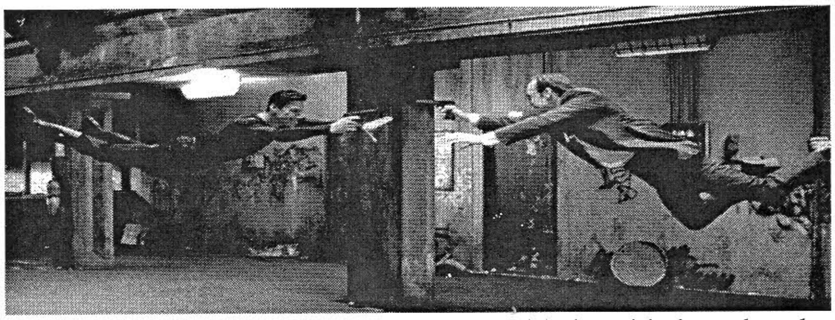
Systems of Particles – Set 4



Neo and Agent Smith are flying towards each other. They collide in mid air and grab onto each other (they stick together).

a) Assume that momentum is conserved in the Matrix and find an expression relating their initial velocities to their final velocity.

m_N v_N v_S m_S $(m_N + m_S)$ v_F

$$m_N v_N - m_S v_S = (m_N + m_S) v_F$$

$$\Rightarrow \boxed{v_F = \frac{m_N v_N - m_S v_S}{m_N + m_S}}$$

b) Let $M_N = 70$ kg, $V_{NI} = 50$ m/s, $M_S = 100$ kg, and $V_{SI} = 35$ m/s. Put these numbers into your expression and solve for their final velocity.

$$v_F = \frac{(70\,kg)(50\,m/s) - (100\,kg)(35\,m/s)}{(70\,kg + 100\,kg)} = \underline{0}$$

c) Calculate the pre-collision and post-collision kinetic energy of the system. Does this system conserve kinetic energy through the collision?

Pre

$K_I = K_N + K_S$

$K_I = \frac{1}{2} m_N v_N^2 + \frac{1}{2} m_S v_S^2$

$=$ a positive number

Post

$K_F = \frac{1}{2}(m_N + m_S) v_F^2$

$\underline{K_F = 0}$

$\boxed{K \text{ is } \underline{\underline{\text{not}}} \text{ conserved}}$

Systems of Particles – Set 3

A 4000 kg railroad car collides and sticks to a chain of three other 4000 kg cars initially sitting at rest on a rough track. The four cars travel together down the rough track for 1.5 m before they stop. Assuming $\mu_k = 0.10$, what is the velocity of the first car at impact?

Answer these important questions before "solving" this problem:

Does the train car conserve momentum throughout the entire problem? Why not?

No. Friction is an external force. $\Rightarrow \Sigma F_{ext} \neq 0$

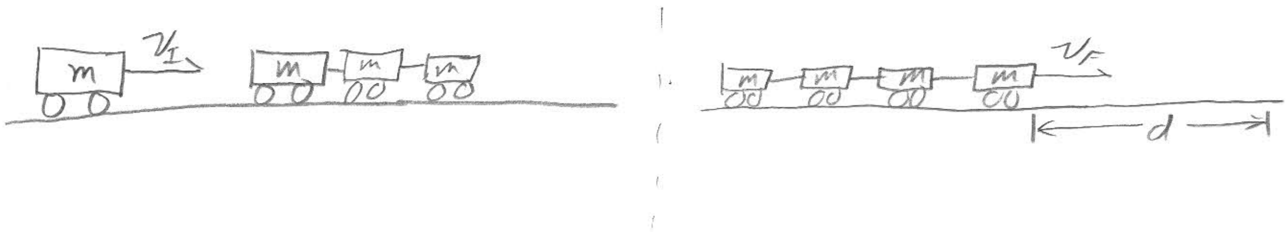
Is there a sub-problem where conservation of momentum can be applied? What is it?

Yes! The inelastic collision.

What other physics principal are you going to use to solve the problem?

Conservation of Energy.

Draw a picture (or pictures) showing the action.



Do the math and solve the problem.

