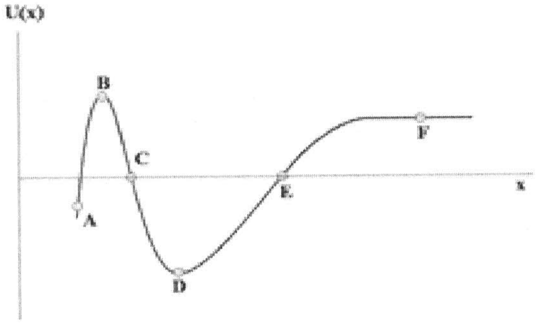
Energy Problems – Set 4

The figure below shows the plot of a potential-energy function for a particle moving along the x-axis.



a) At each point indicated, state whether the corresponding force F_x acting on the particle is positive, negative, or zero.

A: −	D: 0
B: 0	E: −
C: +	F: 0

b) At which point does the force have the greatest magnitude? Explain.

Point A. The slope of the potential curve is greatest there.

c) Identify all points corresponding to stable, unstable, and neutral equilibrium.

Unstable: B
Stable : D
Neutral : F

d) Assuming the particle starts at point A with a large positive velocity, identify the points where the particle's speed is a maximum, minimum, and constant. Explain. (Remember, in order for there to be a potential energy, the force must be conservative.)

Speed is minimum at point B. Maximum potential, minimum K
Speed is max at point D. Min U, max K
Speed is constant at point F. Potential is not changing.

Energy Problems – Set 4

The potential energy functions in a particular physics experiment are $U_1 = Ax^4$ and $U_2 = Ax^3 - Bx$, where A and B are constants.

a) Find the force, F_1, associated with U_1.

$$F = -\frac{dU}{dx}$$

$$F = -\frac{d}{dx}Ax^4 \Rightarrow \underline{F_1 = -4Ax^3}$$

b) Are there any points along the x-axis where the F_1 equals zero? If so, where?

$F = 0$ when $x = 0$

c) Find the force F_x associated with the potential-energy function $U = Ax^3 - Bx$, where A and B are constants.

$$F_2 = -\frac{d}{dx}(Ax^3 - Bx) \Rightarrow F_2 = -(3Ax^2 - B)$$

d) Are there any points along the x-axis where F_2 equals zero? If so, where?

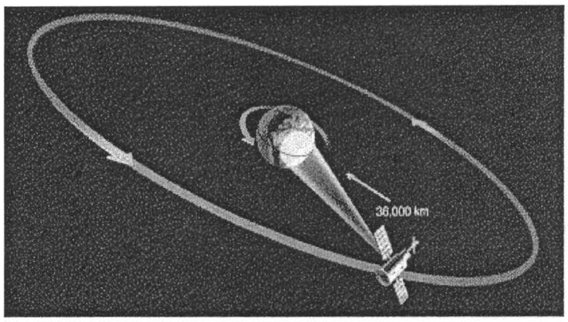
$F_2 = 0$ when: $-(3Ax^2 - B) = 0$

$$\Rightarrow 3Ax^2 = B$$

$$\Rightarrow x^2 = \frac{B}{3A}$$

$$\boxed{x = \pm\sqrt{\frac{B}{3A}}}$$

Energy Problems – Set 4



Satellites in geosynchronous orbit always remain above the same geographic spot on the Earth's surface, making them extremely handy for communications.
Using what you know about Newton's Universal Law of Gravity and the rotation of the Earth:

a) calculate the radius of the geosynchronous orbit.
b) calculate how much energy is required to put a 1000kg communications satellite into geosynchronous orbit.

The radius of the Earth is: $R_{Earth} = 6.37\text{x}10^{6}$ m.
The gravitaional constant: $G = 6.67\text{x}10^{-11}\ m^3/(kg\ s^2)$

a) Assume geosynchronous orbit is circular.
To stay above the same spot on the Earth, it must have a period of 24 hours.
To be going in a circle, it must have a centrally directed force providing an acceleration $\underline{a = r\omega^2}$

$$F = ma$$

$$\frac{G M_{\oplus} m}{r^2} = m r \omega^2 \Rightarrow r^3 = \frac{G M_{\oplus}}{\omega^2}, \quad \omega = \frac{2\pi}{P}$$

$$\Rightarrow r^3 = \frac{G M_{\oplus} P^2}{4\pi^2}$$

$$\boxed{r = \left[\frac{G M_{\oplus}}{4\pi^2} P^2\right]^{1/3}} = \left[\frac{(6.37\times10^{-11})\cdot(5.97\times10^{24})}{4\pi^2}\left[(24\ hrs)(3600\ s/hr)\right]^2\right]^{1/3}$$

$$= 4.1\times10^{7}\ m \cong \boxed{41{,}000\ km}$$

EP4,3

b) Energy on the surface is all potential (ignoring the small amount from Earth's rotation)

$$E_I = U_I = -\frac{GM_\oplus m}{R_\oplus}$$

In orbit, total energy is potential plus kinetic.

