

Systems of Particles – Set 4

1

A block with a mass of $m_1 = 3.5$ kg is placed in front of a spring with spring constant $k = 2.4 \times 10^4$ N/m that has been compressed a distance d . After the spring is released, the block slides without friction to a hanging pendulum. The block then slides into a cup with mass $m_2 = 5$ kg. The cup is hanging from a string with length $l = 1.4$ m. After the collision, the resulting pendulum (cup and block together) swings up and makes a maximum angle $\theta = 26.5^\circ$.

What was the original spring compression d ?



Stage 1

Stage 2

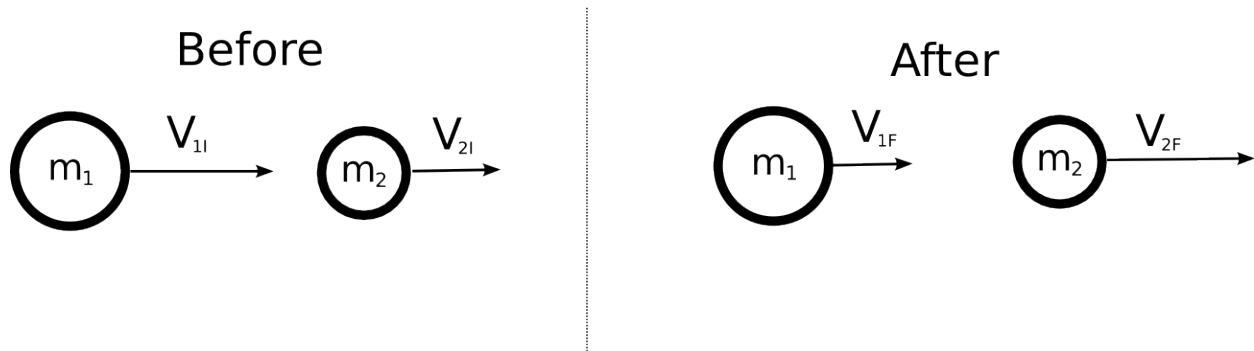
Stage 3

Assemble!

If Neo and Agent Smith conserved energy as well as momentum, they would bounce off of each other and the collision would be *elastic*. Let's derive a general expression relating the initial and final velocities in an elastic collision.

Step 1:

Starting with the picture below, write two equations, one for the *conservation of momentum* and one for the *conservation of kinetic energy*.



Using the two equations above, work out the algebra required to get to the following equation:

$$\frac{V_{1i}^2 - V_{1f}^2}{V_{1i} - V_{1f}} = \frac{V_{2f}^2 - V_{2i}^2}{V_{2f} - V_{2i}} \quad \text{This is waypoint 1}$$

Step 2:

Starting with waypoint 1:

$$\frac{V_{1I}^2 - V_{1F}^2}{V_{1I} - V_{1F}} = \frac{V_{2F}^2 - V_{2I}^2}{V_{2F} - V_{2I}}$$

Perform the required algebra to get to waypoint 2:

$$V_{1I} + V_{1F} = V_{2I} + V_{2F}$$

The following relationship may prove useful:

$$(a+b)(a-b) = (a^2 - b^2)$$

Step 3:

Combine the results of waypoint 2, $V_{1I} + V_{1F} = V_{2I} + V_{2F}$, with the equation for conservation of momentum from part 1 to arrive at waypoint 3:

$$(m_1 - m_2)V_{1I} + 2m_2 V_{2I} = (m_1 + m_2)V_{1F}$$

Step 4:

Solve waypoint 3 to get the general expression for V_{1F} :

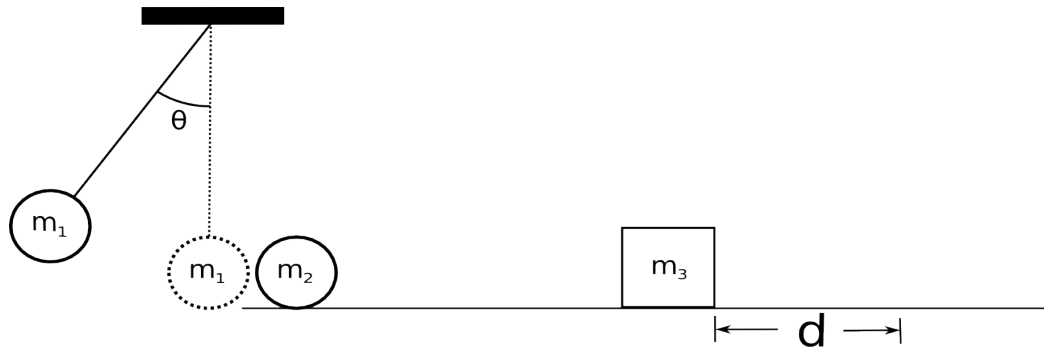
$$V_{1F} = \frac{(m_1 - m_2)}{(m_1 + m_2)} V_{1I} + \frac{2m_2}{m_1 + m_2} V_{2I}$$

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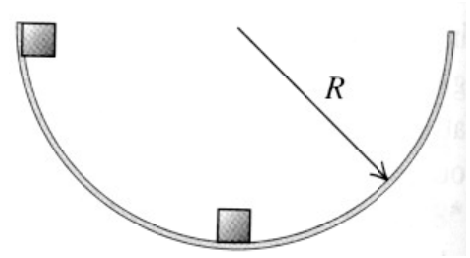
A mass $m_1 = 3$ kg is attached to a string of length $l = 4.0$ m to create a pendulum. The pendulum, initially making an angle θ with the vertical, is released from rest. At the bottom of its swing, it collides elastically with mass $m_2 = 5$ kg. Mass 2 rolls (no friction) and sticks to $m_3 = 5$ kg. The m_2, m_3 combination slides with $\mu_k = 0.3$ a distance $d = 0.2$ m before coming to rest.

What was the original value of θ ?



Extra Work Space

Two masses are released from rest in a frictionless hemispherical bowl of radius R from the positions shown in the figure. Derive an expression for their final height in the case of :



- a) An elastic collision
- b) An inelastic collision
- c) How much bigger than the second mass does the first mass have to be so that the second mass gets out of the bowl.