① Collide

$\vec{P}_I = \vec{P}_F$

$mV_I = 4mV_F$

$V_I = 4V_F$

② conserve Energy

$K_I = \frac{1}{2}4mV_F^2 \quad K_F = 0, \quad W_F = -\mu_k 4mgd$

$\frac{1}{2}4mV_F^2 = \mu_k 4mgd$

$V_F = (2\mu_k gd)^{1/2}$

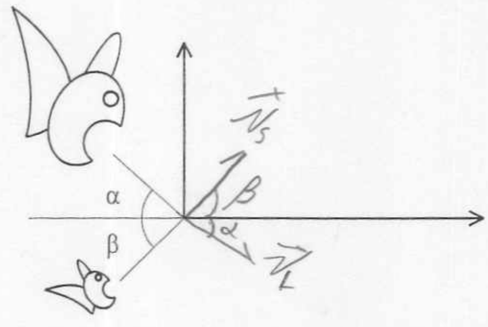
$$\boxed{V_I = (32\mu_k gd)^{1/2}}$$

$$V_I = (32(0.1)(9.8)(1.5))^{1/2} = \boxed{6.9\,m}$$

1. A large fish will soon make a dish of a smaller fish. What is the velocity of the large fish and his dinner immediately after he eats? Give both the magnitude and direction of the final velocity with respect to the x-axis.

$m_{\text{large fish}} = 4.0$ kg
$v_{o\ \text{large fish}} = 1.0$ m/s
$\alpha_{\text{large fish}} = 25.0°$

$m_{\text{small fish}} = 0.20$ kg
$v_{o\ \text{small fish}} = 5.0$ m/s
$\beta_{\text{small fish}} = 50.0°$



Conserve momentum in both axis

$$x: m_L v_L \cos\alpha + m_s v_s \cos\beta = (m_L + m_s) v_F \cos\theta$$

$$y: -m_L v_L \sin\alpha + m_s v_s \sin\beta = (m_L + m_s) v_F \sin\theta$$

Divide y by x to eliminate v_F

$$\frac{-m_L v_L \sin\alpha + m_s v_s \sin\beta}{m_L v_L \cos\alpha + m_s v_s \cos\beta} = \frac{\cancel{(m_L+m_s)}\cancel{v_F}\sin\theta}{\cancel{(m_L+m_s)}\cancel{v_F}\cos\theta}$$

$$\tan\theta = \frac{-m_L v_L \sin\alpha + m_s v_s \sin\beta}{m_L v_L \cos\alpha + m_s v_s \cos\beta} \Rightarrow$$

$$\theta = \tan^{-1}\left[\frac{-(4.0)(1.0)\sin(25) + (0.2)(5)\sin(50)}{(4.0)(1.0)\cos(25) + (0.2)(5)\cos(50)}\right] = \boxed{-12°}$$

Plug back into x (or y) to get v_F

$$v_F = \frac{m_L v_L \cos\alpha + m_s v_s \cos\beta}{(m_L + m_s)\cos\theta} = \frac{(4)(1)\cos 25 + (0.2)(5)\cos(50)}{(4 + 0.2)\cos(-12)} = \boxed{1.0\ \text{m/s}}$$

Systems of Particles – Set 3

A HUGE truck, T and a Prius, P, move towards each other, collide and stick. Let $\mathbf{F_P}$ be the force experienced by the Prius and let $\mathbf{F_T}$ be the force experienced by truck.

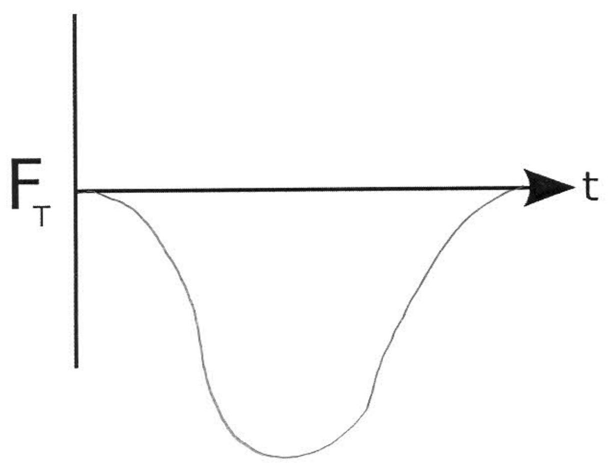
1. $\mathbf{F_P}$ is _____ $\mathbf{F_B}$.
   a) Greater than
   b) The same as
   c) Less than

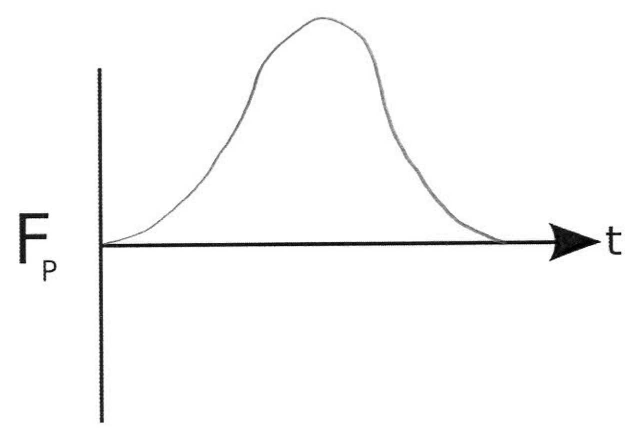
Newton's 3rd Law

2. The amount of time that $\mathbf{F_P}$ is applied is _____ the time that $\mathbf{F_T}$ is applied.
   a) Greater than
   b) The same as
   c) Less than

It's a contact force...
One can't lose contact before the other.

