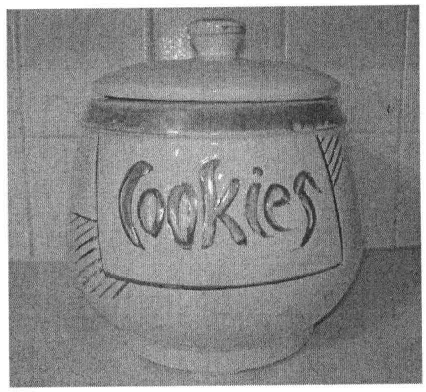
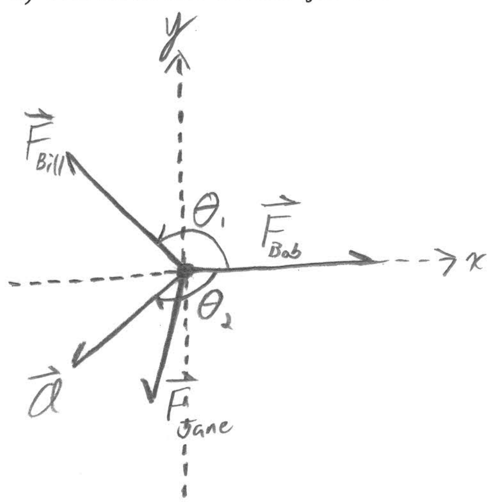
Force Problems – Set 1

Bobby, Billy, and Jane each have their hands in a 2 kg cookie jar. As a result of their tussle, the jar has an acceleration of 8 m/s^2 at an angle of -150° with respect to the x axis. Bobby is pulling with a force of 10 N in the positive x direction. Billy is pulling with a force of 12 N at an angle of 150° degrees with respect to the x axis.



a) Find the x and y components of Jane's force.
b) Find the magnitude and direction of Jane's force.
c) How fast is the cookie jar moving after 20 seconds?
d) Where is the cookie jar after 20 seconds?

a)



$|\vec{F}_{Bill}| = 12\ N$

$|\vec{F}_{Bob}| = 10\ N$

$\theta_1 = 150°$

$\theta_2 = -150°$

$|\vec{a}| = 8\ m/s^2$

$m = 2\ kg$

$\vec{F}_{net} = m\vec{a} \Rightarrow$ Newton's 2nd law

$$\vec{F}_{Bill} + \vec{F}_{Bob} + \vec{F}_{Jane} = m\vec{a}$$

$x: F_{Bill\,x} + F_{Bob\,x} + F_{Jane\,x} = ma_x$

$$|\vec{F}_{Bill}|\cos\theta_1 + |\vec{F}_{Bob}| + F_{Jane\,x} = m|\vec{a}|\cos\theta_2$$

$$\Rightarrow F_{Jane\,x} = m|\vec{a}|\cos\theta_2 - |\vec{F}_{Bill}|\cos\theta_1 - |\vec{F}_{Bob}|$$

$$= (2)(8)\cos(-150) - (12)\cos(150) - (10) = \boxed{-13.5\ N}$$

continued ↓

Cookie jar - continued

$$F_{Janey} = ma_y - F_{Billy} - F_{Boby}$$

$$= m|\vec{a}|\sin\theta_2 - |\vec{F}_{Bill}|\sin\theta_1 - \emptyset$$

$$= (2)(8)\sin(-150) - (12)\sin(150) = \boxed{-14\,N}$$

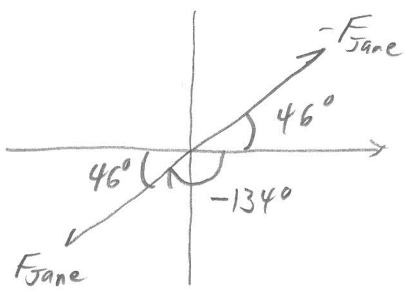
b)

$$|\vec{F}_{Jane}| = \left(F_{Janex}^2 + F_{Janey}^2\right)^{1/2}$$

$$= \left((-13.5)^2 + (-14)^2\right)^{1/2} = \underline{19\,N}$$

$$\theta_{Jane} = \tan^{-1}\left(\frac{14}{13.5}\right) = 46^\circ$$

But we're in the 4th Quadrant



$$\boxed{\theta_{Jane} = -134^\circ}$$

continued

↓

Cookie jar continued

c) $\vec{v} = \vec{v}_0 + \vec{a}t$

$v_x = 0 + |\vec{a}| \cos\theta_2 t$

$v_x = (8)\cos(-150)(20)$

$v_x = -139$ m/s

$v_y = 0 + |\vec{a}| \sin\theta_2 t$

$v_y = (8)\sin(-150)(20)$

$v_y = -80$ m/s

d) $\vec{r} = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$

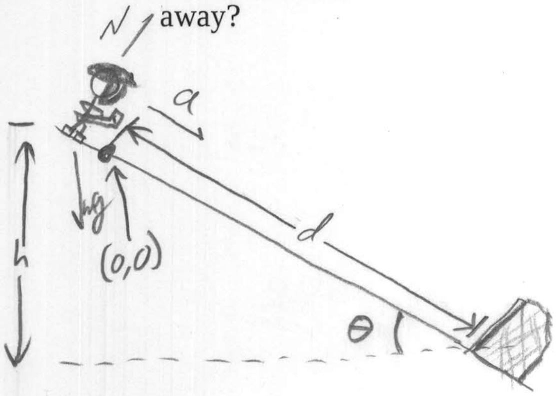
$x = 0 + 0 + \frac{1}{2}(8)\cos(-150)(20)^2$

