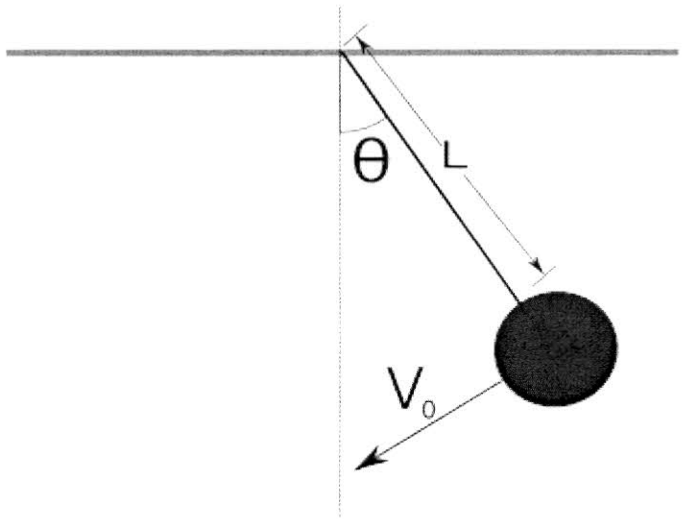
# Oscillation – Set 2

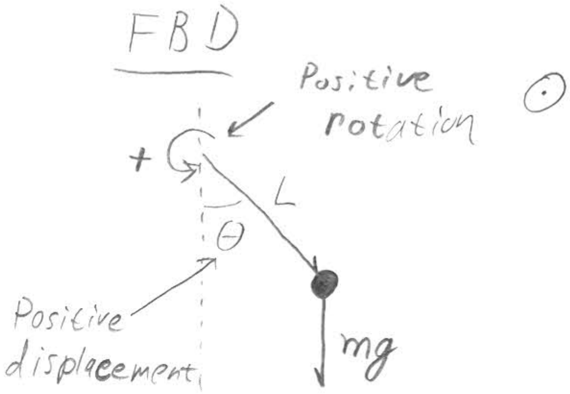
2

Below is a simple pendulum consisting of a massless rod of length $L$ with a point mass if mass $m$ attached to the end.

a) Find the frequency of small oscillations of the pendulum.

b) At t=0, the pendulum makes an angle $\theta_0$ with the vertical and the point mass has a velocity $V_0$. What is ~~the amplitude of the oscillator?~~ Phase angle?





$\vec{L} \times m\vec{g} \Rightarrow \otimes$ gives negative torque by rhr

NSL

$$\sum T = I\alpha,\ I = mL^2$$

$$-\cancel{m}g\cancel{L}\sin\theta = \cancel{m}L^{\cancel{2}}\frac{d^2\theta}{dt^2}$$

$$\Rightarrow \frac{d^2\theta}{dt^2} = -\frac{g}{L}\sin\theta,$$ Almost a SHO.

For small $\theta$, $\sin\theta \cong \theta$ (small angle approximation)

So; For small oscillations: $\boxed{\frac{d^2\theta}{dt^2} = -\frac{g}{L}\theta}$ SHO!

$$\boxed{\omega = \sqrt{\frac{g}{L}}}$$

continued ↓

Oscillation Set 2, P2 continued

b) $\theta(0) = \theta_0$, $v(0) = v_0 = \omega_0 L$

!!Be very careful!!

This angular velocity is not the same $\omega$ as the oscillator frequency, $\sqrt{g/L}$. Let's call $\boxed{\sqrt{g/L} = \omega_F}$