3. Sketch a graph showing a plausible $\mathbf{F_P}$ as a function of time and another graph showing $\mathbf{F_T}$ function of time. Be sure to consider the *sign* of each force.





Forces are equal and opposite.

5. Compare the impulses on the two. What's the same? What's different? $\vec{J} = \int \vec{F} dt = \Delta \vec{p}$

Because the force and the time are the same, magnitude the impulse must also have the same magnitude.

The forces are oppisitely directed, so the implse is also.

4. The *magnitude* of the change in momentum of the Prius is ____ the *magnitude* of the change in momentum of the truck.

a) Greater than
b) The same as
c) Less than

If |implse| is the same, |momentum| is the same:

$|\vec{J}| = |\Delta \vec{p}|$

5. The magnitude of the acceleration of the Prius is _____ the magnitude of the acceleration of the truck.

a) Greater than
b) The same as
c) Less than

The force is the same magnitude

and $\vec{F} = m\vec{a}$.

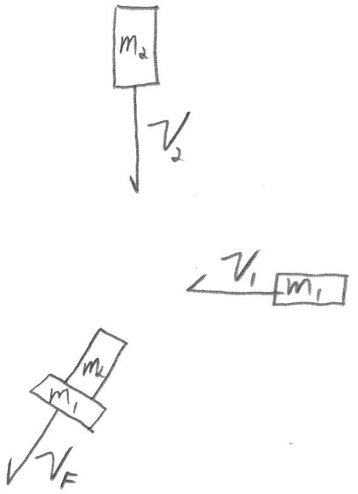
So, if $m\downarrow$, $a\uparrow$

Systems of Particles – Set 3

You are driving West along Summit Ave, lawfully doing the speed limit (50 km/hr) in your new car which (as you've read in the owners manual) has a mass of 1500 kg. Sleepy McSnoozer is driving South along Cleveland in his 1965 Ford pickup truck loaded with bags of cement. His truck (plus cement) weighs 2300 kg. Sleepy runs the red light and smashes into your car. The cars fuse together and skid to a stop.

Certain that Sleepy was speeding, you measure the skid mark and find that the length of the skid is L = 18 m. You look up the rubber/asphalt coefficient of friction and find that it is $\mu_k = 0.6$.

What was Sleepy's velocity? Was he speeding? The speed limit is 50 km/hr.



$\mu_K = 0.6$

$L = 18$

$m_1 = 1500\,kg$

$m_2 = 2300\,kg$

$V_1 = 50\,\frac{km}{hr} \cdot 1\times10^3\,\frac{m}{km} \cdot \frac{1}{3600} \cdot \frac{hr}{s} = 13.9\,m/s$

Two parts, collision and skid. Conserve momentum for collision, Conserve energy to do skid.

<u>Collision</u>

$P_I = P_F$

$x: \; m_1 V_1 = (m_1 + m_2) V_{Fx}$

$y: \; m_2 V_2 = (m_1 + m_2) V_{Fy}$

$\boxed{(1)\; V_{Fx} = \frac{m_1}{(m_1+m_2)} V_1, \quad (2)\; V_{Fy} = \frac{m_2}{(m_1+m_2)} V_2}$

<u>Skid</u>

$U_I = 0 \qquad U_F = 0$

$K_I = \frac{1}{2}(m_1+m_2)|\vec{V}_F|^2 \qquad K_F = 0$

$W_f = -\mu_K (m_1+m_2) g L$

$\frac{1}{2}\cancel{(m_1+m_2)}|\vec{V}_F|^2 = \mu_K \cancel{(m_1+m_2)} g L$

$\boxed{|\vec{V}_F|^2 = 2\mu_K g L \; (3)}$

continued ↓

Systems of particles Set 3, P3 continued

Now: $|\vec{V}_F|^2$ is related to V_{Fx} and V_{Fy} by pythagoras.

