By writing my name above, I affirm that this test represents my work only, without aid from outside sources. In all aspects of this course I perform with honor and integrity.

SHOW YOUR WORK ON ALL OF THE PROBLEMS. YOUR APPROACH TO THE PROBLEM IS AS IMPORTANT AS, IF NOT MORE IMPORTANT THAN, YOUR ANSWER. DRAW **CLEAR AND NEAT PICTURES** SHOWING COORDINATE SYSTEMS AND ALL OF THE RELEVANT PROBLEM VARIABLES. ALSO, **EXPLICITLY** SHOW THE **BASIC EQUATIONS** YOU ARE USING. BE NEAT AND THOROUGH. THE EASIER IT IS FOR ME TO UNDERSTAND WHAT YOU ARE DOING, THE BETTER YOUR GRADE WILL BE.

Moment of Inertia, discrete definition

Moment of Inertia, integral definition

Parallel Axis Theorem

Superposition

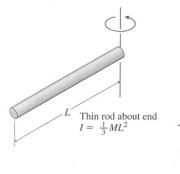
TABLE 10.2 Rotational Inertias

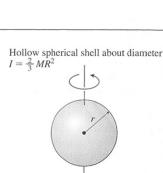




Thin ring or hollow cylinder about its axis  $I = MR^2$ 

> Disk or solid cylinder about its axis  $I = \frac{1}{2}MR^2$



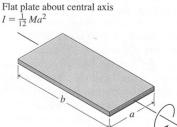


Solid sphere about diameter

 $I = \frac{2}{5}MR^2$ 

Flat plate about perpendicular axis

 $I = \frac{1}{12}M(a^2 + b^2)$ 



 $I = I_{cm} + Md^2$ 

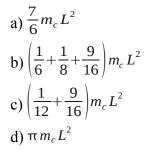
 $I=\int r^2 dm$ 

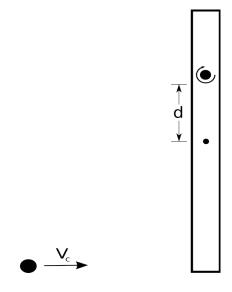
 $I = \sum m_i r_i^2$ 

 $I_{Total} = \sum I_i$ 

- 1) Derivations
- a) (10pts) Starting with the definition of linear Kinetic energy ( $K = \frac{1}{2}mV^2$ ), show that rotational kinetic energy of a rigid body is  $K = \frac{1}{2}I\omega^2$  where  $I = \int r^2 dm$ .
- b) (10pts) Starting with the definition of angular momentum ( $L=m(\vec{r}\times\vec{V})$ ), show that the angular momentum of a rigid body is  $L=I\omega$  where  $I=\int r^2 dm$ .

- 2) Multiple Choice, 4 points each.
- 2.1) A wad of clay with mass  $m_c$  is thrown at a **thin rod** whose length is L and whose mass is  $2m_c$ . The rod is allowed to rotate about a pivot a distance d = L/4 from its center as in the picture below. What is the moment of inertia of the clay, stick combination after the impact?





- 2.2) A disk, a hoop, a solid sphere, and a hollow sphere, all with the same mass and radius, are having a race down an incline plane. Rank them in the order that they will arrive at the bottom of the ramp, 1 = winner, 4 = loser.
  - \_\_\_\_ Disk

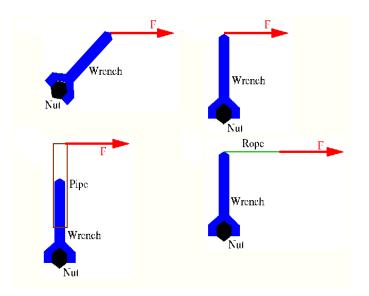
\_\_\_\_ Ноор

\_\_\_\_\_ Solid Sphere

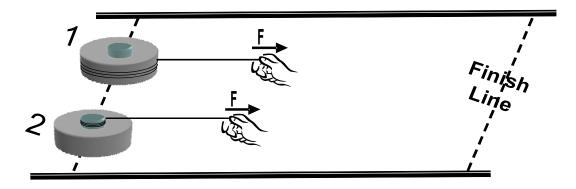
- 2.3) Some children are riding on the outside edge of a merry-go-round. All of the children simultaneously move towards the center. Ignore friction in the rotation of the merry-go-round. When they move:
  - a) the moment of inertia of the system stays constant.
  - b) the angular momentum of the system stays constant.
  - c) the angular velocity of the system stays constant.
  - d) the merry-go-round slows down.