Angular versions of SHO general solution

$$\theta(t) = A\cos(\omega_F t + \phi)$$
$$\omega(t) = -\omega_F A \sin(\omega_F t + \phi)$$

$$\theta_0 = A\cos(\phi)$$
$$\frac{v_0}{L} = -\omega_F A \sin(\phi)$$
$$\Rightarrow \frac{v_0}{L\theta_0} = \frac{-\omega_F \cancel{A} \sin(\phi)}{\cancel{A}\cos(\phi)}$$

$$\Rightarrow \boxed{\tan(\phi) = -\frac{v_0}{\omega_F L \theta_0}}$$

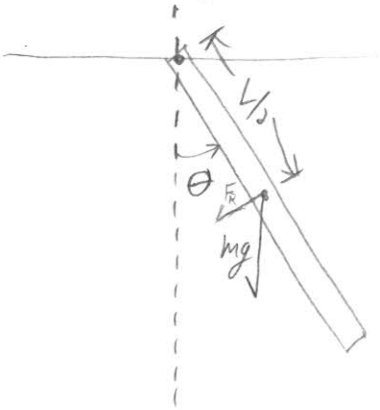
$$\Rightarrow \tan(\phi) = -\left(\frac{L}{g}\right)^{1/2} \frac{v_0}{L\theta_0}$$ plug in $\omega_F$

$$\Rightarrow \boxed{\tan(\phi) = -\frac{v_0}{\theta_0 \sqrt{gL}}}$$

**4**

A meter stick with a mass M is suspended from one end and allowed to swing like a pendulum.

a) What is its **period** of small oscillations?
b) What length *L* does a simple pendulum (a point mass attached to a massless rod) need in order to have the same period?



$I = \frac{1}{3}ML^2, \quad F = -mg\sin\theta, \quad L = 1$ meter

$\sum \tau = I\alpha$

$-(mg\sin\theta)\frac{L}{2} = I\alpha$

$-mgL\sin\theta = 2I\alpha$

$-\cancel{m}g\cancel{L}\sin\theta = 2\cdot\frac{1}{3}\cancel{M}L^{\cancel{2}}\frac{d^2\theta}{dt^2}$

$$\boxed{\frac{d^2\theta}{dt^2} = -\frac{3}{2}\frac{g}{L}\sin\theta}$$

a) For small oscillations, $\sin\theta \cong \theta$

$\Rightarrow \frac{d^2\theta}{dt^2} = -\frac{3}{2}\frac{g}{L}\theta \Rightarrow \omega = \left(\frac{3}{2}\frac{g}{L}\right)^{1/2}$

and $T = \frac{2\pi}{\omega}$, $L = 1$ m, $T = 2\pi\left(\frac{2}{3g}\right)^{1/2} = \boxed{1.6\ s}$

b) From problem 2, the frequency of a simple pendulum is

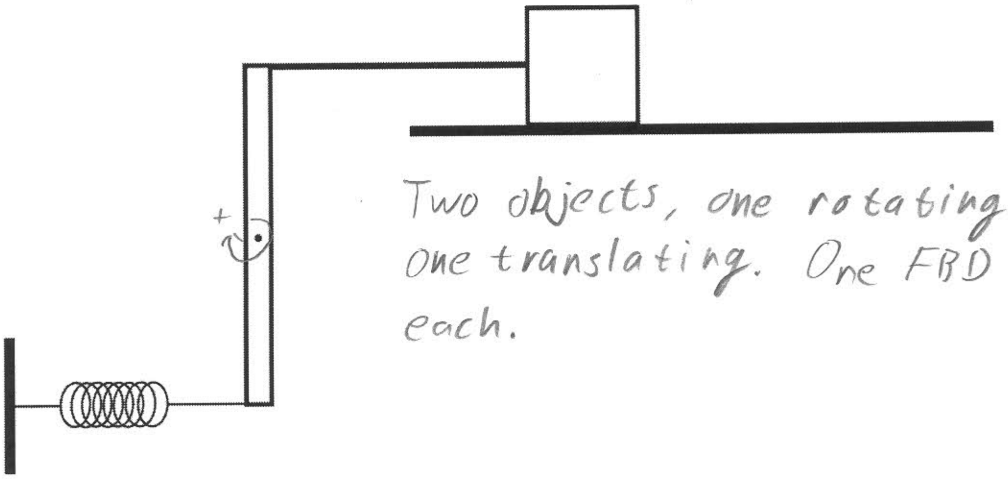
$\omega = \sqrt{g/L_s}$ so $T = \frac{2\pi}{\omega} \Rightarrow T = 2\pi\sqrt{L_s/g}$