$$E_F = U_F + K_F$$

$$= -\frac{GM_\oplus m}{r} + \frac{1}{2}m\underline{v}^2 \leftarrow \text{What's this?}$$

$$\overset{F}{\frac{GM_\oplus \cancel{m}}{r^{\cancel{2}}}} \overset{= ma}{= \cancel{m}\frac{v^2}{\cancel{r}}} \Rightarrow \underline{v^2} = \frac{GM_\oplus}{r}$$

$$\underline{E_F} = -\frac{GM_\oplus m}{r} + \frac{1}{2}\frac{GM_\oplus m}{r} = \boxed{-\frac{1}{2}\frac{GM_\oplus m}{r}}$$

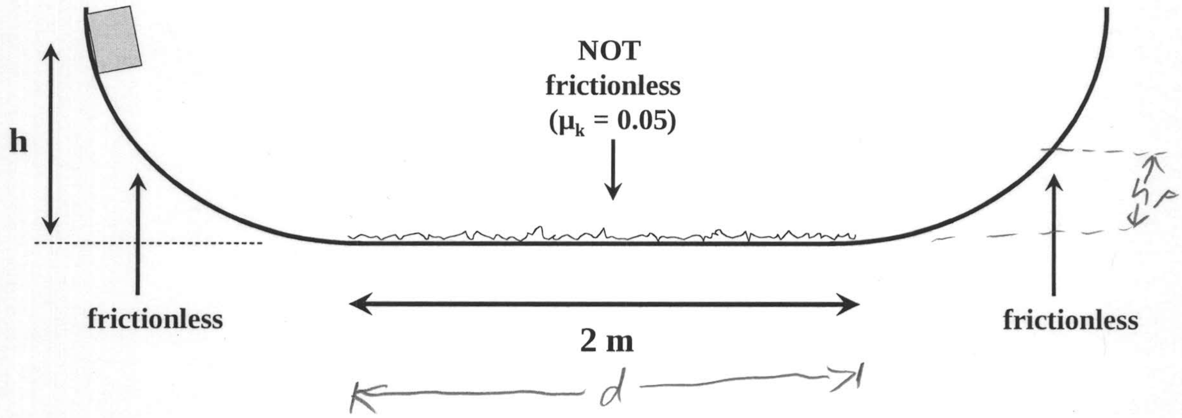
$$\Delta E = E_F - E_I$$

$$= -\frac{1}{2}\frac{GM_\oplus m}{r} + \frac{GM_\oplus m}{r} = \frac{1}{2}\frac{GM_\oplus m}{r}$$

$$\Delta E = \frac{1}{2}\frac{(6.67\times10^{-11})(5.97\times10^{24})(1\times10^{3})}{4.1\times10^{7}} = 4.9\times10^{9}\ \text{J}$$

Energy Problems – Set 4

An 8.75-kg block starts at rest, at height h = 1.0 m, and slides down a frictionless ramp onto a horizontal plane where $\mu_k = 0.05$. If the block has enough energy after passing the plane, it will rise onto another frictionless ramp, and so forth.

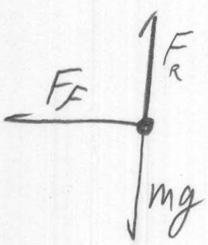


(a) The block is released, makes its first trip to the right hand side, returns to the left hand side, and then returns once more to the right. On this second excursion to the right side, how high up the ramp does the block go?

This is a conservation of Energy problem with Friction.

The simplest solution is to consider the two end points.

It starts a height h on the right and ends a height h_F on the right after crossing the friction 3 times.



F_R does no work, $\vec{F}_R \perp d\vec{s}$

F_F does work on the flat bottom, non-conservative

mg does work, conservative.