$$④ \; |\vec{V}_F|^2 = V_{Fx}^2 + V_{Fy}^2$$

Plugging ④ → ③:

$$⑤ \; V_{Fx}^2 + V_{Fy}^2 = 2\mu_k g L$$

and plugging ① and ② → ⑤

$$\frac{m_1^2}{(m_1+m_2)^2} V_1^2 + \frac{m_2^2}{(m_1+m_2)^2} V_2^2 = 2\mu_k g L$$

and solve for V_1:

$$\frac{m_1^2 V_1^2 + m_2^2 V_2^2}{(m_1+m_2)^2} = 2\mu_k g L$$

$$m_1^2 V_1^2 + m_2 V_2^2 = 2\mu_k g L (m_1+m_2)^2$$

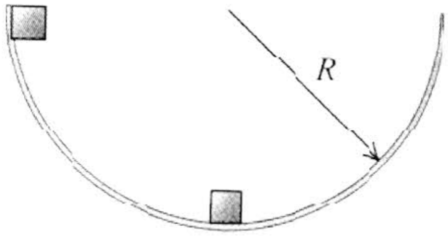
$$\Rightarrow \boxed{V_2 = \left[2\mu_k g L (m_1+m_2)^2 - m_1^2 V_1^2\right]^{1/2} \frac{1}{m_2}}$$

$$V_2 = \left[(2)(0.6)(9.8)(18)(1500+2300)^2 - (1500 \cdot 13.9)^2\right]^{1/2} \frac{1}{2300}$$

$$V_2 = 22.3\,\text{m/s} \cdot 1\times10^{-3}\,\frac{\text{km}}{\text{m}} \cdot 3600\,\text{s/hr} = 80\,\text{km/hr}$$

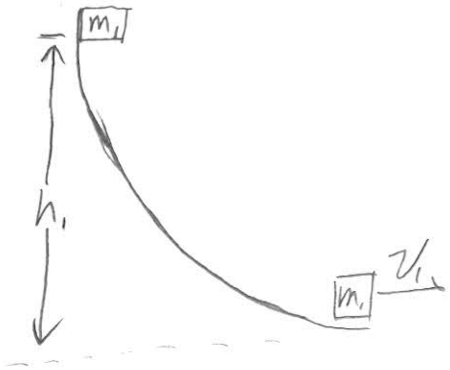
Speeder!

Two masses, m_1 and m_2, are released from rest in a frictionless hemispherical bowl of radius R from the positions shown in the figure. The upper mass collides with and sticks to the lower mass and the two slide up the other side together.



Derive an expression for their final height of the combined masses.

① Conserve Energy



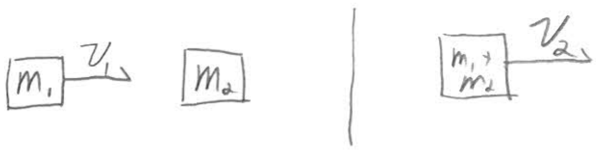
$U_I = m_1 g h_1 \qquad U_F = 0$

$K_I = 0 \qquad K_F = \frac{1}{2} m_1 v_1^2$

$\cancel{m_1} g h_1 = \frac{1}{2} \cancel{m_1} v_1^2$

$\boxed{v_1 = \sqrt{2 g h_1}}$ ①

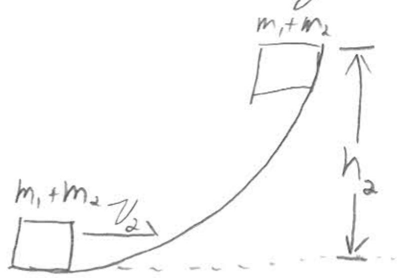
② Conserve momentum



$m_1 v_1 + m_2 (0) = (m_1 + m_2) v_2$

$\boxed{v_2 = \frac{m_1}{(m_1 + m_2)} v_1}$ ②

③ Conserve energy



$U_I = 0 \qquad U_F = (m_1 + m_2) g h_2$

$K_I = \frac{1}{2}(m_1 + m_2) v_2^2 \qquad K_F = 0$

$\Rightarrow \boxed{h_2 = \frac{v_2^2}{2g}}$ ③

Systems of Particles Set 3, P6 continued.

From 3: $h_2 = \dfrac{v_2^2}{2g}$

Plug in 2: $h_2 = \dfrac{1}{2g}\left[\dfrac{m_1}{m_1+m_2}\right]^2 v_1^2$

plug in 1: $h_2 = \dfrac{1}{\cancel{2g}}\left[\dfrac{m_1}{m_1+m_2}\right]^2 \cancel{2g}h_1$

$$\boxed{h_2 = \left[\dfrac{m_1}{m_1+m_2}\right]^2 h_1}$$