\_\_\_\_ Hollow Sphere

2.4) You are trying to turn a nut with a wrench. The same force is applied in each picture. Rank the pictures by torque, 1 = smallest. If any of the torques are the same, give them the same ranking.

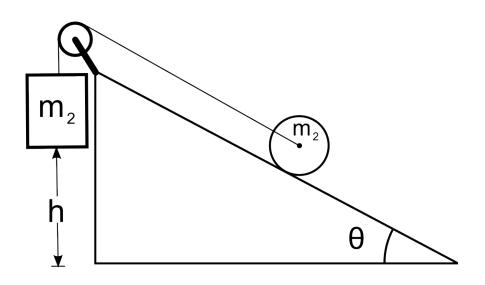


- 2.5) Strings are wound around two identical pucks: one is around its outer rim; the other is around its axle. You pull both pucks from rest by using the same force F. Both pucks start to move on a frictionless surface. Which puck arrives at the finish line first?
  - A) Puck 1
  - B) Puck 2
  - C) They arrive at the same time
  - D) There is not enough information to tell.



3) Mass  $m_2$  is attached to a string that passes over a massless pulley. The other end of the string is attached to the central axle of a cylinder of mass  $m_1$  and radius R. Assuming that  $m_2 \gg m_1$ , the cylinder rolls without slipping up a slope that makes an angle  $\theta$  with the horizontal.

Assuming that the system starts from rest, use **Torque and Kinematics**, to find an expression for the velocity, v, of  $m_2$  after it falls a distance h.

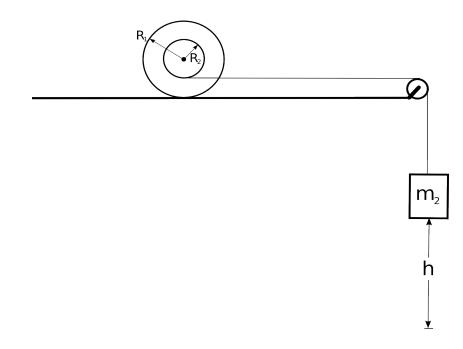


Extra Space

4) Mass  $m_2$  is attached to a string that passes over a massless pulley. The other end of the string is attached to the inner radius of a spool with moment of inertia I, inner radius  $R_1$  and outer radius  $R_2$ .

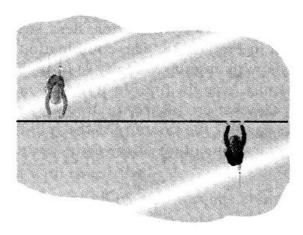
Assuming that the system starts from rest and that the spool rolls without slipping, use **Conservation of Energy**, to find an expression for the velocity of  $m_2$  after it falls a distance h.

*HINT:* The hanging mass and the spool will not have the same velocity so you must write an expression that relates them. It is perhaps easiest to begin by writing an expression that relates h, the distance the mass drops, to d, the distance that the spool moves.



Extra Space

- 5. Two skaters, each with a mass of 50 kg, approach each other along parallel paths separated by 3.0 m. They have equal and opposite velocities of 1.4 m/s. The first skater is holding one end of a long pole with negligible mass. As the skaters pass, the second skater grabs the other end of the pole. Assume that the ice is completely frictionless.
  - a) What is the moment of inertia about the center of mass of the resulting skater-pole system?
  - b) What is the resulting angular velocity of the skater-pole system?



Extra Space