time!" so Pa gets the team hitched up and they take off for Doc Weber's place. Doc's place is 15 miles south and 10 miles east of the homestead. But In his excitement, Pa drives northeast at 18 miles per hour for 1.5 hours. The baby will arrive 4 hours after Ma said "It's time!"



How fast and in what direction does Pa have to drive to make it to Doc's before the baby arrives? Find the magnitude and the direction.



Given

$\vec{r}_F = (10\hat{\imath} - 15\hat{j})\ mi$

$|\vec{v}_1| = 18\ mi/hr$

$\theta_1 = 45^\circ$

$t_1 = 1.5\ hr$

$t_T = 4\ hr$

This is a vector displacement problem with constant velocities.

Vector sum: $\vec{r}_1 + \vec{r}_2 = \vec{r}_F$ (1)

And constant velocity:

$$\vec{r} = \vec{v}t + \vec{r}_0, \quad \vec{r}_0 = (0\hat{\imath} + 0\hat{j})$$

Two legs of the trip: $\vec{r}_1 = \vec{v}_1 t_1$ (2)

$\vec{r}_2 = \vec{v}_2 t_2$ (3)

continued ↓

Ma and Pa continued

combine equations ①, ②, and ③

$$\vec{v}_1 t_1 + \vec{v}_2 t_2 = \vec{r}_F$$

But... we don't know t_2. We know $t_1 + t_2 = t_T$

$$\Rightarrow \underline{t_2 = t_T - t_1}$$

$$\vec{v}_1 t_1 + \vec{v}_2 (t_T - t_1) = \vec{r}_F$$

Now solve for $\vec{v}_2$

$$\boxed{\vec{v}_2 = \frac{\vec{r}_F - \vec{v}_1 t_1}{(t_T - t_1)}}$$

Now we can solve for the components of $\vec{v}_2$

$$x: \ v_{2x} = \frac{r_{Fx} - v_{1x} t_1}{t_T - t_1} , \quad v_{1x} = |\vec{v}_1| \cos(\theta_1)$$

$$v_{2x} = \frac{r_{Fx} - |\vec{v}_1| \cos(\theta_1) t_1}{t_T - t_1} = \frac{10 - (18)\cos(45)(1.5)}{4 - 1.5} = \boxed{-3.6 \text{ mi/hr}}$$

$$y: \ v_{2y} = \frac{r_{Fy} - v_{1y} t_1}{t_T - t_1} , \quad v_{1y} = |\vec{v}_1| \sin(\theta_1)$$

$$v_{2y} = \frac{r_{Fy} - |\vec{v}_1| \sin(\theta_1) t_1}{t_T - t_1} = \frac{-15 - (18)\sin(\theta_1)(1.5)}{4 - 1.5} = \boxed{-14 \text{ mi/hr}}$$

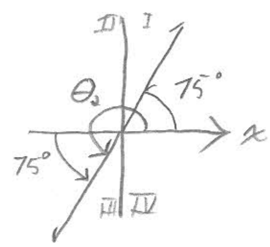
continued ↓

Ma and Pa continued

Finally, calculate magnitude and direction

$$|\vec{V}_2| = (-3.6^2 + -14^2)^{1/2} = \boxed{14.5\ mi/hr}$$

$$\theta_2 = \tan^{-1}\left(\frac{-14}{-3.6}\right)$$



My calculator gives: 75°

The real angle is in Quadrant 3 : $\theta_2 = 180 + 75 = \boxed{255°}$

Just a touch west of south