We want $T_m = T_s$ so: $\cancel{2\pi}\cancel{\sqrt{L_s/g}} = \cancel{2\pi}\left(\frac{2L}{3g}\right) \Rightarrow \frac{L_s}{\cancel{g}} = \frac{2}{3}\frac{1}{\cancel{g}}$

meter stick ↗ simple ↑

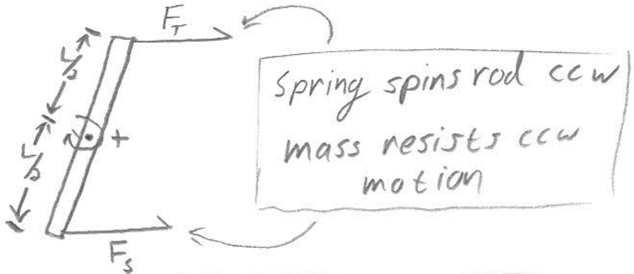
$$\boxed{L_s = \frac{2}{3}L}$$

A block of mass $M$ resting on a frictionless surface is attached to a stiff rod of negligible mass. The other end of the rod is attached to the top of a thin bar of length $l$ mass $M$ that is allowed to rotate about its center. The bottom of a bar is attached to a light spring of spring constant k. The spring is relaxed when the bar is vertical. Find the frequency of small oscillations.

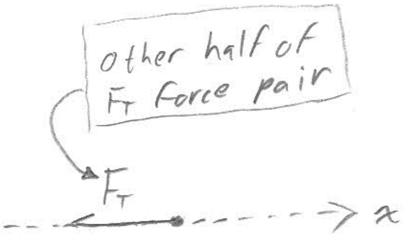


Two objects, one rotating and one translating. One FBD for each.

FBD - Displace rod clockwise.



Spring spins rod ccw
mass resists ccw motion



other half of $F_T$ force pair

$$\sum T = I\alpha \Rightarrow \boxed{-\frac{L}{2}F_s + \frac{L}{2}F_T = I\alpha} \; ①$$

$$\boxed{-F_T = ma} \; ②$$

Subst: ② → ① $-\frac{L}{2}kx - \frac{L}{2}ma = I\alpha$

hmm... rot or tran. Let's go tran...

$\frac{L}{2}\alpha = a \Rightarrow \alpha = \frac{2a}{L}$ , $I = \frac{1}{12}mL^2$

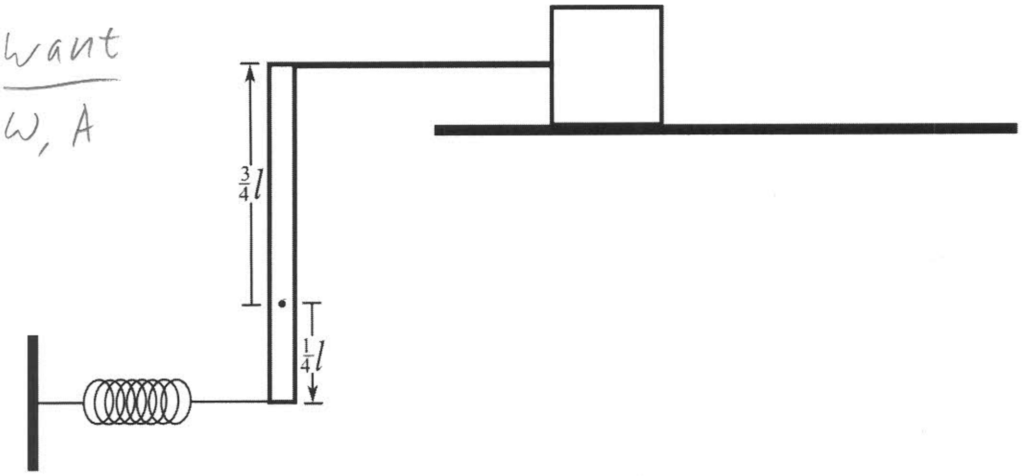
$$\Rightarrow -\frac{\cancel{L}}{\cancel{2}}kx = \frac{\cancel{L}}{\cancel{2}}ma + \frac{1}{\cancel{12}_{6}}m\cancel{L}^{\cancel{2}}\frac{2a}{\cancel{L}} \Rightarrow -kx = ma + \frac{1}{3}ma$$