$* \; U_I = mgh$ $\qquad$ $* \; U_F = mgh_F$ ← solve for h_F

$* \; K_I = K_F = 0$

$W_F = \int_0^d \vec{F}_F \cdot d\vec{s} = -\mu_k mgd$ for one trip

continued ↓

EP4, 6 - Continued

Conserve energy accounting for 3 trips across the friction patch.

$$U_I + K_I + W_F = U_F + K_F$$

$$\cancel{mg}h + 0 + -3\mu_k \cancel{mg} d = \cancel{mg} h_F + 0$$

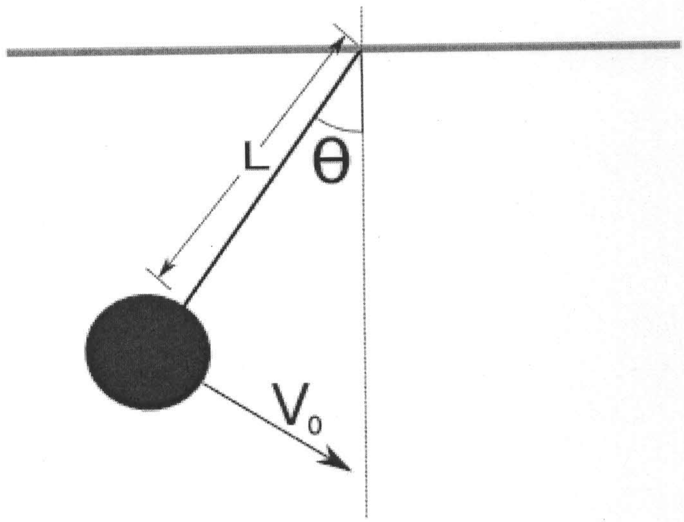
$$\Rightarrow \boxed{h_F = h - 3\mu_k d}$$

$$h_F = 1.0 - (3)(0.05)(2)$$

$$\boxed{h_F = 0.7\,m}$$

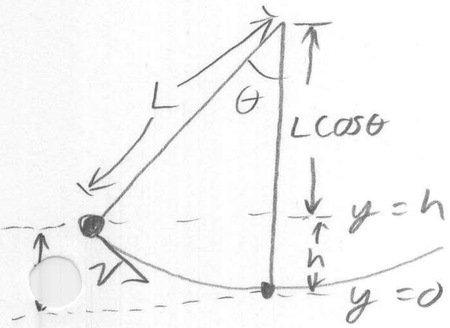
Energy Problems – Set 4

The picture shows a pendulum with a weight of mass m attached to a light (massless) string of length L. The mass has a speed v_0 when the cord makes an angle θ with the vertical.

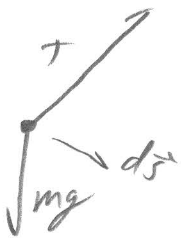


a) Derive an expression for the speed of the mass when it is in its lowest position.

b) What is the minimum value of v0 for the chord to make an angle of 90 on the pendulum's upswing?



$$h = L - L\cos\theta = L(1-\cos\theta)$$



$W_T = 0,\ \vec{T} \perp d\vec{s}$

$U_I = mgh \qquad U_F = 0$

$K_I = \frac{1}{2} m v_0^2 \qquad K_F = \frac{1}{2} m v^2$

$$\cancel{m}gh + \tfrac{1}{2}\cancel{m}v_0^2 = \tfrac{1}{2}\cancel{m}v^2$$

$$\Rightarrow v = \left(v_0^2 + 2gh\right)^{1/2} = \boxed{\left(v_0^2 + 2gL(1-\cos\theta)\right)}$$

EP4,4 continued

b) When $\theta = 90$, $h = L(1-\cos(90)) = L$

$U_I = mg(L - L\cos\theta)$ $\qquad U_F = mgL$

$K_I = \frac{1}{2}mV_0^2$ $\qquad K_F = 0$

$\cancel{m}g(L - L\cos\theta) + \frac{1}{2}\cancel{m}V_0^2 = \cancel{m}gL$

$\frac{1}{2}V_0^2 = \cancel{gL} + gL\cos\theta - \cancel{gL}$

$\boxed{V_0^2 = 2gL\cos\theta}$

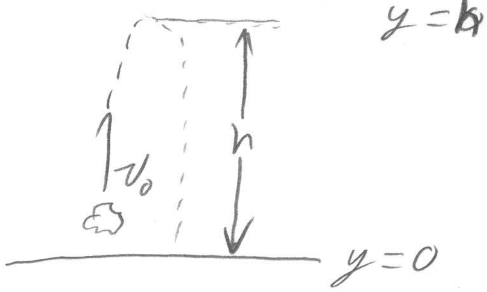
Energy Problems – Set 4

Use work energy to solve the following problem.

A stone of mass m is thrown vertically upward into the air from ground level with an initial speed of v_o. If a constant drag force equal to 20% of the stone's weight acts on the stone throughout its flight, what is the speed of the stone in terms of v_o when it returns to the ground?