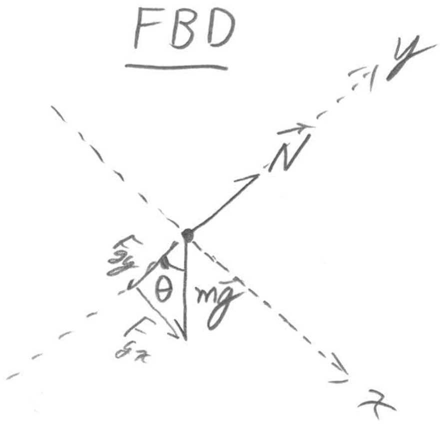
$x = -1.39 \times 10^3$ m

$y = 0 + 0 + \frac{1}{2}(8)\sin(-150)(20)^2$

$y = -800$ m

A terrible earthquake has happened in San Francisco right in the middle of a critical hockey tournament. As a result of the quake, the ice rink is tilted 15° from horizontal. The 80 kg goalie begins to slide down the slope uncontrollably from his net directly into the opposing goalies net. How fast is he when he crosses the opposite goal line 53 m away?





$\theta = 15°$

$m = 80\ kg$

$d = 53\ m$

$V_F = ?$

Find the acceleration

$\vec{F}_{net} = m\vec{a} \Rightarrow NSL$

$x: N_x + F_{gx} = ma_x$

$0 + mg\sin\theta = ma_x \Rightarrow \boxed{a_x = g\sin\theta}$

$y: N_y + F_{gy} = 0$

$N_y - mg\cos\theta = 0 \Rightarrow N_y = mg\cos\theta$ okay... not useful here...

no acceleration in the y

continued ↓

Ice rink continued

We have acceleration, now Find V_F using kinematics.
But there's only action in x:

$x = x_0 + V_{0x}t + \frac{1}{2}a_x t^2$ $\qquad$ $V = V_0 + at$

① $d = 0 + 0 \oplus \frac{1}{2} g \sin\theta \, t^2$ $\qquad$ ② $V_F = 0 + g \sin\theta \, t$

acceleration is in positive x

Solve ① for t and plug into ②

From ①: $t = \left[\frac{2d}{g \sin\theta}\right]^{1/2}$

into ②: $V_F = g \sin\theta \left[\frac{2d}{g \sin\theta}\right]^{1/2}$

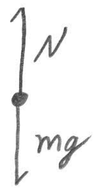
$V_F = (2gd \sin\theta)^{1/2}$ $\qquad$ hmm... $d \sin\theta = h$

$V_F = [(2)(9.8)(53)\sin(15)]^{1/2} = 16.4 \, m/s$

Force Problems – Set 1

You toss a rock straight up into the air by placing it on the palm of your hand (you're not gripping it) then pushing your hand up very rapidly. Draw free body diagrams for the following:

a) As you hold the rock at rest on your palm, before moving your hand.



$N = mg$, no accel.

b) As your hand is moving up but before the rock leaves your hand.

$N > mg$

c) One-tenth of a second after the rock leaves your hand.

No $F_{Aristotle}$

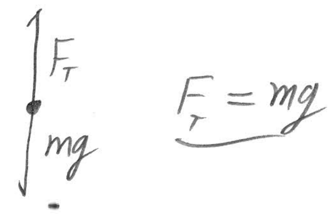
d) After the rock has reached its highest point and is now falling straight down.

Force Problems – Set 1

For the following situations, draw free-body diagrams to indicate all forces acting on the object(s) in question. *Indicate relative magnitudes of forces by drawing long, short, or equal-length vectors.*

a) An elevator suspended by a cable is descending at a constant velocity.

→ No acceleration. Net force is zero

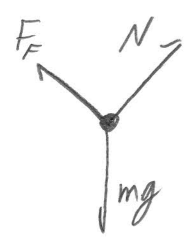


b) A car on a very slippery (frictionless) icy road sliding headfirst into a snow bank, where it gently comes to a rest with no one injured.

no "impelling" force
car is decelerating.

c) A compressed spring is pushing a block across a rough horizontal table.

d) A block is resting on a rough incline plane without sliding

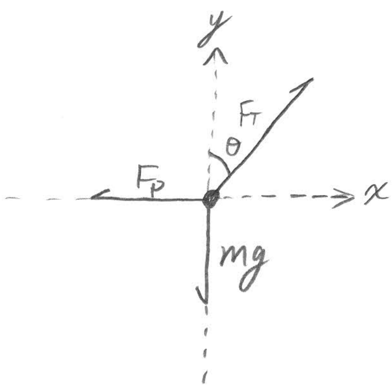


A 30-kg child is seated in a swing of negligible mass. How much horizontal force is required to pull the child and swing aside so that the support rope makes an angle of 32° with the vertical? (The child is held fixed in that position.)

a) What is the child's acceleration?