$$\Rightarrow -kx = \frac{4}{3}ma \Rightarrow \frac{d^2x}{dt^2} = -\boxed{\frac{3}{4}\frac{k}{m}}x \overset{\omega^2}{\longleftarrow}$$

3) (36 points) A block of mass *M* resting on a frictionless surface is attached to a stiff rod of negligible mass. The other end of the rod is attached to the top of a thin bar of length *l* and mass *M* that is allowed to rotate about the point shown in the figure below. The bottom of the bar is attached to a light spring with spring constant k. The spring is relaxed when the bar is vertical.

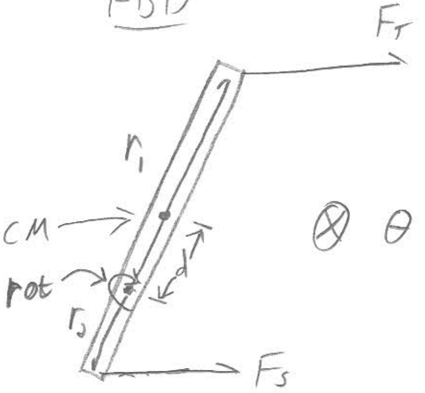
Given
$M$
$l$
$k$
$r_1$
$r_2$

Want
$\omega, A$



a) Find the frequency of small oscillations.
b) As the spring passes equilibrium, the velocity of the block is $V_0$. What is the amplitude of oscillations.

FBD



$\otimes\ \theta$

$T = I\alpha$

$F_T r_1 - k x r_2 = I\alpha$

$\boxed{-M r_1 a - k x r_2 = I\alpha}$

$F = ma$

$-F_T = Ma$

$r_1 = \frac{3}{4}l$
$r_2 = \frac{1}{4}l$
$d = \frac{l}{2} - r_2$
$d = \frac{1}{4}l$

Let's go to rotation. Then: $a = r_1\alpha$ and $x = r_2\theta$

$\Rightarrow -M r_1 r_1 \alpha - k r_2 \theta r_2 = I\alpha$

$-M r_1^2 \alpha - k r_2^2 \theta = I\alpha$

$\boxed{I = I_{cm} + Md^2}$

$I_{cm} = \frac{1}{12} M l^2$

continued ↓

Oscillation Sec 3 P4 continued

$$-Mr_1^2\alpha - kr_2^2\theta = (I_{cm} + Md^2)\alpha$$

* Put it all together

$$-M\,{}^9\!/_{16}\,l^2\alpha - k\frac{1}{16}l^2\theta = \left(\frac{1}{12}Ml^2 + M\frac{1}{16}l^2\right)\alpha$$

$$-\frac{1}{16}k\cancel{l^2}\theta = \left(\frac{9}{16} + \frac{1}{16} + \frac{1}{12}\right)M\cancel{l^2}\alpha$$