Let's handle the trip up and the trip down separately.

Trip up:



$$U_I = 0 \qquad U_F = mgh$$

$$K_I = \frac{1}{2} m v_0^2 \qquad K_F = 0$$

$$W_{NCF} = \int_0^h \vec{F}_d \cdot d\vec{s}\,, \quad \vec{F}_d = -0.2\, mg\,\hat{j}\,, \quad d\vec{s} = dy\,\hat{j}$$

$$= \int_0^h (-0.2\, mg\,\hat{j}) \cdot (dy\,\hat{j}) = -\frac{1}{5} mgh$$

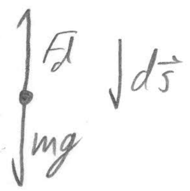
$$U_I + K_I + W_{NCF} = U_F + K_F$$

$$0 + \frac{1}{2} m v_0^2 - \frac{1}{5} mgh = mgh + 0$$

$$\Rightarrow \boxed{\frac{1}{2} m v_0^2 = \frac{6}{5} mgh} \quad (1)$$

EP 4,5

<u>Trip down</u>



$U_I = mgh \qquad K_I = 0$

$U_F = 0 \qquad K_F = \frac{1}{2}mv^2$

$W_{FCF} = \underline{-0.2mgh}$

$$\cancel{m}gh - \frac{1}{5}\cancel{m}gh = \frac{1}{2}\cancel{m}v^2$$

$$\frac{4}{5}gh = \frac{1}{2}v^2$$

$$\underline{v^2 = \frac{8}{5}gh}$$

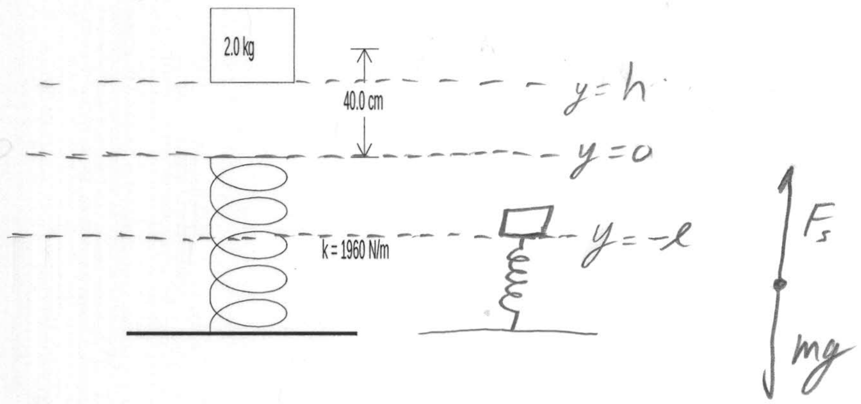
Plug in h from eq 1 : $h = \frac{5}{12}\frac{v_0^2}{g}$

$$v^2 = \frac{\cancelto{2}{8}}{\cancel{5}}\cancel{g}\,\frac{\cancel{5}}{\cancelto{3}{12}}\,\frac{v_0^2}{\cancel{g}}$$

$$v^2 = \frac{2}{3}v_0^2$$

$$\boxed{v = \sqrt{\frac{2}{3}}\,v_0}$$

Use work-energy techniques to solve the following problem. A 2.0 kg block is dropped from a height of 40 cm onto a spring of spring constant k = 1960 N/m. Find the maximum distance the spring is compressed.



as spring is being compressed

Both Forces have a pot. Func.

If we set $y=0$ where the spring is uncompressed, life will be easier, since the spring potential and the grav. potential are intertwined.

$U_I = mgh \qquad U_F = \frac{1}{2}kl^2 - mgl$

$K_I = 0 \qquad K_F = 0$

$U_I + K_I + W_{NCF} = U_F + K_F$

$mgh + 0 + 0 = \frac{1}{2}kl^2 - mgl$ hmm... Quadratic.

$$\frac{1}{2}kl^2 - mgl - mgh = 0$$

$$l = \frac{1}{k}\left[mg \pm \left((mg)^2 + 2kmgh\right)^{1/2}\right]$$

continued ↓

Negative root gives negative value for l.

But the way we have the potential defined, l must be positive. (l would then be above the $y=0$ line)

$$l = \frac{1}{1960}\left((2.0)(9.8) + \left[(2\cdot 9.8)^2 + (2)(1960)(2)(9.8)(0.4\,m)\right]^{1/2}\right)$$

$$\boxed{l = 0.1\ m}$$