$a = 0$, not moving

b) Draw a free body diagram of the system. Choose a coordinate system.



$m = 30$ kg

$\theta = 32°$

$a = 0$

c) Using the picture from part b, write Newton's Second Law for the x-axis and the y-axis. Solve these equations for the required force.

$$\sum \vec{F} = m\vec{a} \Rightarrow \vec{F}_P + \vec{F}_T + m\vec{g} = m\vec{a} \rightarrow 0$$

$$x: \; F_T \sin\theta - F_P = 0 \quad \Rightarrow \quad F_T \sin\theta = F_P$$

$$y: \; F_T \cos\theta - mg = 0 \quad \Rightarrow \quad F_T \cos\theta = mg$$

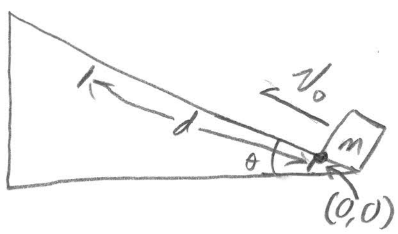
Divide x by y:

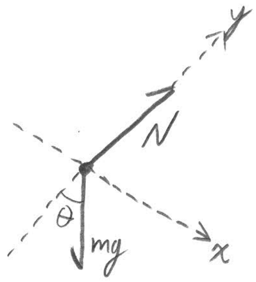
$$\frac{F_T \sin\theta}{F_T \cos\theta} = \frac{F_P}{mg} \Rightarrow \tan\theta = \frac{F_P}{mg} \Rightarrow \boxed{F_P = mg\tan\theta}$$

$$F_P = (30)(9.8)\tan(32) = \boxed{184\,N}$$

A block is given an initial velocity of 5 m/s up a frictionless 20° incline. How far up the incline does the block slide before coming to rest?

a) Draw a free body diagram of the block





$V_o = 5\,m/s$

$\theta = 20°$

$d = ?$

b) Put a coordinate system on your free body diagram and, from the resulting picture, write Newton's second law for the x axis and for the y axis. Solve these equations for the acceleration of the block.

$$x: \; mg\sin\theta = ma_x \Rightarrow a_x = g\sin\theta$$

$$y: \; N - mg\cos\theta = 0$$

c) Use the kinematics equations and the acceleration from part b to find the distance.

$$x = x_0 + V_o t + \tfrac{1}{2} a_x t^2 \qquad V = V_o + a_x t$$

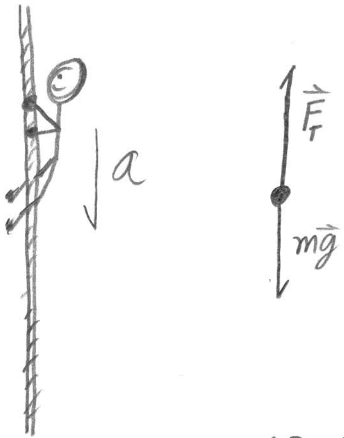
$$-d = 0 - V_o t + \tfrac{1}{2} g\sin\theta\, t^2 \qquad 0 = -V_o + g\sin\theta\, t$$

$$-d = \frac{-V_o^2}{g\sin\theta} + \frac{1}{2}\frac{V_o^2}{g\sin\theta} \qquad t = \frac{V_o}{g\sin\theta}$$

$$\boxed{d = -\frac{V_o^2}{2g\sin\theta}} = \frac{-5^2}{(2)(9.8)\sin(20)} = \boxed{-3.7\,m}$$

A 52 kg circus performer slides down a rope that will break if the tension exceeds 425 N.

a) What happens if the performer hangs stationary from the rope?
b) At what acceleration will the performer just avoid breaking the rope?



$F_{Tmax} = 425 N$
$m = 52\ kg$

NSL: $\Sigma F = ma$

$F_T - mg = ma$

a) Stationary performer, (or performer at constant v):

$a = 0$

$\Rightarrow F_T - mg = 0 \Rightarrow \boxed{F_T = mg}$

$F_T = (52\ kg)(9.8\ m/s^2) = \underline{510 N}$

Rope breaks.

b) Accelerating performer: $F_T = ma + mg < F_{Tmax}$

or: $\boxed{a < \frac{F_{Tmax}}{m} - g} \Rightarrow a < \frac{425N}{52kg} - 9.8\ m/s^2$

$a < -1.6\ m/s^2$ or less.