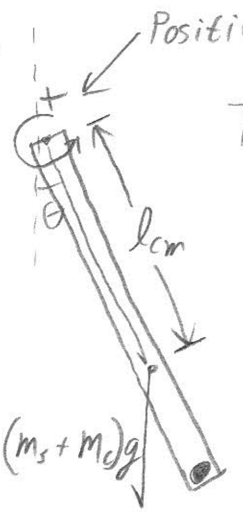
$$\Rightarrow \alpha = -\frac{3}{34}\frac{k}{M}\theta$$

$$\Rightarrow \boxed{\omega = \left[\frac{3}{34}\frac{k}{M}\right]^{1/2}}$$

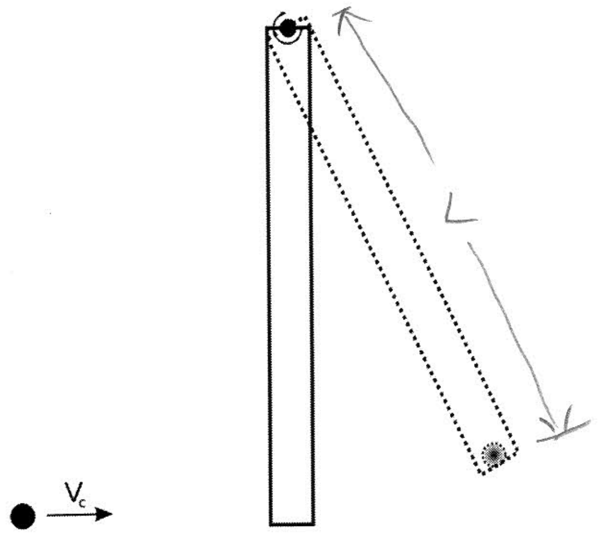
# Oscillation – Set 2

A 1 kg meter stick is hung from its and allowed to pivot. A small wad of clay with a mass of 0.25 kg with a velocity $V_c = 2$ m/s impacts the bottom of the meter stick. Assuming that the resulting oscillations are small:

a) Find the angular frequency of the resulting pendulum.
b) Find the phase angle of the resulting oscillator.
~~c) Find the amplitude of the oscillations.~~



Then, $\vec{r}\times\vec{F}$ is neg.



so: $-(m_s+m_c)g\,l_{cm}\sin\theta = I\alpha$

$$\Rightarrow \boxed{\alpha = \frac{-(m_s+m_c)l_{cm}g}{I}\sin\theta} \quad ①$$

But, for small θ, $\boxed{\sin\theta \cong \theta}$; ②

$$I = I_s + I_c \Rightarrow \boxed{I = \tfrac{1}{3}m_sL^2 + m_cL^2} \quad ③$$

$$\boxed{l_{cm} = \frac{m_s\frac{L}{2} + m_cL}{m_s + m_c}} \quad ④$$

$$\Rightarrow \alpha = \frac{-\cancel{(m_s+m_c)}\,g}{(\frac{1}{3}m_s+m_c)L^{\cancel{2}}}\cdot\frac{(\frac{1}{2}m_s+m_c)\cancel{L}}{\cancel{(m_s+m_c)}}\,\theta$$

$$\Rightarrow \alpha = -\overset{\omega^2}{\boxed{\frac{\frac{1}{2}m_s+m_c}{\frac{1}{3}m_s+m_c}\,\frac{g}{L}}}\,\theta \qquad \boxed{\omega = \left[\frac{\frac{1}{2}m_s+m_c}{\frac{1}{3}m_s+m_c}\,\frac{g}{L}\right]^{1/2}}$$

continued
↓

Oscillation Set 2, P6 - continued.

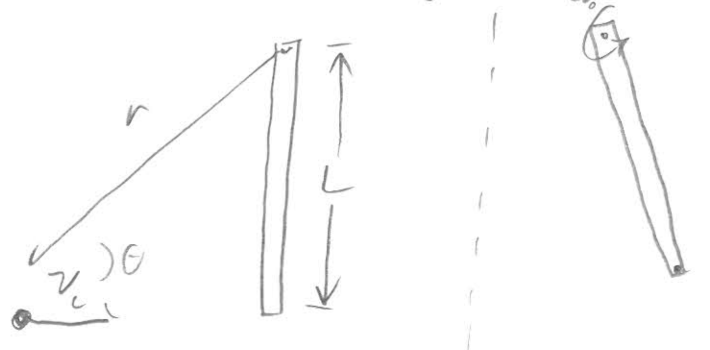
b) $\theta(t) = \theta_{max} \cos(\omega t + \phi)$

$\omega(t) = -\omega\, \theta_{max} \sin(\omega t + \phi)$

NOT the oscillator frequency!

collision at $t = 0$: $\theta(0) = 0$

Conserve angular momentum to find $\omega(0) = \boxed{\omega_0}$



$r m v_c \sin\theta = I\omega_0$

$(\cancel{m_s} + m_c) v_c \cancel{L} = (\tfrac{1}{3} m_s + m_c) L^{\cancel{2}} \omega_0 \Rightarrow \boxed{\omega_0 = \frac{\cancel{m_s} + m_c}{\frac{1}{3} m_s + m_c} \frac{v_c}{L}}$

Find phase angle:

so $\omega_0$ is pos.

$\theta_0 = \theta_{max} \cos(\cancel{\omega t}^{\,0} + \phi) \Rightarrow 0 = \cos(\phi) \Rightarrow \boxed{\phi = \tfrac{\pi}{2}, \textcircled{\tfrac{3\pi}{2}}}$

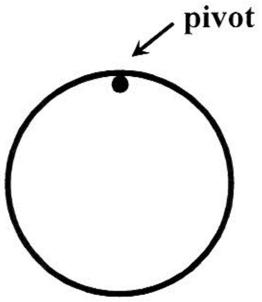
c) $\omega_0 = -\omega\, \theta_{max} \sin(\phi)$, $\phi = \tfrac{3\pi}{2}$ so $\sin(\phi) = -1$

$\Rightarrow \omega_0 = +\omega\, \theta_{max} \Rightarrow \theta_{max} = \frac{\omega_0}{\omega} \Rightarrow \theta_{max} = \frac{m_s + m_c}{\frac{1}{3} m_s + m_c} \frac{v_c}{L} \left[\frac{\frac{1}{3} m_s + m_c}{\frac{1}{2} m_s + m_c} \frac{L}{g}\right]^{1/2}$

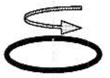
$$\boxed{\theta_{max} = (m_s + m_c)\, v_c/g\, \frac{1}{\left[(\tfrac{1}{3} m_s + m_c)(\tfrac{1}{2} m_s + m_c) L\right]^{1/2}}}$$

# SAMPLE TEST 6
## PHYS 111, FALL 2010, SECTION 1

4. A circular hula-hoop rests on a peg that acts as a pivot point. The hoop is given a small kick so that it oscillates back and forth with small angular displacements. The hoop's mass is



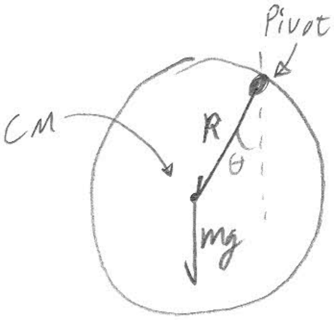
M = 0.80 kg, and its radius is R = 0.6 m.



$I_{cm} = MR^2$

Use Newton's Second Law to find the angular frequency of small oscillations.

The hula-hoop is a physical pendulum.



$$T = I\alpha$$

$$-mgR\sin\theta = I\alpha$$

$$\Rightarrow \alpha = -\frac{mgR}{I}\sin\theta,$$

For small $\theta$, $\sin\theta \cong \theta$

$$\Rightarrow \alpha = -\frac{mgR}{I}\theta \Rightarrow \omega = \left(\frac{mgR}{I}\right)^{1/2}$$

$$I = I_{cm} + Md^2 \Rightarrow I = MR^2 + MR^2 = 2MR^2$$

$$\Rightarrow \boxed{\omega = \left[\frac{\cancel{M}g\cancel{R}}{2\cancel{M}R^{\cancel{2}}}\right]^{1/2} = \left[\frac{g}{2R}\right]^{1/2}}$$

$$\omega = \left[\frac{9.8}{2(0.6)}\right]^{1/2} = 2.9 \text{ rad/s}$$